

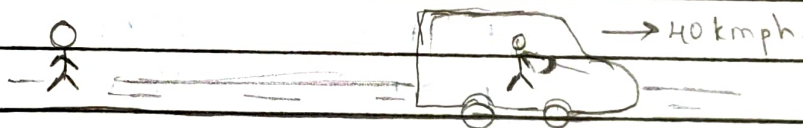
MOTION IN A STRAIGHT LINE

- Kinematics :- It is a branch of physics that deals with motion of a particle without taking into consideration the cause of motion.

Concept of rest and motion

→ It is a relative term, not an absolute term.

- An ~~object~~ is said to be at rest w.r.t an observer if its position is not changing over a period of time.
- An object is said to be in motion w.r.t an observer if its position is changing over a period of time.



Frame of reference

A frame w.r.t which an observer observes the position and motion of an object.

- In previous example, ground was frame of reference.

Reference point & position.

Reference point is a point w.r.t which the position (location) of a particle is given.

Distance - It is the length of actual path travelled
 It is a scalar quantity

SI unit = metre (m)

$$[\text{Distance}] = [L]$$

Distance never decreases with time, it either increases or stays remains constant.

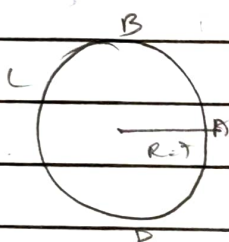
Displacement - It is the length of shortest possible path
 SI unit = metre - It is a vector quantity

$$[\text{Displacement}] = [L]$$

It can increase or decrease with time

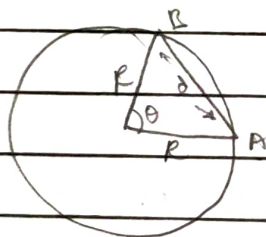
magnitude $|\text{Displacement}| \leq \text{Distance}$

Eg



	Distance	Displacement
$A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$	44m	10m
$A \rightarrow B \rightarrow C$	22m	14m
$A \rightarrow B$	11m	$7\sqrt{2}$ m

$\Rightarrow$



$$\text{Distance (AB)} = R\theta$$

$$\text{Displacement (d)} = 2R \sin \frac{\theta}{2}$$

by cosine rule,

$$\cos \theta = \frac{R^2 + R^2 - d^2}{2 \cdot R \cdot R}$$

$$2R^2 \cos \theta = 2R^2 - d^2$$

$$d^2 = 2R^2 (1 - \cos \theta)$$

$$d^2 = 2R^2 \cdot 2 \sin^2 \frac{\theta}{2}$$

$$d = 2R \sin \frac{\theta}{2} **$$

Resultant displacement
 if given in direction
 $\Delta r = \sqrt{x^2 + y^2}$

Average speed :- $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

Case 1: A particle travels two equal distance with speed v_1 & v_2 .

$$\text{Avg speed} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} \Rightarrow \frac{2}{\frac{1}{v_1} + \frac{1}{v_2}} \Rightarrow \boxed{\frac{2v_1v_2}{v_1+v_2} = V_{\text{avg}}}$$

k³B:- If it travels three equal distance with diff speed.

$$\boxed{V_{\text{avg}} = \frac{2v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}}$$

Case 2: A particle travels with speed v_1 & v_2 for two equal time intervals.

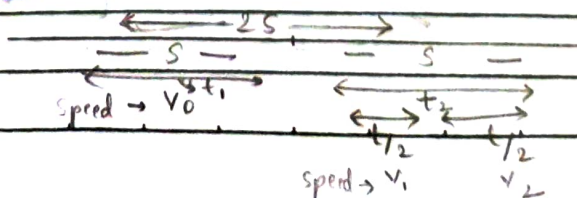
$$V_{\text{avg}} = \frac{v_1t + v_2t}{t+t} \Rightarrow \frac{t(v_1+v_2)}{2t}$$

$$\boxed{V_{\text{avg}} = \frac{v_1+v_2}{2}}$$

k³B :- If it travels with diff speed for three equal time intervals

$$\boxed{V_{\text{avg}} = \frac{v_1+v_2+v_3}{3}}$$

Case 3: A particle covers half the distance with speed v_0 & the remaining distance is covered with speed v_1 for half the time & with speed v_2 for the other half of the time.



$$V_{avg} = \frac{2s}{t_1 + t_2} \Rightarrow \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}}$$

$$s = v_1 \frac{t}{2} + v_2 \frac{t}{2} = (v_1 + v_2) \frac{t}{2}$$

$$t = \frac{2s}{v_1 + v_2}$$

$$V_{avg} \Rightarrow \frac{2v_1(v_1 + v_2)}{2v_1 + v_1 + v_2}$$

Q:- A person travelling on a straight line moves with uniform velocity v_1 for a distance x & with a uniform velocity v_2 for the next $3/2x$ distance. The average velocity in this motion is $50/7$ m/s. If v_1 is 5 m/s then $v_2 = ?$

Soln:-

$$\text{Avg velocity} = \frac{x + 3/2x}{\frac{x}{v_1} + \frac{3x}{2v_2}} = \frac{5x/2}{\frac{2(2v_2 + 3v_1)}{2v_1v_2}}$$

$$\frac{50}{7} = \frac{5x}{2} \times \frac{2v_1v_2}{x(2v_2 + 3v_1)}$$

$$\frac{50}{7} = \frac{5v_1v_2}{2v_2 + 3v_1}$$

$$\frac{50}{7} = \frac{25v_2}{2v_2 + 15}$$

$$100v_2 + 2750 = 175v_2$$

$$2750 = 75v_2$$

$$v_2 = 30 \text{ m/s}$$

$$\frac{450}{18} = 25$$

30

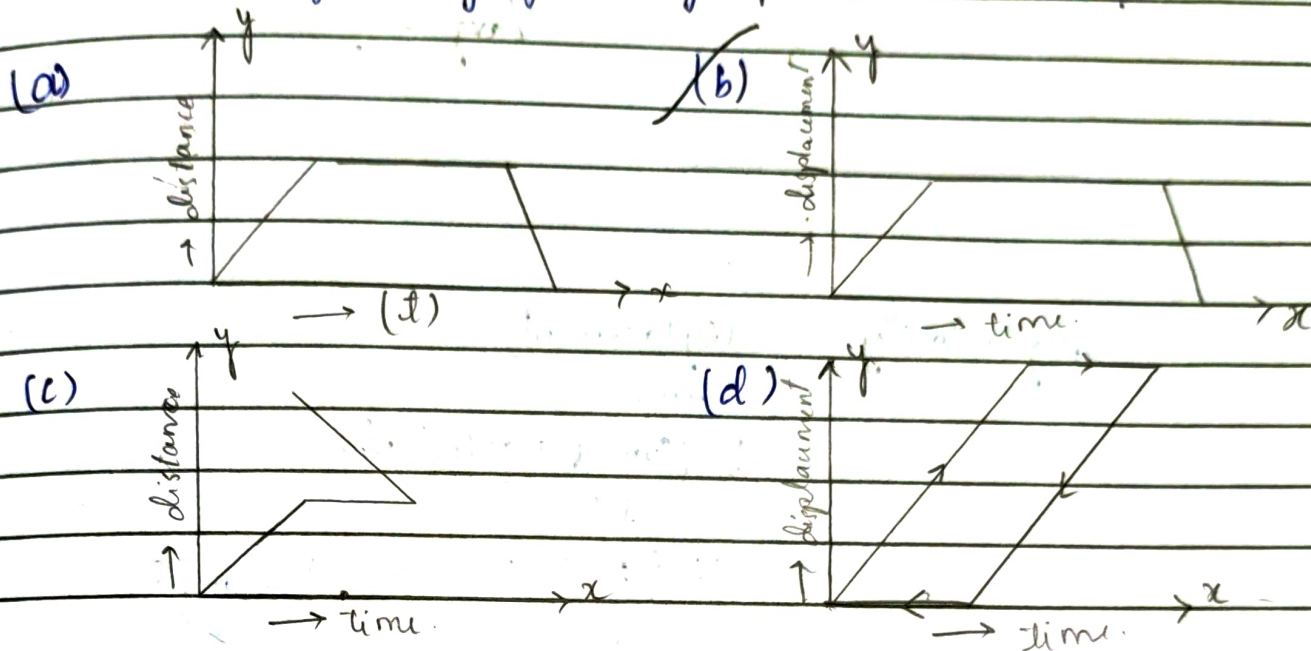
450

2250

75

18

Q:- which of the following graph / graphs are correct?



