

14/04/21

Introduction

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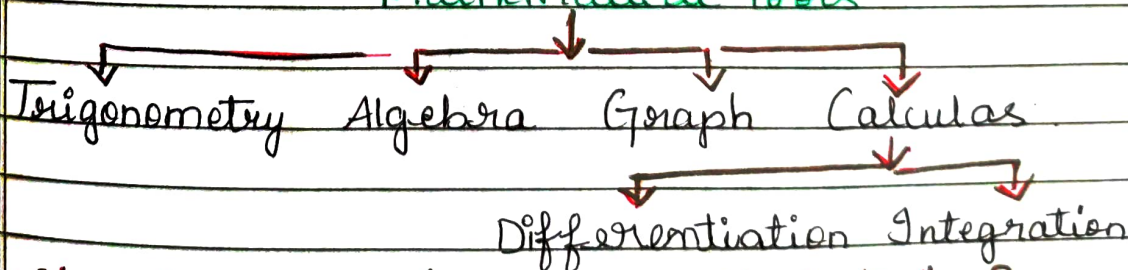
Plan :-

1. Class Notes — (x2) daily
2. Module
3. Additional Questions
4. Test Series

Chapters :-

1. Mathematical Tools
2. Unit & Measurement
3. Motion in a Straight line
4. Motion in a plane
5. Newton's law of Motion & Friction
6. Work power & energy
7. Centre of Mass & collision
8. Rotational Motion
9. Gravitation
10. Mechanical properties of solids
11. Mechanical properties of liquids
12. Thermal properties of Matter
13. Kinetic Theory of gases
14. Thermodynamics
15. Oscillations (Simple Harmonic motion)
16. Wave Motion

Mathematical Tools



1. Why do we study mathematical tools?
Because mathematics is the language of physics
2. Physics = Study of nature & it's law

②

Chapter - 01

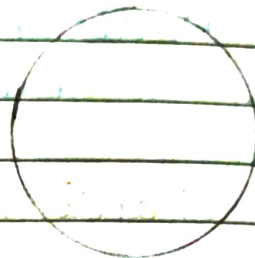
Basics of Trigonometry and Graph Analysis

Angle = SI unit Radian

$$= \frac{\text{Arc length}}{\text{Radius}}$$

$$= \frac{2\pi R}{R}$$

$$= 2\pi$$



$$2\pi \text{ Radian} = 360^\circ$$

$$1^\circ = \frac{2\pi}{360} \text{ Radian}$$

Degree	Radian	$\frac{1}{4} = 0.25$
180°	π	$\frac{1}{8} = 0.125$
90°	$\frac{\pi}{2}$	
45°	$\frac{\pi}{4}$	
120°	$\frac{2\pi}{3}$	
60°	$\frac{\pi}{3}$	
30°	$\frac{\pi}{6}$	

Acute Angle

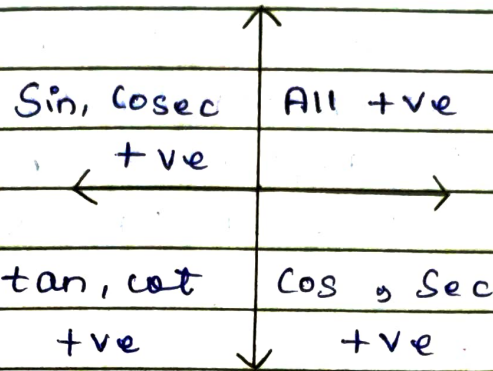
$$0 < \theta < 90^\circ$$

Obtuse Angle

$$90^\circ < \theta < 180^\circ$$

ASTC Rule

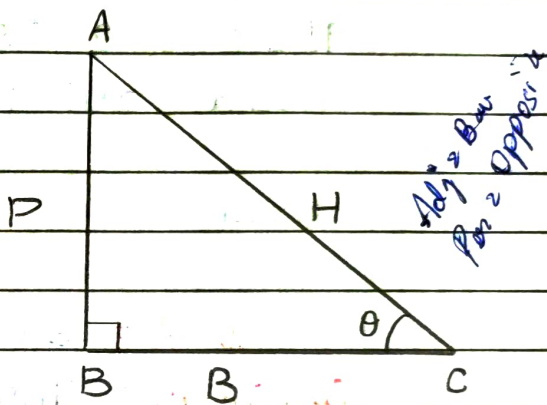
Add ^{Sugar}Salt To Coffee



$$\sin \theta = \frac{P}{H}$$

$$\cos \theta = \frac{B}{H}$$

$$\tan \theta = \frac{P}{B}$$



Angle	0°	30°	37°	45°	60°	53°	90°	120°	135°	150°	180°
Sin	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{3}{5}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	$\frac{3}{4}$	1	$\sqrt{3}$	$\frac{4}{3}$	N.D	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Pythagorean Triplets

(3, 4, 5)	(5, 12, 13)	(7, 24, 25)
(8, 15, 17)	(9, 40, 41)	(11, 60, 61)
(12, 35, 37)	(13, 84, 85)	(16, 63, 65)
(20, 21, 29)	(28, 45, 53)	(33, 56, 65)
(36, 77, 85)	(39, 80, 89)	(48, 55, 73)

Allied Angles :-

$90 \pm \theta$, $180 \pm \theta$, $270 \pm \theta$, $360 \pm \theta$ -- are called allied angles of angle θ

$$90 - \theta$$

$$90 + \theta$$

$$\sin(90 - \theta) = \cos \theta$$

$$\sin(90 + \theta) = \cos \theta$$

$$\cos(90 - \theta) = \sin \theta$$

$$\cos(90 + \theta) = -\sin \theta$$

$$\tan(90 - \theta) = \cot \theta$$

$$\tan(90 + \theta) = -\cot \theta$$

$$\cos(120) = \cos(90 + 30)$$

$$= -\sin 30$$

$$180 - \theta$$

$$180 + \theta$$

$$\sin(180 - \theta) = \sin \theta$$

$$\sin(180 + \theta) = -\sin \theta$$

$$\cos(180 - \theta) = -\cos \theta$$

$$\cos(180 + \theta) = -\cos \theta$$

$$\tan(180 - \theta) = -\tan \theta$$

$$\tan(180 + \theta) = +\tan \theta$$

$$\cos 120 = \cos(180 - 60)$$

$$= -\cos 60$$

$$= -\frac{1}{2}$$

$270 - \theta$

$$\begin{aligned}\sin(270 - \theta) &= -\cos \theta \\ \cos(270 - \theta) &= -\sin \theta \\ \tan(270 - \theta) &= +\cot \theta\end{aligned}$$

$270 + \theta$

$$\begin{aligned}\sin(270 + \theta) &= -\cos \theta \\ \cos(270 + \theta) &= +\sin \theta \\ \tan(270 + \theta) &= -\cot \theta\end{aligned}$$

$360 - \theta$

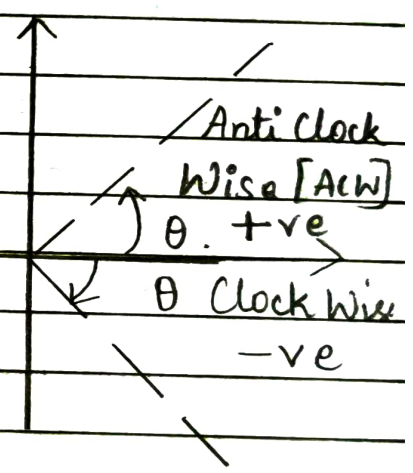
$$\begin{aligned}\sin(360 - \theta) &= -\sin \theta \\ \cos(360 - \theta) &= +\cos \theta \\ \tan(360 - \theta) &= -\tan \theta\end{aligned}$$

$360 + \theta$

$$\begin{aligned}\sin(360 + \theta) &= +\sin \theta \\ \cos(360 + \theta) &= +\cos \theta \\ \tan(360 + \theta) &= +\tan \theta\end{aligned}$$

$-\theta$

$$\begin{aligned}\sin(-\theta) &= -\sin \theta \\ \cos(-\theta) &= \cos \theta \\ \tan(-\theta) &= -\tan \theta\end{aligned}$$



$$\sin x = x - \frac{x^3}{3 \times 2 \times 1} + \frac{x^5}{5 \times 4 \times 3 \times 2 \times 1} - \frac{x^7}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} + \dots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