(a) is wrong $\because$ distance cannot decrease at any point.

(b) is wrong $\because$ a particle cannot be at two positions at the same moment.

(d) is wrong because of the same reason.

• Velocity : $\frac{\text{Displacement}}{\text{Time}}$

$V_{\text{avg}} :- \frac{\Delta x}{\Delta t}$

$\hookrightarrow$ average velocity

$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

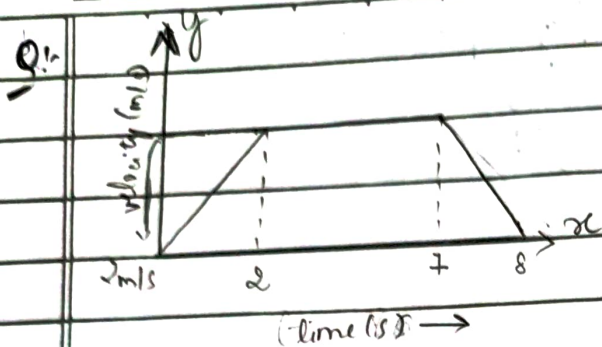
$\hookrightarrow$ instantaneous velocity.

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v dt$$

$[x]_{x_1}^{x_2} = \text{Area under velocity time graph.}$

$[x_2 - x_1] = \Delta x = \int_{t_1}^{t_2} v dt.$

$\Rightarrow V_{\text{avg}} = \frac{\int_{t_1}^{t_2} v dt}{\int_{t_1}^{t_2} dt}$



$V_{avg} = ?$

Avg velocity = $\frac{\text{displacement}}{\text{time}}$

$= \frac{1}{2} (\text{Sum of 11 sides}) \Delta t$

$= \frac{1}{2} (5+8) 2 \Rightarrow \frac{13}{8} \text{ m/s}$

Q:- A particle is moving along x-axis & its position x varies with time as $x = (2t^2 + 4t - 6) \text{ m}$. Find average velocity b/w $t = 0 \text{ sec}$ to $t = 2 \text{ sec}$.

Soln:- $x_i = -6 \text{ m}$ at $t=0$, $x_f = 10 \text{ m}$ at $t=2$

$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{10 - (-6)}{2} = 8 \text{ m/s}$

OR

instantaneous $v = \frac{dx}{dt} = 4t + 4$ ✓ differentiation

$V_{avg} = \frac{\int_{t_i}^{t_f} v dt}{\int_{t_i}^{t_f} dt} = \frac{\int_0^2 (4t + 4) dt}{\int_0^2 dt}$

$\left[\frac{4t^2}{2} + 4t \right]_0^2 \Rightarrow (4 \times 2 + 4 \times 2) - (0 \times 0 + 4 \times 0) = \left[4t^2/2 + 4t \right]_0^2$
 $\Rightarrow \frac{16 - 0}{2} = 8$

$= \frac{16 - 0}{2} \Rightarrow 8 \text{ m/s}$

Retardation / De-acceleration
 → -ve acceleration.

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• Instantaneous speed = $\left| \frac{dx}{dt} \right|$

• Acceleration = $\frac{\text{change in velocity}}{\text{Time taken.}}$

$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$

instantaneous accⁿ = $\lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$

$a_{\text{avg}} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a dt$

$a = \frac{dv}{dt}$

$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a dt$

$v_2 - v_1 = \Delta v = \text{area under}$

$a-t$ graph
 change in velocity

→ Motion with constant acceleration -

$v = u + at$

$s = ut + \frac{1}{2}at^2 \Rightarrow vt = \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$V_{\text{avg}} = \frac{s}{t} = \frac{ut + \frac{1}{2}at^2}{t} \Rightarrow u + \frac{1}{2}at$

$\Rightarrow \frac{2u + at}{2}$

$\Rightarrow \frac{u + u + at}{2}$

avg velocity $V_{\text{avg}} \Rightarrow \frac{u + v}{2} = \frac{s}{t}$

also, $s = \left(\frac{u + v}{2} \right) t$

★ $a = \frac{dv}{dt} \Rightarrow \int_u^v dv = \int_0^t a dt \leftarrow \text{Derivation 1}$

$$\Rightarrow [v]_u^v = a[t]_0^t$$

$$v - u = at$$

$$v = at + u$$

★ $v = \frac{dx}{dt} \Rightarrow \int_0^s dx = \int_0^t v dt$
 $\int_0^s dx = \int_0^t (u + at) dt$

$$s = u \int_0^t dt + a \int_0^t t dt$$

$$s = ut + \frac{at^2}{2}$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

★ $a = \frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{dx}{dx} = \frac{dx}{dt} \cdot \frac{dv}{dx}$

$$a = \frac{v dv}{dx} \Rightarrow \int_u^v v dv = \int_0^s a dx$$

$$\Rightarrow \left[\frac{v^2}{2} \right]_u^v = a [dx]_0^s$$

$$\Rightarrow \frac{v^2 - u^2}{2} = a(s - 0)$$

$$\Rightarrow \boxed{v^2 - u^2 = 2as}$$

$$D_n = \frac{1}{2} a n^2$$

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$$S_n = D_n - D_{n-1} = \frac{a}{2} (2n-1)$$

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Displacement in n^{th} second -

$$S_{n^{\text{th}}} = S_n - S_{n-1}$$

$$\begin{aligned} &= un + \frac{1}{2} a n^2 - \left(u(n-1) + \frac{1}{2} a (n-1)^2 \right) \\ &= \cancel{un} + \frac{1}{2} a n^2 - \cancel{un} + u - \frac{1}{2} a n^2 + an - \frac{a}{2} \\ &= u + an - \frac{a}{2} \end{aligned}$$

$$S_{n^{\text{th}}} = u + \frac{a}{2} (2n-1)$$

Q A body travels 102.5 m in n^{th} second. & 115 m in $(n+2)^{\text{th}}$ second. Find acceleration.

$$\rightarrow 102.5 = u + \frac{a}{2} (2n-1) \rightarrow \textcircled{1}$$

$$\rightarrow 115 = u + \frac{a}{2} (2(n+2)-1)$$

$$115 = u + \frac{a}{2} (2n+3) \rightarrow \textcircled{2}$$

$$u = 115 - \frac{a}{2} (2n+3) \rightarrow \textcircled{2} \text{ [put in } \textcircled{1}]$$

$$102.5 = 115 - \frac{a}{2} (2n+3) + \frac{a}{2} (2n-1)$$

$$-12.5 = \frac{a}{2} (2n-1-2n-3)$$

$$-12.5 = \frac{a}{2} (-4)$$

$$-12.5 = -2a$$

$$a = +6.25 \text{ m/s}^2$$

Q: Find the distance & displacement travelled by a particle in 10s. The particle is under constant retardation for the whole journey.

$$a = 2 \text{ m/s}^2$$



$$S = ut + \frac{1}{2} at^2$$

displacement

$$= 10 \times 10 + \frac{1}{2} \times -2 \times 100$$

$$= 100 - 100 = 0$$

$$v^2 = u^2 + 2as \Rightarrow v^2 = u^2 + 2ad$$

distance

$$0 = 10^2 - 2 \times d + 100$$

$$+ 4d = 100$$

$$d = 25 \text{ m} \Rightarrow 50 \text{ m} \therefore$$

$$v = u + at$$

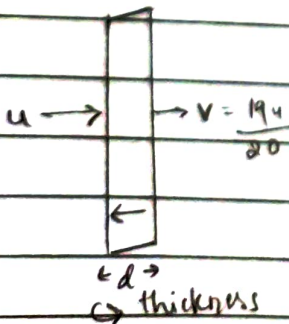
$$0 = 10 + -2 \times t$$

$$2t = 10$$

$$t = 5 \text{ sec}$$

$\therefore$ in 5 sec it will travel 25 m, & in 10 sec it will travel 50 m as force is still acting.

Q: A bullet loses $\frac{1}{20}$ th of its speed after transversing a plank. How many such planks are needed to stop the bullet?



$$v^2 - u^2 = 2as$$

$$\left(\frac{19u}{20}\right)^2 - u^2 = 2 \times (-a) \times d$$

$$\Rightarrow \frac{-39u^2}{400} = -2ad \rightarrow \textcircled{1}$$

$u \rightarrow$

$\rightarrow v = 0$

$\leftarrow d' \rightarrow$

$$v^2 - u^2 = 2as$$

$$0 - u^2 = 2(-a)(d')$$

$$-2ad' = -u^2 \rightarrow \textcircled{2}$$

dividing ① & ②

$$\frac{39}{400} = \frac{d}{d'}$$

$$d' = \frac{400d}{39}$$

$$d' = 10.25d$$

$\therefore$ 11 planks needed.

Q: A car moving with a speed of 50 kmph can be stopped by application of brake after at least 60 m.

If the same car is moving at a speed of 100 kmph the stopping distance? (assume retardation to be same)

$$v^2 = u^2 + 2as$$

$$0 = (50)^2 + 2(-a)(60) \rightarrow \textcircled{1}$$

$$2500 = 120a \rightarrow \textcircled{1}$$

$$2500 = u^2 + 2as$$

$$0 = (100)^2 - 2as$$

$$2as = 100^2 \rightarrow \textcircled{2}$$

divide ① & ②

$$\left(\frac{50}{100}\right)^2 = \frac{120as}{2as}$$

$$\frac{1}{4} = \frac{60}{s}$$

$$\Rightarrow s = 240 \text{ m}$$

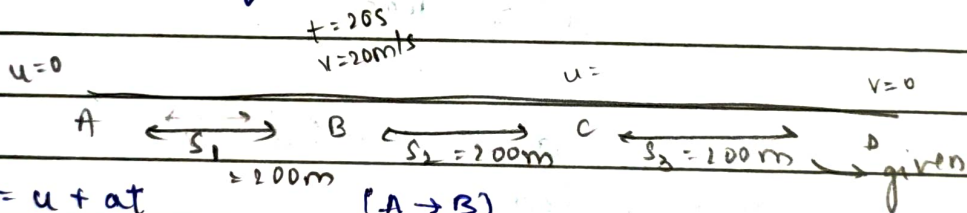
Q:- A car starts from rest and accelerates uniformly for 20s to a velocity of 20 m/s. It then runs at a constant velocity & finally is brought to rest at 600m. with a constant retardation.

The total distance covered is 600m. Find

(i) acceleration

(ii) Retardation

(iii) Total time of journey.



(i) $v = u + at$ (A $\rightarrow$ B)

$$20 = 0 + a(20)$$

$$a = 1 \text{ m/s}^2$$

~~ii~~ $s_1 = ut + \frac{1}{2}at^2$

$$= \frac{1}{2} \times 1 \times 400 = 200 \text{ m}$$

~~iii~~ total distance = 600m

$$s_2 = 600 - (s_1 + s_3) = 200 \text{ m}$$

(ii) $v^2 = u^2 + 2as$ ($c \rightarrow D$)

$0 = 20^2 + 2(a') \times 200$

$a' = -1 \text{ m/s}^2$

$\therefore \text{retardation} = 1 \text{ m/s}^2$

$v = u + at$

$0 = 20 + (-1)(t)$

$t = 20 \text{ sec}$

(iii) time taken ($B \rightarrow C$)

$v^2 = u^2 + 2as$

$(20)^2 = 400 + 2(a)(200)$

$400 = 400 + 400a \Rightarrow a = 0$

$s = ut + \frac{1}{2}at^2$

$200 = 20t \Rightarrow t = 10 \text{ sec} \text{ (B} \rightarrow \text{C)}$

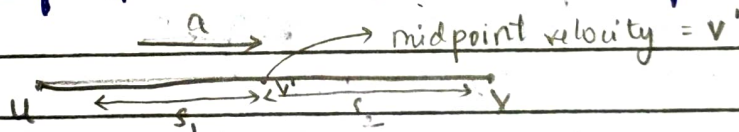
$\therefore \text{total time taken}$

$\Rightarrow (A \rightarrow B) + (B \rightarrow C) + (C \rightarrow D)$

$\Rightarrow (20 + 10 + 20) \text{ sec}$

$\Rightarrow \underline{\underline{50 \text{ seconds}}}$

Q: A particle moving with constant a has initial velocity u . After some time its velocity becomes v . Find the velocity of particle at the midpoint of its path.



$v' \rightarrow u$ $(v')^2 = u^2 + 2as$
 $(v')^2 - u^2 = 2as \rightarrow \textcircled{1}$

$u \rightarrow v$ $v^2 = u^2 + 2a(2s)$
 $v^2 - u^2 = 4as \rightarrow \textcircled{2}$

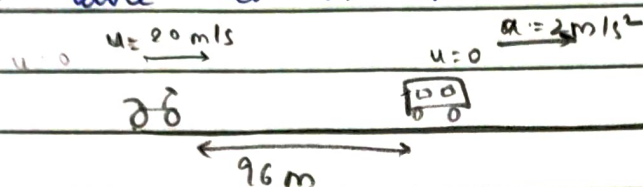
Dividing $\textcircled{1}$ & $\textcircled{2}$

$\frac{(v')^2 - u^2}{v^2 - u^2} = \frac{1}{2}$

$\Rightarrow 2(v')^2 - 2u^2 = v^2 - u^2$

$v' = \sqrt{\frac{v^2 + u^2}{2}}$

Q:- A bus starts from rest, moves with an acceleration of 2 m/s^2 . A cyclist 96m behind the bus starts simultaneously towards the bus, at a constant speed of 20 m/s . After what time the cyclist will be able to overtake the bus.



(cyclist) ~~$v = u + at$~~

$\rightarrow d = s \times t$

$$20 \times t = \frac{1 \times 2 \times t^2}{2} + 96$$

$$t^2 - 20t + 96 = 0$$

$$t = 8 \text{ sec} \text{ \& } 12 \text{ sec}$$

Concept :- A particle starts from rest and moves with uniform acceleration. The ratio of displacement travelled by the particle is

(i) 1 sec : 2 sec : 3 sec $\therefore 1 : 4 : 9$

(ii) 1st sec : 2nd sec : 3rd sec $\therefore 1 : 3 : 5$

(iii) ratio of time for equal displacements

$$s = ut + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}}$$

$$2s = ut + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{4s}{a}}$$

$$3s = ut + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{6s}{a}}$$

Ratio $\Rightarrow 1 : \sqrt{2}-1 : \sqrt{3}-\sqrt{2} : \sqrt{4}-\sqrt{3} : \sqrt{5}-\sqrt{4}$

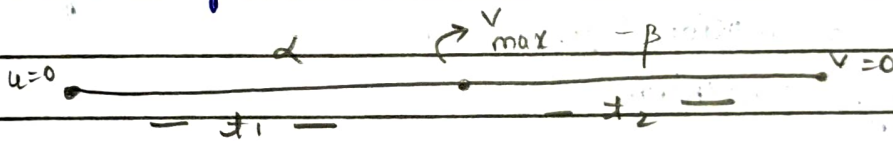
Q: A small toy starts moving from rest under a constant acceleration, if it travels a distance of 10m in 't' seconds, the distance travelled by the toy in the next 't' seconds will be.

→ 30 sec, because,
 $s_{1st} : s_{2nd} :: 1 : 3$

Q: A car accelerates from rest at a constant rate ' α ' for some time after which it decelerates at a constant rate ' β ' to come to rest. If the total time elapsed is 't' seconds. Find first

(i) maximum velocity.

Let t_1 be time for accelerated motion & t_2 be the time for retarded motion.



$$v = u + at$$

$$V_{max} = 0 + \alpha t_1$$

$$t_1 = \frac{V_{max}}{\alpha} \rightarrow (1)$$

$$v = u + at$$

$$0 = V_{max} + (-\beta)(t_2)$$

$$t_2 = \frac{V_{max}}{\beta} \rightarrow (2)$$

$$t = t_1 + t_2$$

$$t = \frac{V_{max}}{\alpha} + \frac{V_{max}}{\beta}$$

$$t = \frac{\alpha V_{max} + \beta V_{max}}{\alpha \beta}$$

$$V_{max} = \frac{\alpha \beta t}{\alpha + \beta}$$

(ii) total distance travelled.

$$s_1 = ut + \frac{1}{2}at^2$$

$$= \frac{1}{2}(\alpha)(t_1)^2$$

$$= \frac{\alpha t_1^2}{2}$$

$$s_1 = \frac{\alpha t_1^2}{2}$$

$$s_2 = vt - \frac{1}{2}at^2$$

$$= -\frac{1}{2}(1-\beta)(t_2)^2$$

$$= \frac{\beta t_2^2}{2}$$

$$s_2 = \frac{\beta t_2^2}{2}$$

$$s_1 + s_2 = \frac{\alpha t_1^2}{2} + \frac{\beta t_2^2}{2}$$

$$s_1 + s_2 = \frac{\alpha t_1^2}{2} + \frac{\beta t_2^2}{2}$$

$$= \frac{\alpha t_1^2 + \beta t_2^2}{2}$$

$$= \frac{\alpha t_1^2 + \beta t_2^2}{2}$$

$$= \frac{\alpha^2 t_1^2 + \beta^2 t_2^2 + 2\alpha\beta t_1 t_2}{2\alpha + 2\beta}$$

$$= \frac{\alpha^2 t_1^2 + \beta^2 t_2^2 + 2\alpha\beta t_1 t_2}{2\alpha + 2\beta}$$