Small Angle Approximation:-

($x \ll 1$) $\rightarrow$ Radian

$$\sin x \approx x$$

$$\cos x \approx 1$$

$$\tan x \approx x$$

* Few Identities

1. $\sin^2 \theta + \cos^2 \theta = 1$

2. $\sec^2 \theta = 1 + \tan^2 \theta$

3. $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

4. $\sin 2\theta = 2 \sin \theta \cos \theta$

5. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$1 + \cos 2\theta = 2 \cos^2 \theta$$

$$1 - \cos 2\theta = 2 \sin^2 \theta$$

6. $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$$1 + \cos \theta = 2 \cos^2 \left[\frac{\theta}{2} \right]$$

$$1 - \cos \theta = 2 \sin^2 \left[\frac{\theta}{2} \right]$$

7. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

8. $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

9. $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

10. $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

11. $\sin C + \sin D = 2 \sin \left[\frac{C+D}{2} \right] \cos \left[\frac{C-D}{2} \right]$

$$12 \quad \sin C - \sin D = 2 \sin \left(\frac{C-D}{2} \right) \cos \left(\frac{C+D}{2} \right)$$

$$13 \quad \cos C + \cos D = 2 \cos \left[\frac{C+D}{2} \right] \sin \left[\frac{C-D}{2} \right]$$

$$14 \quad \cos C - \cos D = 2 \sin \left[\frac{C+D}{2} \right] \sin \left[\frac{D-C}{2} \right]$$

$$15 \quad \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$16 \quad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\begin{aligned} \text{Ex } \sin 15^\circ &= [\sin 45^\circ - \sin 30^\circ] \\ &= \left[\frac{1}{\sqrt{2}} - \frac{1}{2} \right] \\ &= \left[\frac{2 - \sqrt{2}}{2\sqrt{2}} \right] \end{aligned}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{2 \cdot \sqrt{6}}{2\sqrt{2}} - \frac{2 \cdot \sqrt{2}}{2\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\sin(A-B) = \frac{2 \cdot \sqrt{3}}{4\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

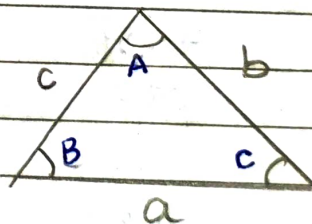
Ex

$$\begin{aligned}
 \cos 15^\circ &= \cos(60-45) \\
 &= \cos 60 \cos 45 + \sin 60 \sin 45 \\
 &= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \\
 &= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} \\
 &= \frac{\sqrt{3}+1}{2\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \tan 15^\circ &= \tan(60-45) \\
 &= \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} \\
 &= \frac{\sqrt{3} - 1}{1 + \sqrt{3}(1)} \\
 &= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}
 \end{aligned}$$

Sin rule.

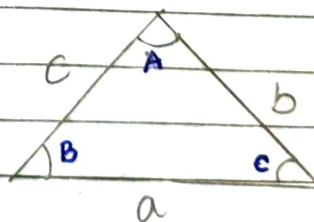
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



cos rule

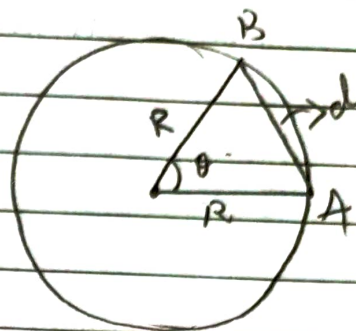
$$\cos B = \frac{a^2 + c^2 - b^2}{2 \times a \times c}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c}$$



Displacement (d)

$$\cos \theta = \frac{R^2 + R^2 - d^2}{2RR}$$



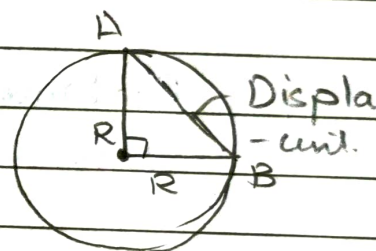
$$2R^2 \cos \theta = 2R^2 - d^2$$

$$d^2 = 2R^2(1 - \cos \theta)$$

$$d^2 = 2R^2 \times 2 \sin^2 \left[\frac{\theta}{2} \right]$$

$$d = 2R \sin \frac{\theta}{2}$$

$$AB = 2R \sin \frac{\theta}{2}$$



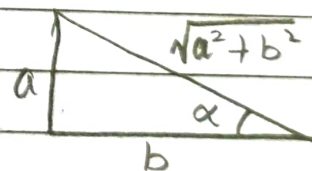
$$-\sqrt{a^2+b^2} \leq a \sin \theta + b \cos \theta \leq \sqrt{a^2+b^2}$$

$$(a \sin \theta + b \cos \theta) \times \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

$$= \sqrt{a^2+b^2} \left(\frac{a}{\sqrt{a^2+b^2}} \sin \theta + \frac{b}{\sqrt{a^2+b^2}} \cos \theta \right)$$

$$= \sqrt{a^2+b^2} (\sin \alpha \sin \theta + \cos \alpha \cos \theta)$$

$$= \sqrt{a^2+b^2} \cos(\theta - \alpha)$$

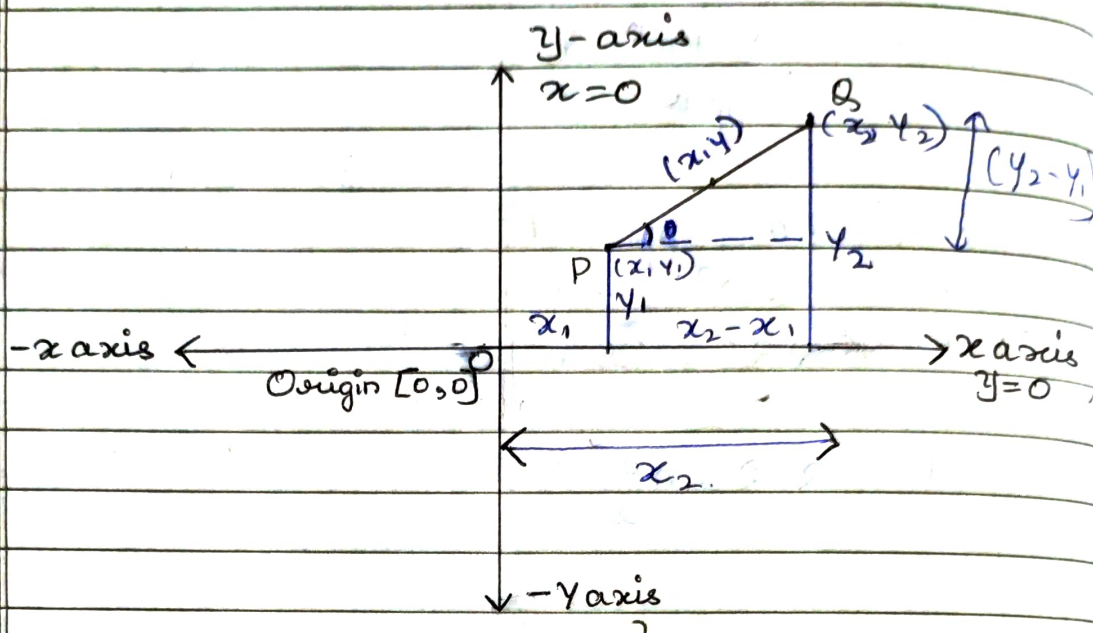


$$\text{Max: } +\sqrt{a^2+b^2}$$

$$\text{Min: } -\sqrt{a^2+b^2}$$

Find range of $3 \cos \theta - 4 \sin \theta$
 $(-5, 5)$

Coordinate Geometry



$$\text{Distance } PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Slope} = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$$

3D Geometry

$$P(x_1, y_1, z_1) \text{ and } Q(x_2, y_2, z_2)$$

Distance Formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of straight line PQ

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Q Find the equation of a straight line passing through points (1, 2) and (3, 5)

Sol

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{3 - 1} = \frac{3}{2}$$

$$y = mx + c$$

↓
Slope y-intercept

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$$y - 2 = \frac{5-2}{3-1} (x-1)$$

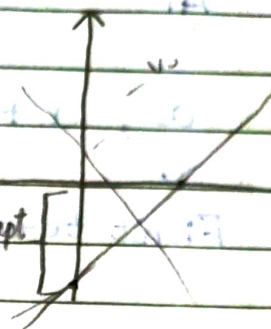
$$y - 2 = \frac{3}{2} (x-1)$$

$$2y - 4 = 3x - 3$$

$$2y = 3x - 3 + 4$$

$$2y = 3x + 1$$

$$y = \frac{3x+1}{2} \quad \frac{3x+1}{2} = \frac{3x}{2} + \frac{1}{2}$$



Quadratic Equations

$$ax^2 + bx + c = 0$$

→ x_1

→ x_2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 + x_2 = -\frac{b}{a}, \quad x_1 \cdot x_2 = \frac{c}{a}$$

$$\Delta = b^2 - 4ac$$

- 1 $\Delta > 0$ = 2 Real & distinct roots
- 2 $\Delta = 0$ = 2 Equal & real roots
- 3 $\Delta < 0$ = 2 No Real Roots (Imaginary roots)

APArithmetic Progression $a, a+d, a+2d, a+3d, \dots, a+(n-1)d$ $\downarrow$

*

First term

Common difference

 $\downarrow$
nth term

*

Sum of n terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [a + a_n]$$

*

Sum of first n natural numbers

$$1 + 2 + 3 + 4 + 5 + \dots + n$$

$$= \frac{n(n+1)}{2}$$

*

Sum of square of first n natural number

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \dots + n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