• Motion with constant acceleration under gravity

Equation of motion -

Sign convention

+ve $\rightarrow$ a, v, s

-ve $\rightarrow$ a, v, s

$a = -g \rightarrow$ acceleration due to gravity

$g = 10 \text{ m/s}^2$ (or) 9.81 m/s^2

• Time of ascend :

A ball is thrown vertically upward with initial velocity u . Time of ascend is the time taken by the ball to reach its maximum height & $v=0$.

$$t_{asc} = \frac{u}{g} \quad (v = u + at)$$

maximum height reached by the ball

$$h_{max} = \frac{u^2}{2g} \quad (v^2 = u^2 + 2as)$$

accⁿ due to gravity on the surface of moon = $9/10$ or $10/6$

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• Time of descend:

A ball is dropped from a building of height 'h'. Time of descend is the time taken by the ball to reach the ball. $u=0$.

$$t_{des} = \sqrt{\frac{2h}{g}} \quad (s = ut + \frac{1}{2}at^2)$$

• Speed of The ball just before striking the ground.

$$v = \pm \sqrt{2gh} \quad (v^2 = u^2 + 2as)$$

$$\text{speed} = \sqrt{2gh}$$

$$\text{velocity} = -\sqrt{2gh}$$

Q: A person is standing on top of a 25 m high building, throws a ball upwards with a velocity of 20 m/s. Find the time taken by the ball to touch the ground & max^m height reached by the ball from ground.

$$h_{\max} = \frac{400}{20} \Rightarrow 20 \text{ m}$$

$$\text{from ground} \rightarrow 20 \text{ m} + 25 \text{ m} = 45 \text{ m}$$

$$[t] \quad s = ut + \frac{1}{2}at^2 \Rightarrow -25 = 20t + \frac{1}{2}(-10)(t)^2$$

Δ displacement of ball

$$\Rightarrow 5t^2 - 20t - 25 = 0$$

$$\Rightarrow t^2 - 4t - 5 = 0$$

$$t = -1 \text{ or } 5$$

$$\therefore t = 5 \text{ seconds.}$$

$\nearrow$ about building (0)

$$[t] \Rightarrow t_{\dots} = \sqrt{\frac{2h/g}{\frac{u^2}{2g}}} = \sqrt{\frac{2 \times 20}{10}} \Rightarrow 2 \text{ sec} \quad \Rightarrow 5 \text{ seconds}$$

$$\Rightarrow t_{des} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} \Rightarrow 3 \text{ sec}$$

$\hookrightarrow$ from highest point to ground.

Q. A stone is dropped from a bridge at a height of 180 m over a river. After 3 seconds, a ball is thrown straight downward. What should be the initial velocity of ball so that both stone & ball reaches simultaneously in the river.

$$t_{\text{stone}} = \sqrt{2h/g} = 6 \text{ sec}$$

∴ for ball, $6 - 3 \Rightarrow 3$ seconds are left to reach the river

$$s = ut + \frac{1}{2}at^2$$

$$-180 = u(3)^2 + \frac{1}{2}(-10)(9)$$

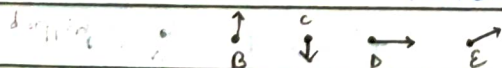
$$-180 = 3u - 45$$

$$3u = -135$$

$$u = -45 \text{ m/s}$$

Important results.

- The speed with which a ball is projected upward = The speed with which it comes back to the point of projection.
- Speed at any point on the path is same even the ball is in upward or downward motion.
- The motion is independent of mass.
- Time taken by Ball 'B' is max^m & time taken by ball 'C' is minimum.



- The speed with which the ball strikes the ground is same in all cases. A & D. & in B & C.
 (Due to conservation of energy)

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Q. A balloon starts rising from the ground with an acceleration of 1.25 m/s^2 . After 8 seconds of rise a stone is released from balloon. After realising from balloon, time taken by the stone to reach the ground is ?

Initial velocity of stone is the velocity of the balloon at the point of time when stone was released i.e

$$v = u + at \text{ [of balloon]}$$

$$v = 1.25 \times 8$$

$$v = 10 \text{ m/s.}$$

$$u \text{ of stone} = 10 \text{ m/s}$$

displacement of stone, displacement travelled by balloon from ground

$$s = ut + \frac{1}{2}at^2 \text{ [balloon]}$$

$$s = 0 + \frac{1}{2} \times 1.25 \times 8 \times 8$$

$$s = 40 \text{ m}^2$$

∴ Displacement of stone will -40 m .

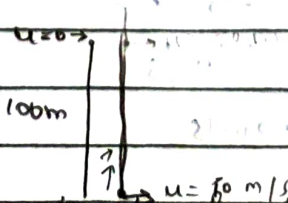
$$-40 = ut + \frac{1}{2}at^2$$

$$-40 = (10)(t) + \frac{1}{2}(-10)(t^2)$$

$$-40 = 10t + (-5t^2)$$

$$5t^2 - 10t - 40 = 0 \rightarrow \boxed{t = 4 \text{ sec}}$$

Q. A particle is dropped from height 100 m & another particle is projected vertically upward with 50 m/s along the same line from the ground. Find the position from ground where two particles meet each other.



$$s = ut + \frac{1}{2}at^2$$

u put mod to this

$\therefore$ must be downward & distance should not be negative

$$s_1 = 0 + \frac{1}{2}(-10)t^2 \quad [\text{for dropped thing}]$$

$$s_2 = 50t + \frac{1}{2}(-10)t^2 \quad [\text{for projected thing}]$$

$s =$

$$100 = \left| \frac{1}{2}(-10)t^2 \right| + 50t + \frac{1}{2}(-10)t^2$$

$$0 = +5t^2 + 50t - 5t^2 - 100$$

$$100 = 50t$$

$$t = 2 \text{ seconds}$$

$$s_2 = 50(2) + \frac{1}{2}(-10)(4)$$

$$s_2 = 100 - 20 = 80 \text{ m}$$

Q:- A particle is dropped from a tower. It is found that it travels 45m in the last second of its journey. (i) Find the height of tower.

$$s_n = u + a(2n-1)$$

$$-45 = 0 + \frac{-10}{2}(2n-1)$$

$$+45 = 5(2n-1)$$

$$9 + 1 = 2n \Rightarrow n = 5 \text{ seconds}$$

$$t = \sqrt{\frac{2h}{g}} \Rightarrow 5^2 = \frac{2h}{10} \Rightarrow h = 125 \text{ m}$$

(ii) Find speed of particle just before striking the ground.

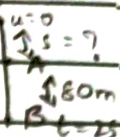
$$v = u + at$$

$$v = 0 + (-10)(5)$$

$$v = -50 \text{ m/s}$$

$$\therefore \text{speed} = 50 \text{ m/s}$$

Q: A particle falling under gravity covers two points A & B separated by 80m & 2 sec. The distance of upper point A from the starting point is ?



O → A

$$v^2 = u^2 + 2as$$

$$v^2 = 2(-10)(-s)$$

$$v = -\sqrt{20s} \quad [- \because \text{downward direction}]$$

A → B

$$s = ut + \frac{1}{2}at^2$$

$$-80 = -\sqrt{20s}(2) + \frac{1}{2}(-10)(2)^2$$

$$-80 = -2\sqrt{20s} - 20$$

$$2\sqrt{20s} = 60$$

$$\sqrt{20s} = 30$$

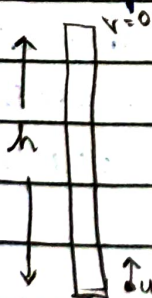
$$20s = 900$$

$$s = 45 \text{ m}$$

Air resistance:

• let us retardation due to air resistance is 'a':

→ ball is moving vertically upward

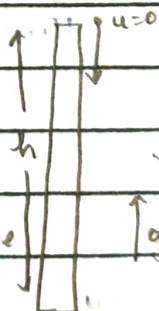


$$\text{time}_{\text{asc}} = \sqrt{\frac{2h}{g+a}}$$

gravit.

air resistance

→ ball is moving vertically downward.



$$t_{\text{des}} = \sqrt{\frac{2h}{g-a}}$$

$$s = vt - \frac{1}{2}at^2$$

$$= -\frac{1}{2}(g^2)t^2$$

$$h = \frac{gt^2}{2} \quad t = \sqrt{\frac{2h}{g}}$$

Same.