*

Sum of cube of first n natural numbers

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \dots + n^3$$

$$= \left[\frac{n(n+1)}{2} \right]^2$$

Geometric Progression [GP]

$a, ar, ar^2, ar^3, \dots, ar^{n-1}$
1st term Common Ratio

If r is greater than 1 / less than 1

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad (\text{if } r > 1)$$

$$= \frac{a(1 - r^n)}{1 - r} \quad (\text{if } r < 1)$$

Sum of ∞ terms
 $\frac{a}{1-r}$

$S_\infty = \frac{a}{1-r}$

 (if $r < 1$)

→ Sum of ∞ terms

Ex: Find sum of $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} \dots$

Sol $S_\infty = \frac{1}{1 - \frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

Ex Find sum of $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} \dots$

$S_\infty = \frac{a}{1-r} = \frac{1}{1 - (-\frac{1}{4})} = \frac{1}{1 + \frac{1}{4}} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$ — (1)

$$\frac{1}{4} = \frac{1}{4} = \frac{1}{4} \times \frac{16}{16} = \frac{16}{60}$$

From 1 & 2

$$\frac{16}{15} - \frac{16}{60}$$

$$\frac{16}{15} - \frac{4}{15}$$

$$= \frac{12}{15} = \frac{4}{5}$$

Binomial Expression

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2 \times 1} + \frac{n(n-1)(n-2)x^3}{3 \times 2 \times 1} + \frac{n(n-1)(n-2)(n-3)x^4}{4 \times 3 \times 2 \times 1}$$

$$n = 2$$

$$(1+x)^2 = 1 + 2x + x^2$$

$$(1+x)^3 = 1 + 3x + 3x^2 + x^3$$

If

$$x \ll 1$$

$$x = 0.01$$

← Binomial Approximation

Binomial Approximation

$$\text{If } x \ll 1$$

$$x \geq 0.01$$

$$(1+x)^n \approx 1 + nx$$

$$(1-x)^n \approx 1 - nx$$

$$(1+x)^{-n} \approx 1 - nx$$

$$(1-x)^{-n} \approx 1 + nx$$

Binomial Approximation

$$x \ll 1$$

$$x \geq 0.01$$

$$(1+x)^n \approx 1 + nx$$

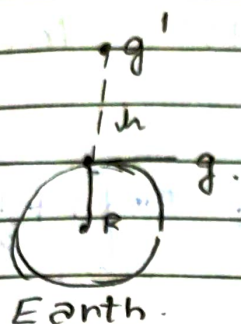
$$(1+x)^n \approx 1 + nx$$

$$(1-x)^n \approx 1 - nx$$

$$(1+x)^{-n} \approx 1 - nx$$

$$(1-x)^{-n} \approx 1 + nx$$

Ex



$$g' = \frac{gR^2}{(h+R)^2}$$

if $h \ll R$

Find acceleration due to gravity at a height h ($h \ll R$)

$$g' = \frac{gR^2}{R^2 \left[\frac{h}{R} + 1 \right]^2}$$

$$= \frac{g}{\left[1 + \frac{h}{R} \right]^2}$$

$$= g \left[1 + \frac{h}{R} \right]^{-2}$$

$$\frac{h}{R} \ll 1$$

$$= g \left(1 - \frac{2h}{R} \right)$$

The mass of a body moving with speed v is given by
 $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ where m_0 = rest mass of body.
 c = Speed of light.

Find the value of mass m at speed $v = 3 \times 10^7$ m/s ($m_0 = 10 \text{ kg}$ + $c = 3 \times 10^8 \text{ m/s}$)

$$\frac{v^2}{c^2} = \frac{1}{10} = 0.1 \quad (0.1)^2 = 0.01$$

$$m = m_0 \left[1 - \frac{v^2}{c^2} \right]^{-\frac{1}{2}} = m_0 \left[1 + \frac{1}{2} \frac{v^2}{c^2} \right]$$

$$= 10 \left[1 - 0.01 \right]^{-\frac{1}{2}} = 10 \left[1 + \frac{1}{2} \times 0.01 \right]$$