Q. Air resistance provides a constant retardation of 6 m/s^2 to a ball projected vertically upwards. Find the ratio of time of ascent to the time of descent.

$$t_{\text{asc}} = \sqrt{\frac{2h}{16}}$$

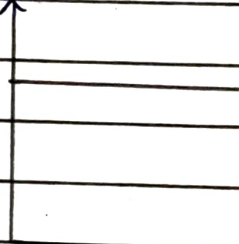
$$t_{\text{desc}} = \sqrt{\frac{2h}{4}}$$

$$\frac{t_{\text{asc}}}{t_{\text{desc}}} = \frac{\sqrt{2h/16}}{\sqrt{2h/4}} = \frac{1}{2} \Rightarrow 1:2$$

Graphs:-

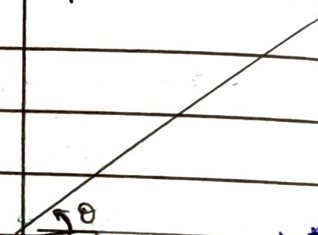
1) Distance / Displacement vs time graph. (slope = velocity)

Displacement



velocity
time

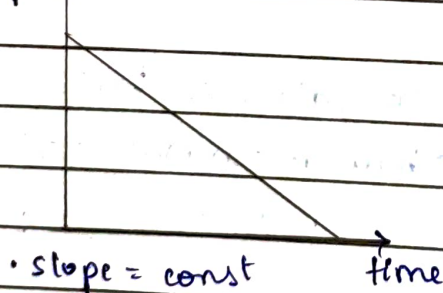
Displacement



time

velocity = 0

Displacement



• slope = const
 → velocity = -ve

Displacement



slope = -ve & decreasing time
 velocity & accⁿ = -ve

slope = constant

velocity = +ve

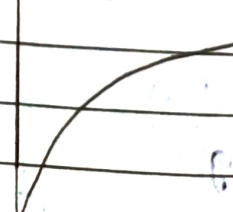
displacement



• velocity is increasing
 • acceleration = +ve & constant

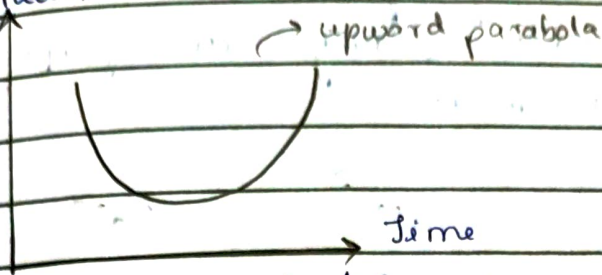
If $x = kt^2$, $v = 2kt$ & $a = 2k$

Displacement



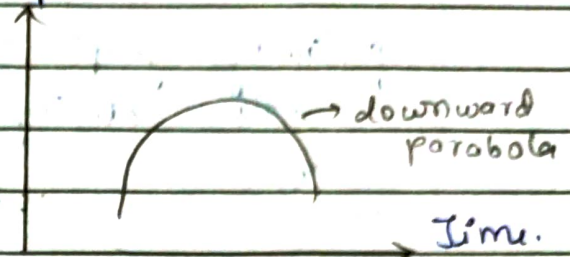
slope & vel = decreasing

Displacement

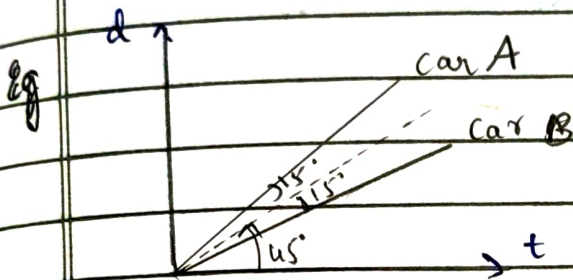


accⁿ is constant & +ve

Displacement



accⁿ is constant & -ve.



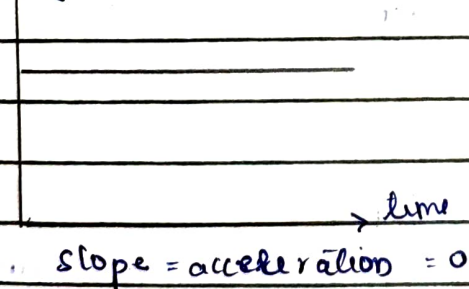
$$\frac{v_A}{v_B} = \frac{\tan(45 + 15)}{\tan(45 - 15)} = \frac{3}{1}$$

2nd velocity time graph

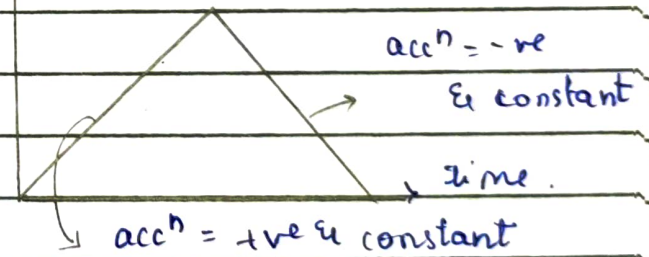
Slope = acceleration

Area = displacement = change in position

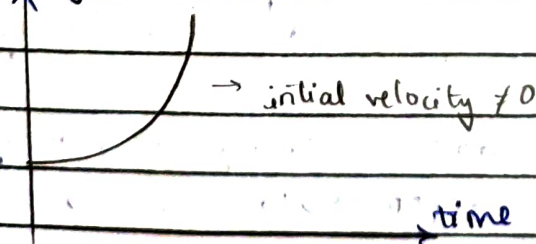
velocity



velocity



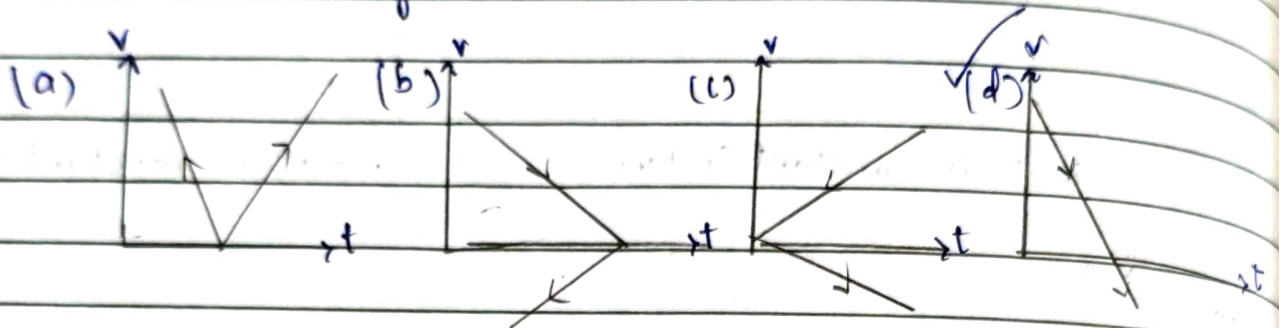
velocity



accⁿ is changing =

non-uniformly accelerated motion

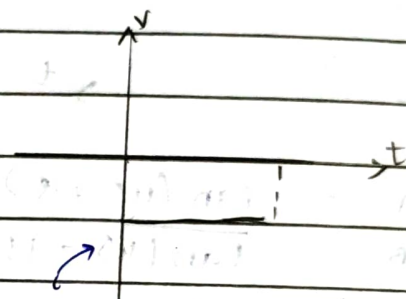
Q: A ball is thrown vertically upwards, which of the following graph represents velocity-time graph of the ball during its motion