$$= 10.005 = 10 \left[1 + 0.005 \right]$$

$$= 10 \left[1.005 \right]$$

$$= 10.05 \text{ kg}$$

Graph

1. Straight line

- General form of equation of a straight line

$$y = mx + c \rightarrow y \Rightarrow \text{intercept}$$

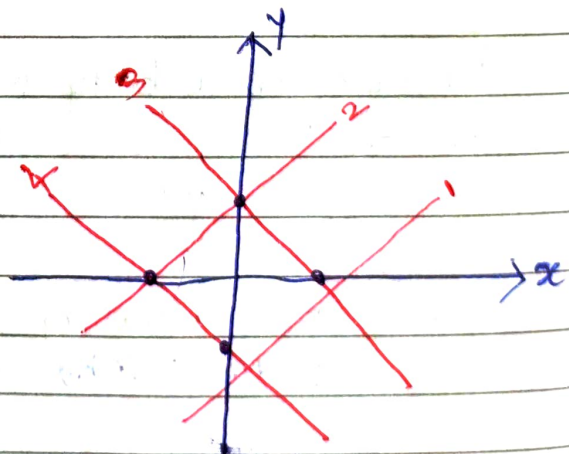
↓
Slope = tan θ

$$y = 2x + 1 \quad \text{--- (2)}$$

$$y = 2x - 1 \quad \text{--- (1)}$$

$$y = -2x + 1 \quad \text{--- (3)}$$

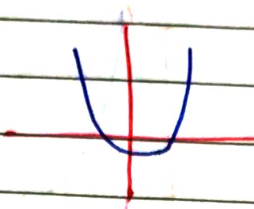
$$y = -2x - 1 \quad \text{--- (4)}$$



Quadratic Equation

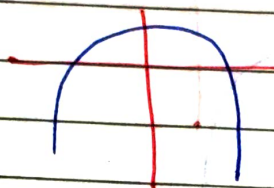
$$ax^2 + bx + c = 0$$

$$a > 0 :$$



upward parabola

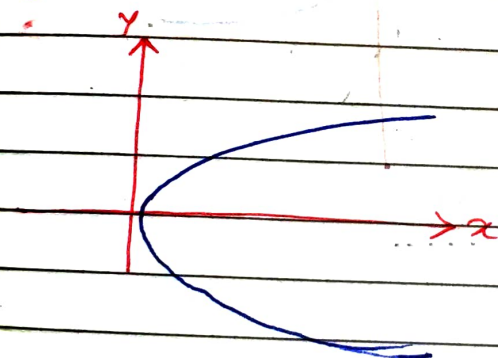
$$a < 0 :$$



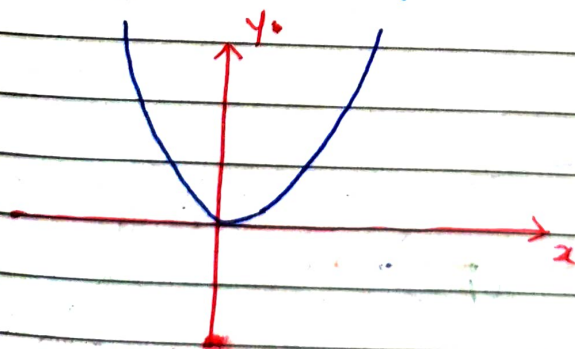
downward parabola

Parabola

$$y^2 = 4x$$



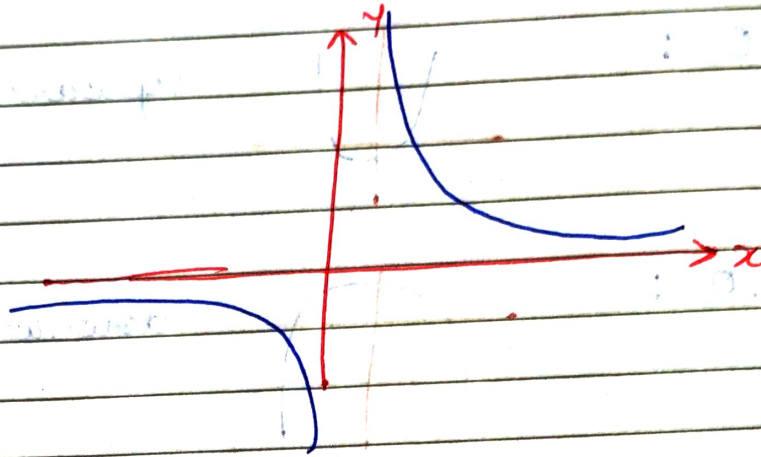
$$x^2 = 4y$$



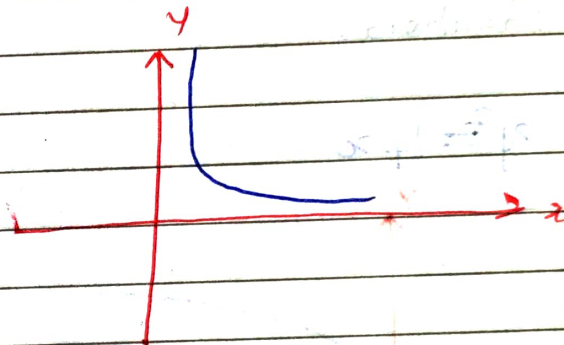
Rectangular Hyperbola

$$xy = \text{constant}$$

$$\text{or } y = \frac{1}{x}$$



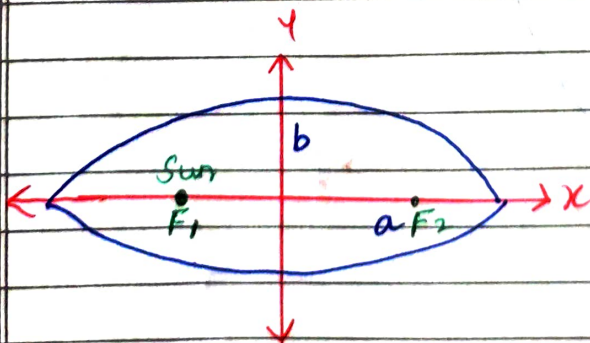
$$y = \frac{1}{x^2}$$



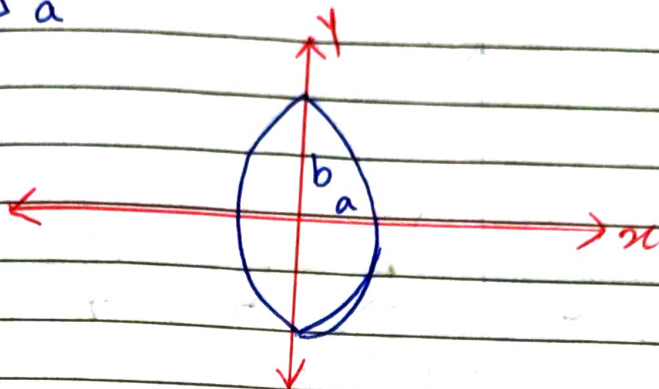
Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a > b$$

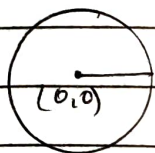


$$b > a$$



Circle.

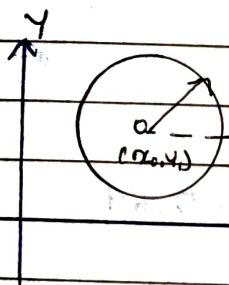
$$x^2 + y^2 = a^2$$



Centre $(0,0)$

Radius $= a$

$$(x - x_0)^2 + (y - y_0)^2 = a^2$$

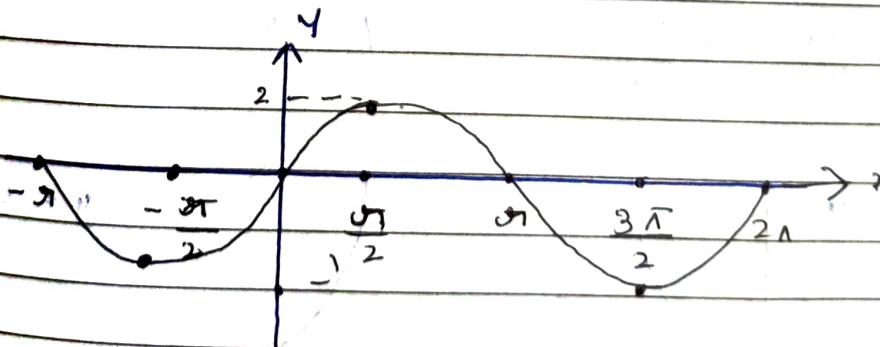


Centre (x_0, y_0)

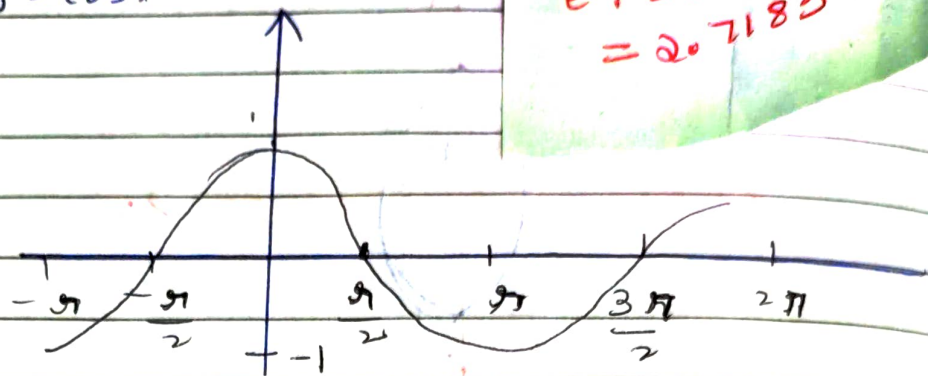
radius $= a$

Some Trigonometric Function

$$y = \sin x$$



$$y = \cos x$$

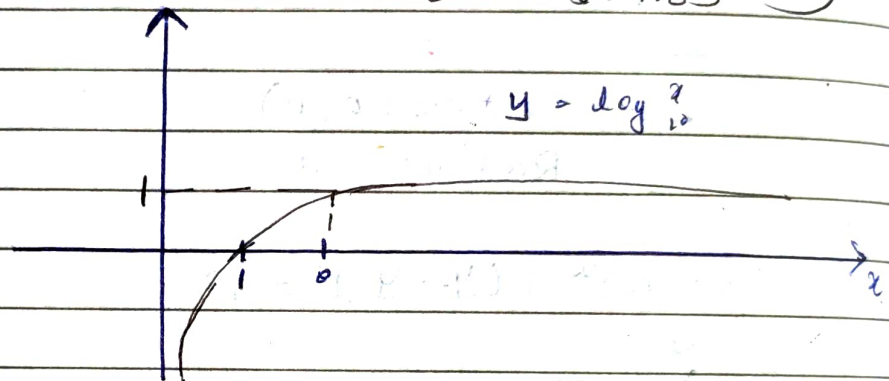


e : Euler Number
 $= 2.7183$

Logarithm Function

$$y = \log_e x = \ln x$$

$[e$: Euler Number
 $= 2.7183]$



$$\ln x = 2.303 \log_{10} x$$

$$\log_{10} 2 = 0.3010 \quad ; \ln 2 = 0.69$$

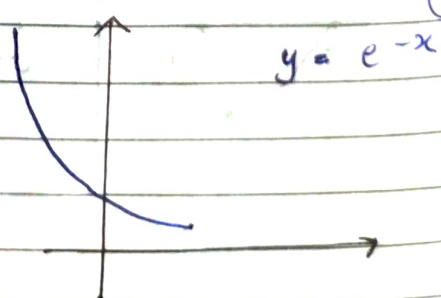
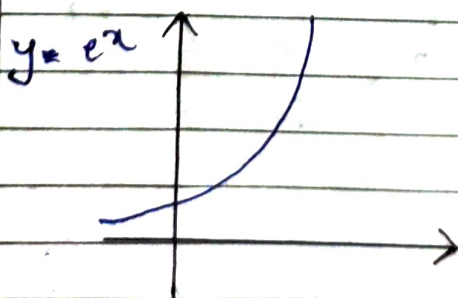
$$\log_{10} 3 = 0.47 \quad ; \ln 3 = 1.082$$

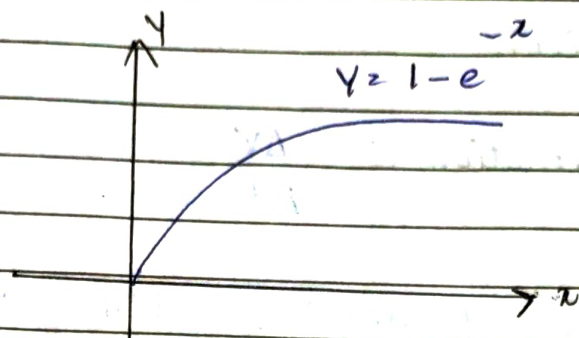
Exponential Function:

$$y = \log_e x \quad ; \quad y = e^x$$

$$x = e^y$$

$y = e^x$ VIP
 $y = \log_e x = x = e^y$

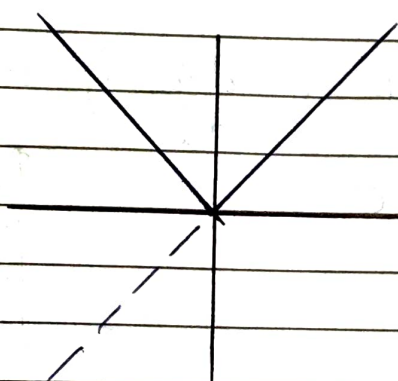




VIP

Modulus Function (Mod Function)

$$y = |x|$$



$$y = \begin{cases} x & : x \geq 0 \\ -x & : x < 0 \end{cases}$$

Calculus

1. Differentiation

$$10 - 6 = 4$$

Finite change (Δ) = $\Delta(10 - 6)$

$$\Delta = 4$$

$$10 - 9.999 = 0.001$$

Infinity change (d)