HRB:



displacement = distance

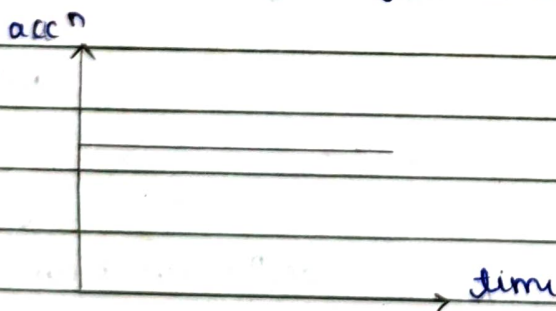


distance = |displacement|

3. Acceleration - time graph.

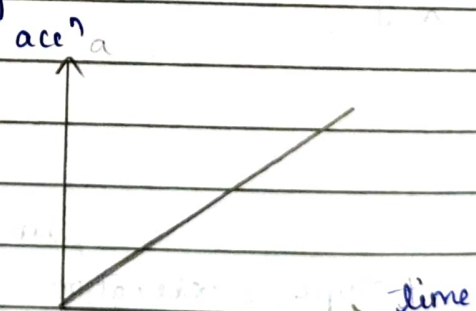
slope = Jerk

Area = change in velocity



slope = 0

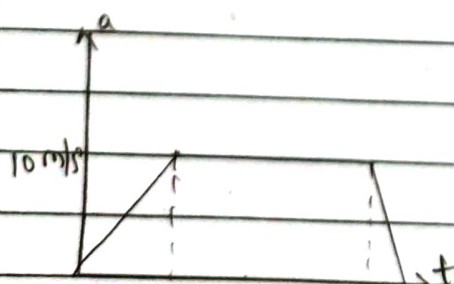
accⁿ = constant



Slope = constant

accⁿ = increasing

Q:



area = change in velocity

$$\frac{1}{2} (20+10) 10 = v - 10$$

$$v = 160 \text{ m/s}$$

If initial vel = 10 m/s. Find v

Acceleration - displacement curve.

$$a = \frac{dv}{dt} \cdot \frac{dx}{dx} = \frac{dx}{dt} \cdot \frac{dv}{dx} = v \frac{dv}{dx} = a$$

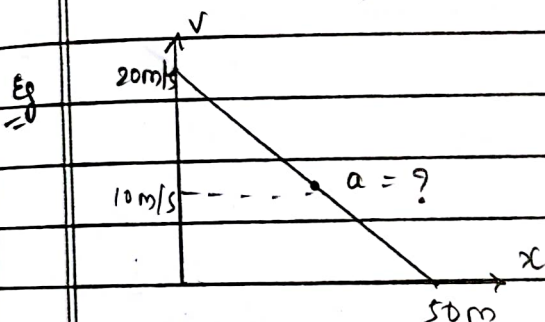
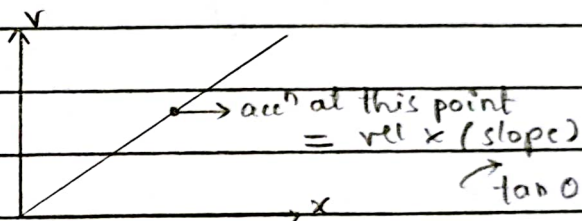
$$a = v \frac{dv}{dx} \rightarrow \int a dx = \int v dv$$

↑ area under a-x graph = $\frac{v^2 - u^2}{2}$

velocity - i.e. area under a-x graph gives $\frac{v^2 - u^2}{2}$

Velocity - displacement curve.

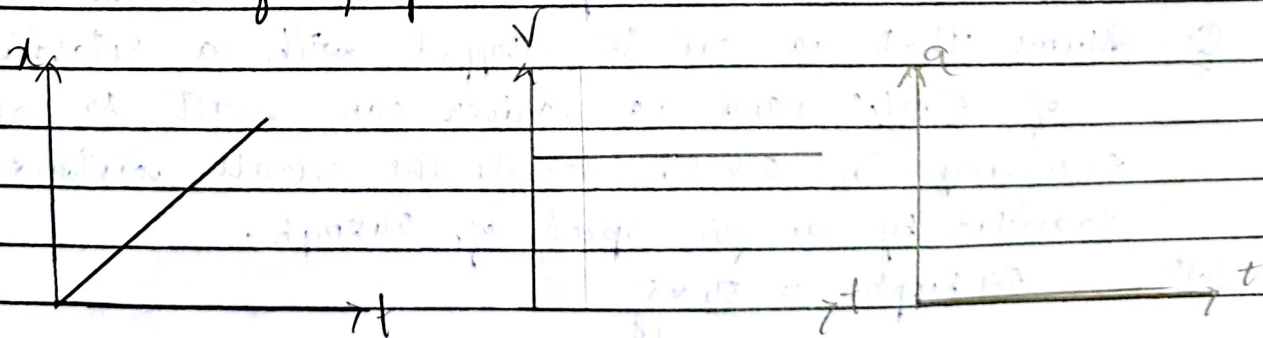
$$a = v \frac{dv}{dx}$$

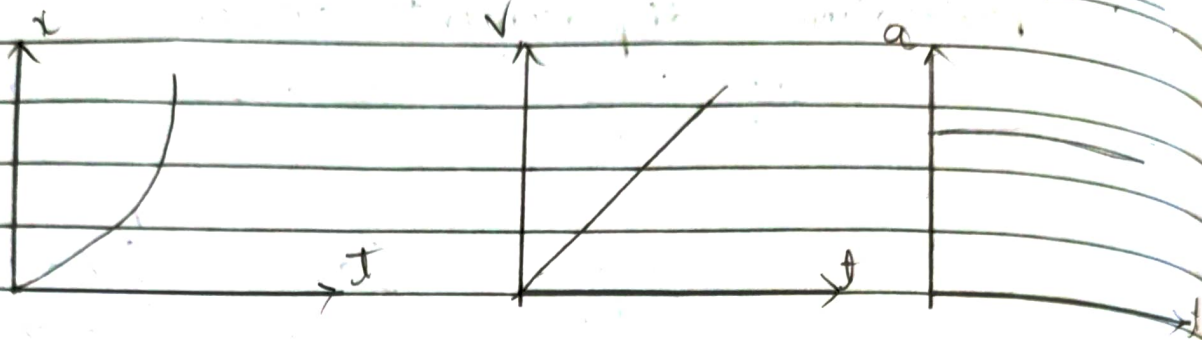


slope = $\tan \theta = \frac{-20}{50} = \frac{-2}{5}$

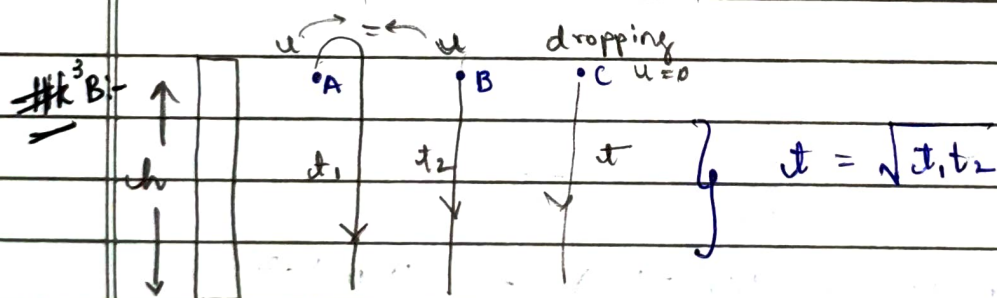
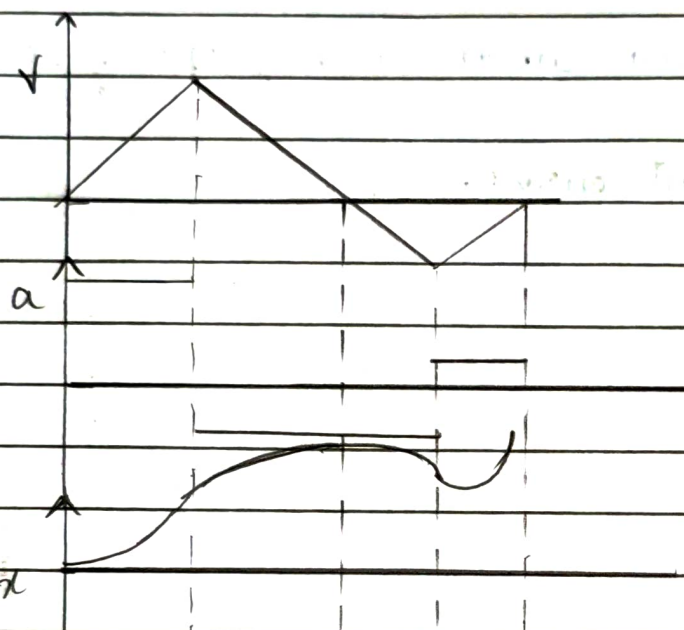
$$a = v \frac{dv}{dx} = 10 \left(\frac{-2}{5} \right) = -4 \text{ m/s}^2$$

Conversion of graph.





eg:



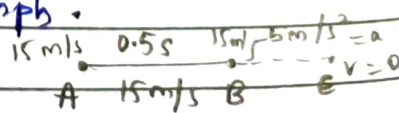
Concept of reaction time -

It is a time a person takes to think & react. Assume that a car is stopped with a retardation of 5 m/s^2 and a driver can react to an emergency in 0.5 s , calculate the overall distance travelled by car for speed of 54 kmph .

Soln

$$54 \text{ kmph} = \frac{54 \times 5}{18}$$

$$= 15 \text{ m/s}$$



distance from A $\rightarrow$ B
 $= s \times t$
 $= 15 \times 0.5 = 7.5 \text{ m}$

distance from B $\rightarrow$ C
 $v^2 = u^2 + 2as$
 $0 = (15)^2 + 2(-5)(s)$
 $225 = 10s$
 $s = 22.5 \text{ m}$

$\therefore$ overall distance travelled $= 22.5 + 7.5$
 $= 30 \text{ m}$

Concept of maximum separation

Q: A car and a truck move in a straight line at the same instant of time. The car moves with a speed of 40 m/s (constant) and truck starts with a constant acceleration of 4 m/s^2 from rest. Find the time elapsed before the truck catches the car and the maximum separation between them prior to it and the time at which this occurs

Solⁿ: For Max^m At the moment of max^m separation, the speed of truck = speed of car.

For Truck, $v = 40$

$40 = 0 + (4)t$

$t = 10 \text{ seconds}$

max^m separation $= d_{\text{car}} - d_{\text{truck}}$
 $= (40 \times 10) - \left(\frac{1}{2} \times 4 \times 10^2\right)$
 $= 400 - 200$
 $= 200 \text{ m}$

For truck to catch the car,

$$40 \times t = 0 \times t + \frac{1}{2} \times 4 \times t^2$$

$$40t = 2t^2$$

$$0 = 2t(t - 20)$$

$$t = 20 \text{ seconds}$$

Concept of minimum separation :-

Q:- The two particles moving along x-axis. Particle, P_1 is 40m behind particle, P_2 . P_1 starts with velocity 12m/s and acceleration 4m/s^2 whereas P_2 starts with velocity 4m/s and acceleration 12m/s^2 in the same direction. Find the time when separation b/w them is minimum and the minimum separation b/w them.

Soln:- For minimum separation, both particles will have equal velocity. ($v_1 = v_2$)

$$12 + 4 \times t = 4 + 12 \times t$$

$$t = 1 \text{ seconds}$$

Minimum separation =

$\Rightarrow$ Og separⁿ + dist travelled by P_2 in t - dist travelled by P_1 in t

$$\Rightarrow \left(40 + \left(4 + \frac{1}{2} \times 4 \times 1 \right) \right) - \left(12 \times 1 + \frac{1}{2} \times 12 \times 1 \right)$$

$$\Rightarrow 36 \text{ m.}$$

or

$$\text{Separation} = \left(40 + ut + \frac{1}{2}at^2 \right) - \left(12t + \frac{1}{2} \times 4 \times t^2 \right)$$

for min^m separation,

$$\frac{ds}{dt} = 4 + 12t - 12 - 4t = 0$$

$$8t - 8 = 0$$

$$t = 1$$

$$\frac{d^2s}{dt^2} = 8 = \text{ve} = \text{minima.}$$

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$$\frac{ds}{dt} = 0$$

$$\frac{ds}{dt} \quad \text{put } t = 1$$

$$S_{\min} = (40 + 4 + 6) - (12 \div 2)$$

$$= 50 - 14$$

$$= 36 \text{ m}$$

Q. Two ships 1 and 2 move with constant velocities 3m/s and 4m/s along two mutually perpendicular straight tracks towards the intersection point O. At the moment $t=0$ the ships were located at the distances 120m & 200m from O. How soon will the distance b/w ships become the shortest and what is it equal to?

$$\text{let time} = t$$

$$D_1 = 3t, \quad D_2 = 4t$$

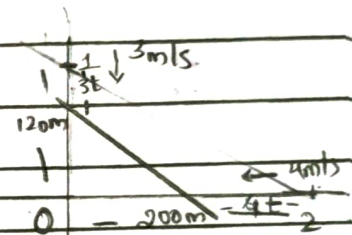
$$S = \sqrt{(120-3t)^2 + (200-4t)^2}$$

$$\frac{ds}{dt} = \frac{1}{2} \left[\frac{2(120-3t)(-3) + 2(200-4t)(-4)}{\sqrt{(120-3t)^2 + (200-4t)^2}} \right] = 0$$

$$\Rightarrow -360 + 9t - 800 + 16t = 0$$

$$\Rightarrow 25t - 1160 = 0$$

$$t = \frac{1160}{25} \text{ sec} = 46.4 \text{ seconds}$$



Motion with variable acceleration :-

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt}$$

$$a = \frac{dv}{dt} \cdot \frac{dx}{dx} \Rightarrow v \frac{dv}{dx}$$

Q: The motion of a particle moving along a straight line is described as $x = 8 + 12t - t^3$. x in 'm' & t in 's'. Find retardation of particle when its velocity is '0'.

$$v = \frac{dx}{dt} = 8 + 12t - 3t^2$$

$$0 = -3t^2 + 12$$

$$0 = -t^2 + 4$$

$$t = 2$$

$$a = \frac{dv}{dt} = -6t = -12 \text{ m/s}^2$$

∴ Retardation = 12 m/s^2

Q: If the velocity of a car is given by $(150 - 10x)^{1/2}$, find acceleration.

$$a = v \frac{dv}{dx}$$

$$= (150 - 10x)^{1/2} \times \frac{1}{2} (150 - 10x)^{-1/2} (-10)$$

$$= \frac{-10}{2} = -5 \text{ m/s}^2$$

Q: $t = x^2 + x$. The accⁿ = 9

$$\frac{dt}{dx} = 2x + 1 \Rightarrow \frac{dx}{dt} = \frac{1}{2x + 1} \Rightarrow v = (2x + 1)^{-1}$$

$$\frac{dv}{dx} = -1(2x + 1)^{-2}(2)$$

$$a = -2(2x + 1)^{-2}$$

$$a = v \cdot \frac{dv}{dx}$$

$$a = (2x+1)^{-1} \times -2(2x+1)^{-2}$$

$$a = \frac{-2}{(2x+1)^3}$$

A particle moves in a straight line, its motion is described as $x^2 = 1+t^2$, then its accⁿ at any time, t is x^{-n} . find n .

$$v = \frac{dx}{dt} = \frac{d}{dt} (\sqrt{1+t^2})$$

$$= \frac{1}{2} (1+t^2)^{-1/2} \times 2t$$

$$= \frac{2t}{2(1+t^2)^{1/2}} \Rightarrow \frac{t}{(1+t^2)^{1/2}}$$

$$a = \frac{dv}{dt} = \frac{\frac{d}{dt} \left(\frac{t}{(1+t^2)^{1/2}} \right)}{(1+t^2)^{1/2}}$$

$$= \frac{(1+t^2)^{1/2} - t^2}{(1+t^2)^{3/2}}$$

$$= \frac{1+t^2 - t^2}{(1+t^2)^{3/2} (1+t^2)^2}$$

$$= \frac{1}{(1+t^2)^{7/2}} \Rightarrow (1+t^2)^{-7/2}$$

$$\Rightarrow (1+t^2)^{-3/2}$$

$$\Rightarrow x^{-3}$$

$$\therefore \boxed{n=3}$$

Q: The acceleration of a particle is given as $a = 6t + 2$.
 at $t = 0$, x is 1m & initial velocity = 2m/s. Find displacement
 of the particle at $t = 3$ seconds.

$$a = 6t + 2$$

$$\frac{dv}{dt} = 6t + 2$$

$$\int_2^v dv = \int_0^t (6t + 2) dt$$

$$[v]_2^v = \left[\frac{6t^2}{2} + 2t \right]_0^t$$

$$v - 2 = (3t^2 + 2t) - 0$$

$$v = 3t^2 + 2t + 2$$

$$\frac{dx}{dt} = 3t^2 + 2t + 2$$

$$\int_1^x dx = \int_0^3 (3t^2 + 2t + 2) dt$$

$$x - 1 = \left[t^3 + t^2 + 2t \right]_0^3$$

$$x - 1 = 27 + 9 + 6 - 0$$

$$x - 1 = 42$$

$$x = 43 \text{ m}$$

Q: The position of a particle moving in a straight line
 along x-axis varies with time as $x = (12t - 3t^2) \text{ m}$,
 where t is in second. The distance travelled by
 particle in first 5 sec is ?

$$v = \frac{dx}{dt} = 12 - 6t$$

$$a = \frac{dv}{dt} = -6$$

particle is moving & taking a

u-turn.