Very very small

$$d(10 - 9.999)$$

$$= 0.001$$

Insignificant

$\frac{\Delta y}{\Delta x} \rightarrow$ change in y with respect change in x . Change in y wrt x

$$\Delta x \rightarrow 0$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} \leftarrow \text{differentiation of } y \text{ wrt } x.$$

Wrt
With
Respect
to

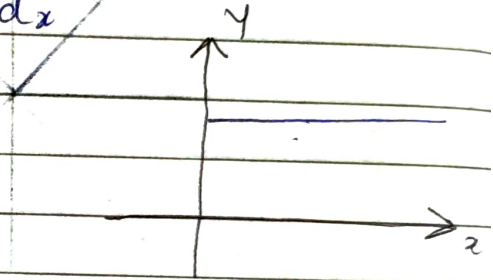
Acceleration

$$\text{Avg. Acceleration} = \frac{\Delta v}{\Delta t}$$

$$\text{Instantaneous Acceleration} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Rules for differentiation

1. $\frac{dy}{dx}$, $y = \text{constant} = \frac{dy}{dx} y = \text{constant}$
 $\frac{d}{dx} \text{constant} = 0$



Rules For Differentiation

1. $\frac{d}{dx} \text{constant} = 0$

2. $\frac{dy}{dx} \Rightarrow y = \text{constant}$

3. $\frac{d(\text{const})}{dx} = 0$

4. $\frac{d}{dx} (x^n) = nx^{n-1}$

2. Power Rule. $\frac{d}{dx} (x^n) = nx^{n-1}$

$\frac{d}{dx} (x^2) = 2x$

$\frac{d}{dx} \left(\frac{1}{x}\right)$

(i) $\frac{d}{dx} x^2 = 2x$ (ii) $\frac{d}{dx} \left(\frac{1}{x}\right) = \frac{d}{dx} (x^{-1}) = -1 x^{-2} = -\frac{1}{x^2}$

$\frac{d}{dx} \sqrt{x} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ $\frac{d}{dx} \left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx} x^{-\frac{1}{2}}$

$= \frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2\sqrt{x}}$

Excellent
 Shashant
 23/04/26

$$\frac{2}{3}x^{\frac{1}{2}}$$

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Constant Multiple Rule

$$y = f(x) \quad \therefore \frac{d}{dx} (c \cdot f(x)) = c \frac{d}{dx} f(x)$$

$$y = x^2 + x - 5$$

$$\frac{d}{dx} \left(\frac{2}{x^2} \right) = 2 \frac{d}{dx} \left(\frac{1}{x^2} \right) = 2 \frac{d}{dx} x^{-2}$$

$$= 2 \cdot (-2) x^{-2-1} = \frac{-4}{x^3}$$

Product Rule

$$\frac{d}{dx} [\underbrace{f(x)}_I \cdot \underbrace{g(x)}_{II}] = \underbrace{f(x)}_I \frac{d}{dx} (g(x))$$

$$+ g(x) \frac{d}{dx} f(x)$$

$$u = f(x), v = g(x)$$

$$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx} \left(x^{\frac{3}{2}} \right) = \frac{d}{dx} (x \cdot \sqrt{x}) = x \frac{d}{dx} \sqrt{x} + \sqrt{x} \frac{d}{dx} (x)$$

$$= x \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} + \sqrt{x}$$

$$= \frac{3}{2} x^{\frac{1}{2}}$$

$$= \frac{\sqrt{x}}{2} + \sqrt{x}$$

$$= \frac{3}{2} \sqrt{x}$$

$$= \frac{3}{2} \sqrt{x}$$

Division Rule

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f(x) \frac{d}{dx} (g(x)) - g(x) \frac{d}{dx} (f(x))}{[g(x)]^2}$$

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} (g(x))}{[g(x)]^2}$$

$$\frac{d}{dx} \left[\frac{u}{v} \right] = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Ex $\frac{d}{dx} \sqrt{x} = \frac{d}{dx} \left(\frac{x}{\sqrt{x}} \right)$

$$\frac{1}{2} x^{-\frac{1}{2}}$$

$$g(x) = x$$

$$f(x) = \sqrt{x}$$

$$\frac{1}{2\sqrt{x}}$$

$$\frac{\sqrt{x} \frac{d}{dx} x - x \frac{d}{dx} \sqrt{x}}{(\sqrt{x})^2}$$

$$\frac{\sqrt{x} \times 1 \times x^0 - x \times \frac{1}{2} x^{-\frac{1}{2}}}{x}$$

$$= \frac{\sqrt{x} - \frac{\sqrt{x}}{2}}{x} = \frac{\sqrt{x} - \frac{\sqrt{x}}{2}}{x} \times \frac{1}{x}$$

$$= \frac{2\sqrt{x} - \sqrt{x}}{2} \times \frac{1}{x} = \frac{\sqrt{x}}{2} \times \frac{1}{x}$$

$$= \frac{\sqrt{x}}{2} \times \frac{1}{\sqrt{x} \times \sqrt{x} \times \sqrt{x}}$$

$$= \frac{\sqrt{x}}{2} \times \frac{1}{x} = \frac{\sqrt{x}}{2} \times \frac{1}{\sqrt{x} \times \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2} x^{-\frac{1}{2}}$$

6 Additional / Subtraction Rule.

$$\frac{d}{dx} (f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} (g(x))$$

$$\text{Ex: } \frac{d}{dx} (x^2 - 2x + 1) = \frac{d}{dx} (x^2) - \frac{d}{dx} 2x + \frac{d}{dx} 1$$

$$= 2x - 2 + 0 = 2x - 2.$$

$$\text{E: } \frac{d}{dx} \left(x - \frac{1}{x} \right) = \frac{d}{dx} x - \frac{d}{dx} \frac{1}{x}$$

$$= 1 + \frac{1}{x^2}$$

$$\text{Ex: } \frac{d}{dx} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) = \frac{d}{dx} \sqrt{x} + \frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right)$$

$$= \frac{1}{2} x^{-\frac{1}{2}} + \frac{-1}{2} x^{-\frac{3}{2}} = \frac{1}{2} \left(x^{-\frac{1}{2}} - x^{-\frac{3}{2}} \right)$$

$$\frac{d}{dx} \left(\frac{x+1}{x-1} \right) = \frac{(x-1)[1][x+1] - [x+1][1][x-1]}{(x-1)^2}$$

$$= \frac{x^2 - 1^2 - x^2 - 1^2}{(x-1)^2}$$

$$= \frac{-2x^2}{(x-1)^2}$$

Differentiation Of Trigonometric Function

$$a \quad \frac{d}{dx} (\sin x) = \cos x$$

$$b \quad \frac{d}{dx} (\cos x) = -\sin x$$

$$c \quad \frac{d}{dx} (\tan x) = \sec^2 x$$

$$d \quad \frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$$

$$e \quad \frac{d}{dx} (\sec x) = \sec x \cdot \tan x$$

$$f \quad \frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$$

$$\textcircled{a} \quad \sin x = x - \frac{x^3}{3 \times 2 \times 1} + \frac{x^5}{5 \times 4 \times 3 \times 2 \times 1} - \frac{x^7}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} + \dots$$

$$\textcircled{b} \quad \frac{d}{dx} (\sin x) = 1 - \frac{x^2}{2 \times 1} + \frac{x^4}{4 \times 3 \times 2 \times 1} - \frac{x^6}{6 \times 5 \times 4 \times 3 \times 2 \times 1} + \dots$$

$$\textcircled{c} \quad \cos x = 1 - \frac{x^2}{2 \times 1} + \frac{x^4}{4 \times 3 \times 2 \times 1} - \frac{x^6}{6 \times 5 \times 4 \times 3 \times 2 \times 1} + \dots$$

$$\textcircled{d} \quad \frac{d}{dx} (\cos x) = 0 - \frac{x}{1} + \frac{x^3}{3 \times 2 \times 1} - \frac{x^5}{5 \times 4 \times 3 \times 2 \times 1} + \dots = -(\sin x)$$



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$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Differentiation of exponential & logarithmic function.

$$\frac{d}{dx}(e^x) = e^x \quad \left| \quad \frac{d}{dx}(\ln x) = \frac{1}{x} \right|$$

$$\text{Ex. } \frac{d}{dx}(e^x \ln x) = \frac{d}{dx} e^x \cdot \frac{d}{dx} \ln x$$

$$e^x \left(\frac{1}{x} \right) + \ln x \cdot e^x$$

$$= e^x \left(\frac{1}{x} + \ln x \right)$$

$$\text{Ex } \frac{d}{dx}(\sin^2 x) = \frac{d}{dx} 2 \sin x \cos x$$

$$\sin x \cos x + \sin x \cos x = 2 \sin x \cos x$$

Chain Rule :

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$a = \frac{dr}{dt} \times \frac{dx}{dx} = \frac{dr}{dx} \cdot \frac{dx}{dt} \quad a = r \frac{dr}{dx}$$

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad x: \text{positive}$$

$$E=1 \quad y = \sqrt{x^2+1} \rightarrow y = \sqrt{u}, \quad u = x^2+1$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$u = x^2+1 \rightarrow \frac{du}{dx} = 2x$$

$$= \frac{1}{2\sqrt{u}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}$$

$$\text{Method: } y = \sqrt{x^2+1}$$

$$\frac{dy}{dx} = \frac{1}{2} (x^2+1)^{-\frac{1}{2}} \times 2x = \frac{x}{\sqrt{x^2+1}}$$

$$Ex: \frac{d}{dx} (\sin^2 x)$$

$$= 2 \sin x \frac{d}{dx} \sin x = 2 \sin x \cos x$$

$$y = A \sin(\omega t + \phi) \quad A, \omega + \phi \rightarrow \text{constant}$$

$$\frac{dy}{dt} = A \cos(\omega t + \phi) \times (\omega + 0)$$

$$y = \sin x \quad u$$

$$\frac{dy}{dx} = (\cos x) \times 1 = \cos x$$

$$y = \sin u \quad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex $y = e^{5x}$

$$u = (5x)^5$$

$$y = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= e^u \cdot 5$$

$$= 5x e^{5x}$$

$$\frac{dy}{dx} = e^{5x} \times 5 = 5e^{5x}$$

* Note $\frac{d}{dx} \{ \sin(Ax+B) \} = A \cos(Ax+B)$

* $\frac{d}{dx} \{ \cos(Ax+B) \} = -A \sin(Ax+B)$

* $\frac{d}{dx} e^{ax} = ae^{ax}$

* $\frac{d}{dx} \sin^2(Ax+B) = 2 \sin(Ax+B) \cos(Ax+B) \times A$

$$\frac{d}{dx} \left[\frac{1}{ax+b} \right] = \frac{-1}{(ax+b)^2} \times a = -\frac{a}{(ax+b)^2}$$

Ex $\frac{d}{dx} (\sqrt{2x+1}) = \frac{1}{2} (2x+1)^{-\frac{1}{2}} \cdot (2+0)$

$$= \frac{1}{\sqrt{2x+1}}$$

Ex $\frac{d}{dx} \sqrt{x^2+1} = \frac{1}{2} (x^2+1)^{-\frac{1}{2}} (2x+0) = \frac{x}{\sqrt{x^2+1}}$

$\frac{1}{2} (x^2+1)^{-\frac{1}{2}} (2x+0)$
 $= \frac{1}{2} \times 2x \times \frac{1}{\sqrt{x^2+1}}$

• Double Differentiation

$$y = f(x)$$

$$1^{st} \text{ differentiation} \rightarrow \frac{dy}{dx} \rightarrow \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$= \frac{d^2 y}{dx^2}$$

Ex $y = x^2 - 2x + 1$

$$\frac{dy}{dx} = 2x - 2 + 0$$

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (2x - 2)$$

$$= 2 - 0$$

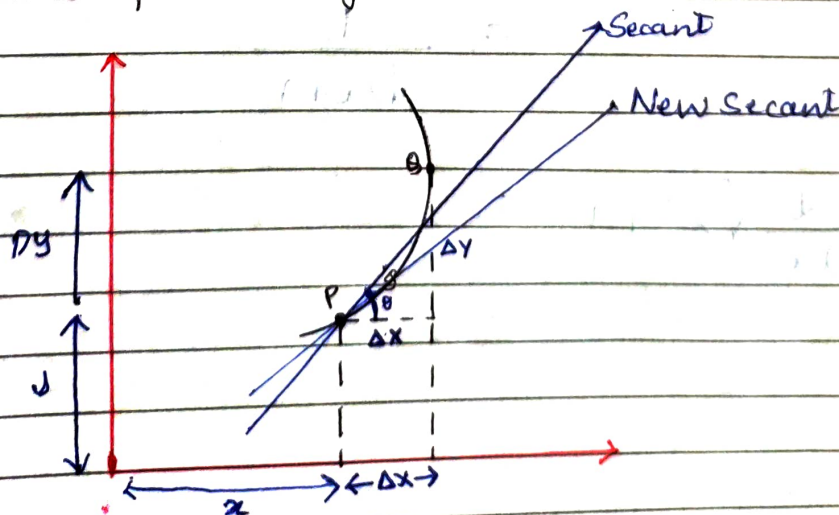
$$= 2$$

Ex $y = e^x \ln x$

$$\frac{dy}{dx} = e^x \left(\frac{1}{x} \right) + \ln x \cdot e^x$$

$$\frac{d^2 y}{dx^2} = e^x \left[-\frac{1}{x^2} \right] + \frac{1}{x} e^x + \ln x \cdot e^x + e^x \cdot \frac{1}{x}$$

Secant & A tangent at a point



Bringing point Q closer to the point P

$\Delta x \rightarrow 0$ [Point Q is very near to point P]

Secant becomes tangent at point

$$\tan \theta = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Slope of y-x graph = $\tan \theta = \frac{dy}{dx}$

Concepts of Local Maxima

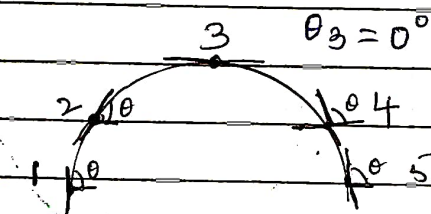
$$\theta_1 = 60^\circ \quad \sqrt{3}$$

$$\theta_2 = 30^\circ \quad \frac{1}{\sqrt{3}}$$

$$\theta_3 = 0^\circ \quad 0$$

$$\theta_4 = 150^\circ \quad -\frac{1}{\sqrt{3}}$$

$$\theta_5 = 120^\circ \quad -\sqrt{3}$$



$$\frac{dy}{dx} = 0$$

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) < 0$$

Slope is decreasing

Ex $y = x^3 - 2x^2 + x - 1$; Find Local Maxima

$$\frac{dy}{dx} = 3x^2 - 4x + 1 = 0$$

$$3x^2 - 3x - x + 1 = 0$$

$$3x(x-1) - 1(x-1) = 0$$

$$(x-1)(3x-1) = 0$$

$$x = 1 \quad \text{or} \quad x = \frac{1}{3}$$

$$\frac{d^2y}{dx^2} = 6x - 4$$

$$\frac{d^2y}{dx^2} \bigg|_{x = \frac{1}{3}} = -2$$

$$\frac{d^2y}{dx^2} \bigg|_{x=1,2}$$

$$y\left(x = \frac{1}{3}\right) = \frac{1}{27} - \frac{2}{9} + \frac{1}{3} - 1$$

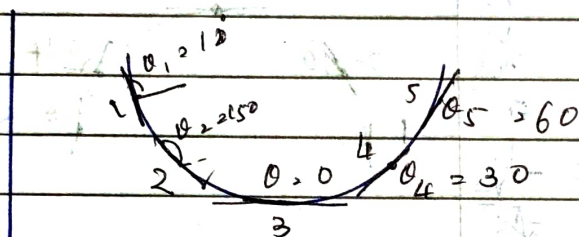
$$= \frac{1-6+9-27}{27}$$

$$= -\frac{23}{27} \rightarrow \text{Local Maximum}$$

$$y(x=1) = 1-2+1-1$$

$$\uparrow = -1$$

Local Minimum



Slope is increasing hence: $\frac{d}{dx}(\text{slope}) > 0$

Q Find the maximum & minimum values of the function $y = x^3 - 3x^2 + 6$ for $0 \leq x \leq 5/2$

$$\frac{dy}{dx} = 3x^2 - 6x = 0 \rightarrow 3x(x-2) = 0$$

$$x = 0 \text{ or } 2$$

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$\frac{d^2y}{dx^2} \bigg|_{x=0} = -6$$