$u = 12 \text{ m/s}$ & $v = 0$ [when $t = 0$ & 2 respectively]

$$S = (12)(2) + \frac{1}{2}(-6)(4)$$

$$= 12 \text{ m}$$

for 5 seconds

$$s = (12 \times 5) + \frac{1(-6)(5)(5)}{2} = -15 \text{ m}$$

$$\text{Total distance} = 12 + 12 + 15 = 39 \text{ m}$$

The motion of a particle is presented by $\frac{dv}{dt} = 6 - 3v$. If the particle is at rest at $t = 0$. Then

(i) The speed of a particle is 2 m/s when $a = 0$

$$\frac{dv}{dt} = 0 = 6 - 3v \Rightarrow v = 2 \text{ m/s}$$

(ii) After a long time, the particle moves with a constant velocity of 2 m/s

$$\int_0^v \frac{dv}{6 - 3v} = \int_0^t dt$$

$$\int \frac{dx}{ax+b} = \frac{\ln(ax+b)}{a}$$

$$\left[\frac{\ln(6 - 3v)}{-3} \right]_0^v = t$$

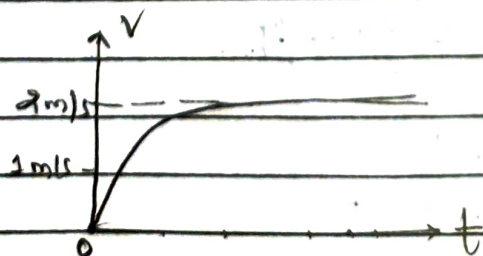
$$\ln(6 - 3v) - \ln(6) = -3t \Rightarrow \ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$$

$$\ln\left(\frac{6 - 3v}{6}\right) = -3t$$

$$\frac{6 - 3v}{6} = e^{-3t}$$

$$1 - \frac{v}{2} = e^{-3t}$$

$$v = 2(1 - e^{-3t})$$



$$\text{if } t = 0, e^0 = 1 \\ 2(1 - 1) = 0$$

$$\text{if } t = \infty, e^{-\infty} = 0 \\ 2(1 - 0) = 2$$

(iii) The magnitude of final acceleration is 6 m/s^2

$$v = 2(1 - e^{-3t})$$

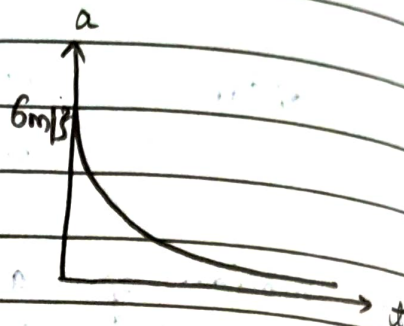
$$a = \frac{dv}{dt} = 2(0 - (-3)(e^{-3t}))$$

$$= 6e^{-3t}$$

Final accⁿ $\rightarrow t = \infty$

$$= 6 \times 0 = 0 \text{ m/s}^2$$

$\rightarrow$ statement is wrong.



(iv) The speed is 0.5 m/s , when the acceleration is half of its initial value.

$$\text{acc}^n = 3 \text{ m/s}^2$$

$$v = 2(1 - e^{-3t})$$

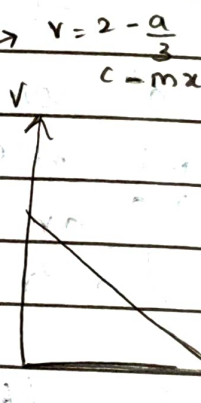
$$v = 2\left(1 - \frac{a}{6}\right)$$

$$v = 2\left(1 - \frac{3}{6}\right)$$

$$v = 2\left(\frac{1}{2}\right)$$

$$v = 1 \text{ m/s}$$

$\rightarrow$ Statement is wrong.

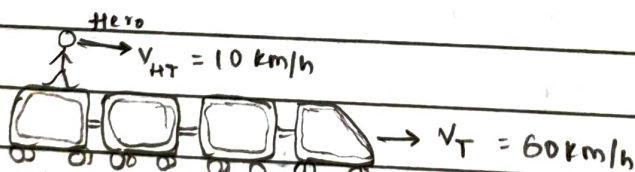


Relative Motion:

$$V_{AB} = V_A - V_B$$

A wrt B

Eg.



find velocity of hero wrt ground (V_{HG})

$$V_{HT} = V_{HG} - V_{TG}$$

$$10 = V_{HG} - 60$$

$$V_{HG} = \underline{\underline{70 \text{ km/h}}}$$

Ex³:-

$$V_{rel} = U_{rel} + a_{rel}t$$

$$s_{rel} = U_{rel}t + \frac{1}{2}a_{rel}t^2$$

$$V_{rel}^2 = U_{rel}^2 + 2a_{rel}s_{rel}$$

$$h_1 = \frac{2u}{g}, \quad t_{1,2} = \frac{2h}{g}$$

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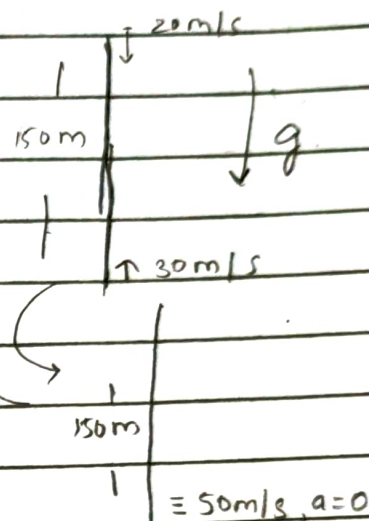
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Q: A ball is thrown vertically upward with a speed of 20 m/s from the top of a building 150 m high and simultaneously another ball is thrown vertically upward with a speed of 30 m/s from the foot of the building along the same line. find the time when they will meet.

$$S_{rel} = U_{rel}t + \frac{1}{2}a_{rel}t^2$$

$$150 \text{ m} = 50 \times t + \frac{1}{2}(0)(t^2)$$

$$t = 3 \text{ sec}$$



Q: a-t graph is given of a particle which starts from rest. The distance covered by particle in first 4 sec is ?

→ Eqⁿ of straight line:-

$$\frac{a-0}{0-1} = \frac{2-0}{t-1}$$

$$a = -2t + 2$$

$$\frac{dv}{dt} = -2t + 2$$

$$\int dv = \int (-2t + 2) dt$$

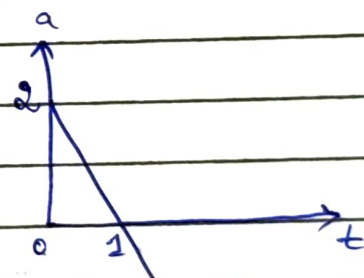
$$v = -t^2 + 2t$$

$$\frac{dx}{dt} = -t^2 + 2t \Rightarrow \int dx = \int (-t^2 + 2t) dt$$

$$x = \left[-\frac{t^3}{3} + t^2 \right]_0^4 + \left[-\frac{t^3}{3} + t^2 \right]_0^1$$

$$x = \left| -\frac{8}{3} + 4 \right| + \left| \left(-\frac{64}{3} + 16 \right) - \left(-\frac{8}{3} + 4 \right) \right|$$

$$x = \frac{4}{3} + \frac{20}{3} \Rightarrow \frac{24}{3} \Rightarrow 8$$



Q: The water drops fall at regular intervals from a tap 5m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant.

$$t \text{ for first drop to touch the ground} = \sqrt{\frac{2h}{g}} = 1 \text{ sec.}$$

In 1 sec, 2 drops have already fallen.

$\therefore$ Drops fall in the interval of 0.5 s

Distance covered by second drop in 0.5 sec

$$\Rightarrow s = \frac{1}{2}(-10)(0.5)^2$$

$$= -1.25 \text{ m}$$

from ground, distance is 3.75 m

$t = mx^2 + nx$, find retardation

$$\frac{dt}{dx} = \frac{2mx+n}{v} = \frac{1}{v}$$

$$\frac{1}{x} = \frac{2mx+n}{v}$$

$$dv = -1(2m)$$

$$dv = -2m \quad (2mx+n)^2$$

$$a = v \cdot \frac{dv}{dx}$$

$$= \frac{1}{2mx+n} \cdot \frac{-2m}{(2mx+n)^2} = \frac{-2m}{(2mx+n)^3} \times \frac{-2m}{\sqrt{3}}$$

$$\text{Retardation} = 2mv^3$$

Excellent
 Shashank
 16/06/20