$\uparrow$ Maximum

$$y|_{x=0} = 6$$

$$\frac{d^2y}{dx^2} \bigg|_{x=2} = 6$$

$\downarrow$ minimum

$$y|_{x=2} = -6$$

$y = \frac{kqx}{(R^2 + x^2)^{3/2}}$ $k, q, R = \text{constant}$
 at what of x , y is
 Maxⁿ or minⁿ | what is maxⁿ & minⁿ
 value of y

$$E = \frac{kqx}{(R^2 + x^2)^{3/2}} \quad \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dE}{dx} = 0 = kq \left[\frac{(R^2 + x^2)^{3/2} - x \cdot 3(R^2 + x^2)^{1/2} \cdot 2x}{(R^2 + x^2)^3} \right]$$

$$(R^2 + x^2)^{3/2} - 6x^2(R^2 + x^2)^{1/2} = 0$$

$$(R^2 + x^2)(R^2 + x^2)^{1/2} = 3x^2(R^2 + x^2)^{1/2}$$

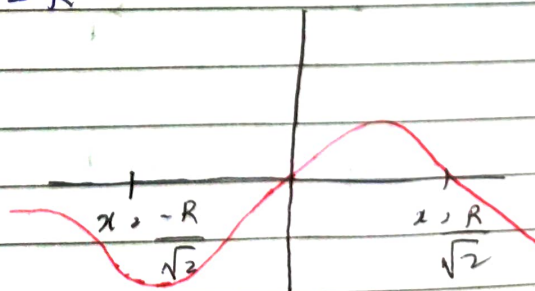
$$R^2 + x^2 = 3x^2 \rightarrow R^2 = 2x^2$$

$$x = \pm \frac{R}{\sqrt{2}}$$

$$E_{x=\frac{R}{\sqrt{2}}} = \frac{kqR}{\sqrt{2}} \cdot \frac{1}{\left(R^2 + \frac{R^2}{2}\right)^{3/2}} = \frac{kqR}{\sqrt{2} \cdot \frac{3R^2}{2} \cdot \sqrt{\frac{3R^2}{2}}}$$

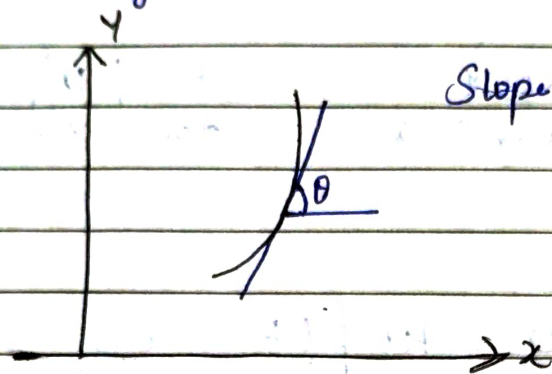
$$= \frac{kqR}{\sqrt{2} \cdot \frac{3R^2}{2} \cdot \frac{\sqrt{3}R}{\sqrt{2}}} = \frac{2kq}{3\sqrt{3}R^2}$$

$$E_{x=-\frac{R}{\sqrt{2}}} = \frac{-R}{\sqrt{2}} = \frac{-2kq}{3\sqrt{2}R^2}$$



Application of differentiation in physics

$\frac{dy}{dx} \rightarrow$ change in y with respect to change in x



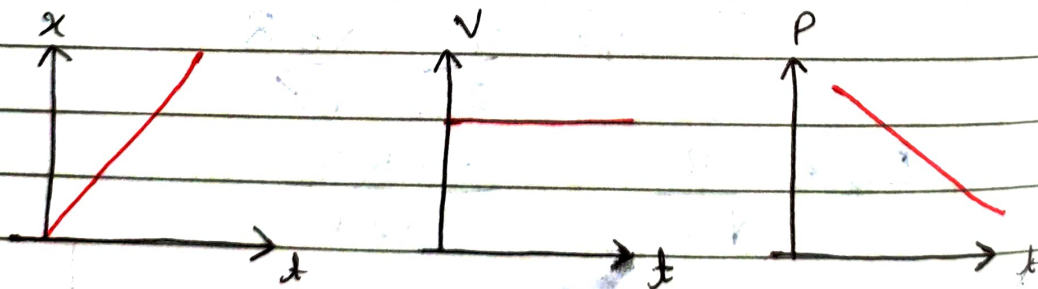
Slope $\equiv \tan \theta = \frac{dy}{dx}$

$v = \frac{dx}{dt}$ change in position wrt change in time $\int dx = \int v dt$

$a = \frac{dv}{dt}$ change in velocity wrt change in time

$f = \frac{dp}{dt}$ change in momentum wrt change in time. (P)

$i = \frac{dq}{dt}$ change in charge wrt change in time



Slope :-

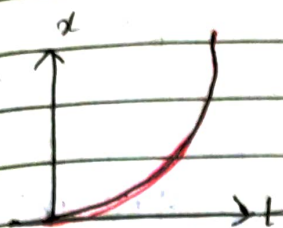
Velocity :- Constant & +ve

Slope :-

Acceleration constant & zero

Slope :- For

Constant & Negative



→ Velocity is increasing

→ Acceleration is positive

A particle is moving in a straight line, with its velocity as function of time is given as $V = 3t^2 - 2t + 1$. Find the acceleration of the particle at $t = 2$ sec.

$$\begin{aligned} \rightarrow V &= 3t^2 - 2t + 1 & 6t - 2 + 1 \\ V &= 6t - 2t + 1 & 12 - 10 + 1 \\ &= 6(2) - 2(2) & \\ &= 12 - 2 & \\ &= 10 & \end{aligned}$$

A particle is moving in a straight line the variation of position x as a function of time (t) is given as $x = t^3 - 6t^2 + 20t + 15$

The velocity of the particle when its acceleration becomes zero

$$\begin{aligned} \rightarrow V &= \frac{dx}{dt} = 3t^2 - 12t + 20 & 3t - 12t + 20t \\ a &= \frac{dv}{dt} = 6t - 12 = 0 & 2 \text{ sec} & 11t \end{aligned}$$

$$3(2) - 12 + 20 =$$

$$3(2)^2 - 12(2) + 20$$

$$3(4) - 24 + 20$$

$$12 - 24 + 20 \rightarrow -24 + 32 \rightarrow 8$$

$$a = \frac{dv}{dt} \times \frac{dx}{dx} = \frac{dx}{dt} \cdot \frac{dv}{dx} = \frac{v dv}{dx}$$

A particle moving in a straight line, its velocity varies $V = 4\sqrt{x}$. Find the acceleration of the particle at $t = 2$ sec.

$$\rightarrow \frac{dv}{dx} = 4\sqrt{x} = 4 \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

$$a = v \frac{dv}{dx} = \frac{2}{\sqrt{x}}$$

$$= 4\sqrt{x} \cdot \frac{2}{\sqrt{x}} = 8 \text{ m/s}$$

Area of circle $A = \pi R^2$

$$\frac{dA}{dR} = \pi \cdot 2R = 2\pi R$$

$$\frac{dA}{dt} = 2\pi R \frac{dR}{dt} = \frac{dA}{dt} = 2\pi R \frac{dR}{dt}$$

Volume of a sphere $V = \frac{4}{3}\pi R^3$

$$\frac{dV}{dR} = \frac{4}{3} \pi 3R^2 = 4\pi R^2$$

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$$

$$R = f(t)$$

$$\frac{dV}{dt} = \frac{4}{3} \pi 3R^2 \frac{dR}{dt}$$

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt} \quad \leftarrow \frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$$

$$V = \frac{4}{3} \pi R^3$$

$$\frac{dV}{dt} = \frac{4}{3} \pi 3R^2 \frac{dR}{dt}$$

Find the rate of change of area of a circle at the instant it's radius is $\frac{2}{5}m$ & the radius is changing with time as $\frac{dR}{dt} = 5$.

$$\frac{dA}{dt} = \pi R \frac{dR}{dt}$$

$$= \pi \times \frac{2}{5} \times 5 = 2\pi \text{ m}^2/\text{sec}$$

Find the Speed of other end of ladder at this inst

$$\frac{dy}{dt} = -v_B$$

$$v_A = \frac{dx}{dt}$$

NLM

$$x^2 + y^2 = l^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

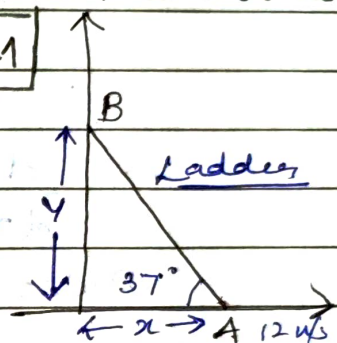
$$2x v_A + 2y (-v_B) = 0$$

$$v_B = \frac{x}{y} v_A$$

$$\cot 37^\circ \cdot v_A$$

$$= \frac{4}{3} \times 12$$

$$= 16 \text{ m/s}$$



$\frac{d\theta}{dt} = 0.14 \text{ rad/sec}$, Find v_B when $\theta = 45^\circ$

$$\tan \theta = \frac{y}{x}$$

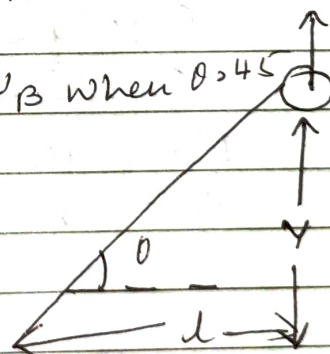
$$y = x \tan \theta$$

$$\frac{dy}{dx} = \sec^2 \theta$$

$$\frac{dy}{dt} = \sec^2 \theta \frac{dx}{dt}$$

$$v_B = 100 \times \sec^2 45^\circ \cdot 0.14$$

$$\theta = 45^\circ = 14 \times 2 = 28 \text{ m/s}$$



Integration.

→ It is reverse of differentiation

$$\rightarrow y = f(x)$$

$$\rightarrow \frac{dy}{dx} = f'(x)$$

$$\rightarrow \int f'(x) dx = \int dy$$

$$\rightarrow \int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\rightarrow \frac{(n+1)x^n}{n+1} = x^n$$

$$\rightarrow \frac{d}{dx} \left(\frac{x^{n+1}}{n+1} \right) = x^n$$

Rules :- 1. Summation / Subtraction.

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

11. Constant Multiple [K]

$$\int K(f(x)) dx = K \int f(x) dx$$

Indefinite Integration

Formulas

$$1) \int x^n dx = \frac{x^{n+1}}{n+1} + C \rightarrow \text{Constant}$$

$$\boxed{n \neq -1}$$
$$\boxed{\int dx = x}$$

Exan

$$\int x^2 dx = \frac{x^3}{3}$$

$$\int \sqrt{x} dx = \frac{x^{3/2}}{3/2} = \frac{2}{3} x^{3/2}$$

$$\int \frac{dx}{\sqrt{x}} = \int x^{-1/2} dx = \int \frac{x^{1/2}}{\frac{1}{2}} = 2x^{1/2}$$

$\downarrow$
 $2\sqrt{x}$

Q2) $\int \frac{1}{x} dx = \int \frac{dx}{x} = \ln x$ $\frac{1}{2} x$

Ex $\int \left(\frac{1}{x^2} - \frac{1}{x} + 1 \right) dx$

$(-x^{-2} + x^{-1} + 1) dx$ $-x^{-2} - x^{-1} + 1$
 $-2x^{-1} + x^0 + 1$ $-x^{-2+1} - x^{-1+1} + 1$
 $-x^{-1} + 1$ $\frac{-x^{-1} - x^0 + 1}{3}$

$\frac{x^2}{2} - \frac{1}{x}$

$\frac{x^3}{3} + x$

Sol $\int \left(\frac{1}{x^2} - \frac{1}{x} + 1 \right) dx$ $\int x^0 dx = \frac{x^{0+1}}{0+1} = x$

$= \int x^{-2} dx - \int \frac{dx}{x} + \int dx$

$= \int \frac{x^{-1}}{-1} dx - \ln x + x$

3 $\int e^x dx = e^x$

4 $\int \sin x dx = -\cos x$

5 $\int \cos x dx = \sin x$

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Substitution Rule

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}$$

Examples:- $\int \sqrt{2x-1} dx = \frac{(2x-1)^{3/2}}{3/2 \times 2} = \frac{2x-1}{3}^{3/2}$

$$\int \frac{dx}{1-x} = \frac{\ln(1-x)}{-1} = - \int \frac{dx}{x} = -\ln x$$

$$\int \sin(2x+1) dx = \frac{-\cos x(2x+1)}{2}$$

$$\int e^{-2x} dx = \frac{e^{-2x}}{-2}$$

$$\int (\sqrt{x-1}) - \frac{1}{\sqrt{x+1}} + \sin(2x-1) - e^{2x} dx$$

$$\frac{x^{3/2}-1}{3/2} - \frac{1}{-x^{1/2}+1} - \frac{\cos(2x-1)}{2} - \frac{e^{2x}}{2}$$

$$\frac{2}{3} x = 1^{3/2} = \frac{(1-x)^{1/2}}{\frac{1}{2}(-1)} - \frac{\cos(2x-1)}{2} + \frac{e^{2x}}{2}$$

$$= -2((1-x)^{1/2})$$

$$\int \frac{6}{2x+5} dx = 6 \int \frac{dx}{2x+5} = \frac{6 \ln 2x+5}{2} = 3 \ln 2x+5$$

$$6 \int \sec^2 x dx = \tan x \quad \frac{(3x+1)^4}{3(4)}$$

$$7 \int \csc^2 x dx = -\cot x \quad \frac{3x+1^4}{12} + c$$

$$8 \int \sec x \tan x dx = \sec x$$

$$9 \int \csc x \cot x dx = -\csc x$$

$$5a \int \cos^2 x dx =$$

$$\cos 2x = 2\cos^2 x - 1$$

$$2\cos^2 x = 1 + \cos 2x$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\int \left(\frac{1 + \cos 2x}{2} \right) dx = \frac{1}{2} \left[\int 1 dx + \int \cos 2x dx \right]$$

$$= \frac{1}{2} \left[x + \frac{\sin 2x}{2} \right]$$

Definite Integration

$$y = f(x)$$

$\int y dx \rightarrow$ Area under $y-x$ curve

$x = x_2 \rightarrow$ Higher limit

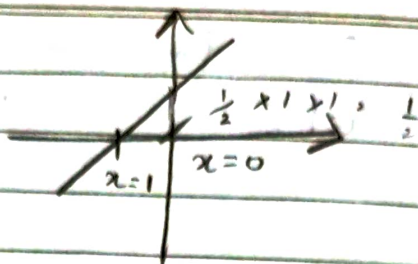
$$\int f(x) dx = [g(x)]$$

$x = x_1 \rightarrow$ Lower limit

$$g(x_2) - g(x_1)$$

$$y = x + 1$$

$$\int_{x=-1}^{x=0} y \, dx$$



$$x=0$$

$$= \int (x+1) \, dx$$

$$x=-1$$

$$= \int_{-1}^0 x \, dx + \int_{-1}^0 1 \, dx = \left[\frac{x^2}{2} \right]_{-1}^0 + \left[x \right]_{-1}^0$$

$$= \left[\frac{x^2}{2} \right]_{-1}^0 + \left[x \right]_{-1}^0$$

$$= -\frac{1}{2} [(1) + (1)]$$

$$= -\frac{1}{2} + 1$$

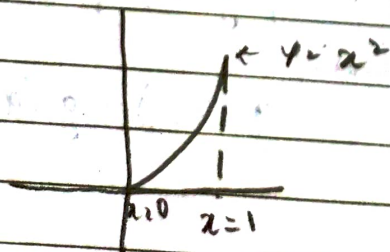
$$\rightarrow \frac{1}{2}$$

Find area enclosed b/w $y = x^2$ & x -axis with x varies from $x=0$ to 1

$$\int_0^1 x \, dx + \int_0^1 1 \, dx = \left[\frac{x^2}{2} \right]_0^1 + \left[x \right]_0^1$$

$$= \frac{1}{2} [(1) + (1)]$$

$$\int y \, dx = \int x^2 \, dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} [1^3 - 0^3] = \frac{1}{3}$$



Find area enclosed b/w $y=x^2$ with y-axis $x \in [0, 1]$ $y=1$

$$\rightarrow \int_{y=0}^1 x dy = \int_{y=0}^1 \sqrt{y} dy = \left[y^{3/2} \right]_0^1 = \frac{3}{2}$$

$$= \frac{2}{3} \left[1^{2/3} - 0^{2/3} \right] = \frac{2}{3} u$$

$$\int \sqrt{1 + \sin x} dx$$

$$\rightarrow \sqrt{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}$$

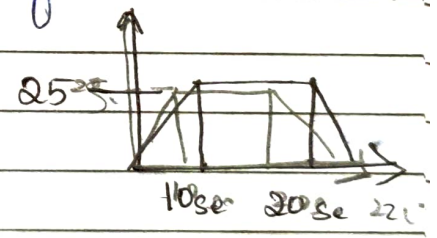
$$\sqrt{\left(\sin \frac{x}{2} \right)^2 + \left(\cos \frac{x}{2} \right)^2 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}$$

$\rightarrow$

The velocity time graph of a car moving in a straight line is shown in fig. The displacement of the car in first 5 sec is

$$\frac{1}{2} \times [25 + 10] \times 20$$

$$\frac{1}{2} \times 35 \times 20 = 350m$$



Area under a curve.

$$v = \frac{dx}{dt} \quad dx = \int v dt$$

$$a = \frac{dv}{dt} \quad \int dx = \int a dt$$

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To calculate average value

$$\left[\begin{array}{l} v_{avg} = \frac{\Delta x}{\Delta t} \\ \rightarrow \text{Velocity} \end{array} \right.$$

$$u = \frac{dx}{dt}$$

Acceleration

$$\left[\begin{array}{l} v_{avg} = \frac{\int v dt}{\int dt} \end{array} \right.$$

$$\int dx = \int v dt$$

$$\Delta x = \int v dt$$

$$\left[\begin{array}{l} I_{avg} = \frac{\int i dt}{\int dt} \end{array} \right.$$

Current

$$\left[\begin{array}{l} f_{avg} = \frac{\int F dt}{\int dt} \\ \rightarrow \text{Force} \end{array} \right.$$

Power of Fundamental physical quantity is called dimensions.

$$[Force] = [MLT^{-2}]$$

Dimension of Mass in force = 1

length = 1

Time = -2

Current = 0