

Question 1

If two vertices of a triangle are $A(3, 1, 4)$ and $B(-4, 5, -3)$ and the centroid of the triangle is $G(-1, 2, 1)$, then the third vertex C of the triangle is

Options:

A. $(2, 0, 2)$

B. $(-2, 0, 2)$

C. $(0, -2, 2)$

D. $(2, -2, 0)$

Answer: B

Solution:

Let $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{g}$ be the position vectors of A, B, C and G respectively.

$$\bar{a} = 3\hat{i} + 1\hat{j} + 4\hat{k},$$

$$\bar{b} = -4\hat{i} + 5\hat{j} - 3\hat{k},$$

$$\bar{g} = -\hat{i} + 2\hat{j} + \hat{k},$$

G is centroid of $\triangle ABC$.

$$\therefore \bar{g} = \frac{\bar{a} + \bar{b} + \bar{c}}{3}$$

$$3\bar{g} = \bar{a} + \bar{b} + \bar{c}$$

$$3(-\hat{i} + 2\hat{j} + \hat{k}) = 3\hat{i} + \hat{j} + 4\hat{k} - 4\hat{i} + 5\hat{j} - 3\hat{k} + \bar{c}$$

$$\begin{aligned}\therefore \bar{c} &= -3\hat{i} + 6\hat{j} + 3\hat{k} - 3\hat{i} - \hat{j} - 4\hat{k} + 4\hat{i} - 5\hat{j} + 3\hat{k} \\ &= -2\hat{i} + 0\hat{j} + 2\hat{k}\end{aligned}$$

$$\therefore \text{Third vertex } C \equiv (-2, 0, 2)$$

Question 2

In a Binomial distribution with $n = 4$, if $2P(X = 3) = 3P(X = 2)$, then the variance is

Options:

A. $\frac{36}{169}$

B. $\frac{144}{169}$

C. $\frac{9}{169}$

D. $\frac{16}{169}$

Answer: B

Solution:

$$P(X = 3) = {}^4C_3 p^3 (1 - p) = 4p^3 (1 - p)$$

$$P(X = 2) = {}^4C_2 p^2 (1 - p)^2 = 6p^2 (1 - p)^2$$

$$\text{Given } 2P(X = 3) = 3P(X = 2)$$

$$\therefore 8p^3 (1 - p) = 18p^2 (1 - p)^2$$

$$\therefore 8p = 18(1 - p)$$

$$\therefore p = \frac{9}{13}$$

$$\text{Variance} = np(1 - p)$$

$$= 4 \times \frac{9}{13} \left(1 - \frac{9}{13} \right)$$

$$= \frac{144}{169}$$

Question 3

Let two non-collinear vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves, so that at any time t the position vector $\overrightarrow{OP}$, where O is origin,

is given by $\hat{a} \sin t + \hat{b} \cos t$, when P is farthest from origin O , let M be the length of $\overline{OP}$ and $\hat{u}$ be the unit vector along $\overline{OP}$, then

Options:

A. $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = (1 + \hat{a} \cdot \hat{b})^{\frac{1}{2}}$

B. $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = (1 + \hat{a} \cdot \hat{b})^{\frac{1}{2}}$

C. $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = (1 + 2\hat{a} \cdot \hat{b})^{\frac{1}{2}}$

D. $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = (1 - 2\hat{a} \cdot \hat{b})^{\frac{1}{2}}$

Answer: A

Solution:

$$M = |\overrightarrow{OP}|$$

$$\begin{aligned} M &= \sqrt{(\hat{a} \sin t + \hat{b} \cos t)^2} \\ &= \sqrt{(\hat{a} \sin t)^2 + (\hat{b} \cos t)^2 + 2(\hat{a} \sin t) \cdot (\hat{b} \cos t)} \\ &= \sqrt{\sin^2 t + \cos^2 t + \hat{a} \cdot \hat{b}(2 \sin t \cos t)} \\ &= \sqrt{1 + \hat{a} \cdot \hat{b}(\sin 2t)} \end{aligned}$$

Maximum value of $\sin 2t = 1$

$$\therefore 2t = \sin^{-1}(1)$$

$$\therefore t = \frac{\pi}{4}$$

$$\begin{aligned} \therefore M &= \sqrt{1 + \hat{a} \cdot \hat{b}(1)} \\ &= (1 + \hat{a} \cdot \hat{b})^{\frac{1}{2}} \end{aligned}$$

$$\text{Now, } \hat{u} = \frac{\overline{OP}}{|\overline{OP}|}$$

$$\begin{aligned} &= \frac{\hat{a} \sin t + \hat{b} \cos t}{|\hat{a} \sin t + \hat{b} \cos t|} \\ &= \frac{\hat{a} \left(\frac{1}{\sqrt{2}} \right) + \hat{b} \left(\frac{1}{\sqrt{2}} \right)}{\left| \hat{a} \left(\frac{1}{\sqrt{2}} \right) + \hat{b} \left(\frac{1}{\sqrt{2}} \right) \right|} \end{aligned}$$

Unit vector of OP is

$$\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$$

Question 4

The number of solutions in $[0, 2\pi]$ of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is

Options:

A. 2

B. 4

C. 6

D. 8

Answer: D

Solution:

$$16^{\sin^2 x} + 16^{\cos^2 x} = 10$$

$$16^{\sin^2 x} + 16^{1 - \sin^2 x} = 10$$

$$16^{\sin^2 x} + \frac{16}{16^{\sin^2 x}} = 10$$

$$\text{Let } 16^{\sin^2 x} = t$$

$$\therefore t + \frac{16}{t} = 10$$

$$\therefore t^2 - 10t + 16 = 0$$

$$\Rightarrow t = 2 \text{ and } t = 8$$

$$\text{Now, } 16^{\sin^2 x} = 2 \text{ and } 16^{\sin^2 x} = 8$$

$$2^{4 \sin^2 x} = 2^1 \text{ and } 2^{4 \sin^2 x} = 2^3$$

$$\therefore 4 \sin^2 x = 1 \text{ and } 4 \sin^2 x = 3$$

$$\therefore \sin^2 x = \frac{1}{4} \text{ and } \sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \frac{1}{2} \text{ and } \sin x = \pm \frac{\sqrt{3}}{2}$$

$$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \text{ and } x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\therefore \text{ number of solutions} = 8.$$

Question 5

The differential equation of all parabolas, whose axes are parallel to Y-axis, is

Options:

A. $y_3 = 1$

B. $y_3 = 0$

C. $y_3 = -1$

D. $yy_3 + y_1 = 0$

Answer: B

Solution:

Parabola whose axes are parallel to Y-axis. Vertex is not $(0, 0)$

Equation becomes

$$(x - h)^2 = 4b(y - k)$$

Differentiating w.r.t. x , we get

$$2(x - h) = 4b \left(\frac{dy}{dx} \right)$$

Again differentiating w.r.t. x , we get

$$2 = 4b \left(\frac{d^2y}{dx^2} \right)$$

Again differentiating w.r.t. x , we get

$$0 = 4b \left(\frac{d^3 y}{dx^3} \right)$$

$$\therefore \frac{d^3 y}{dx^3} = 0$$

i.e., $y_3 = 0$

Question 6

The value of c of Lagrange's mean value theorem for $f(x) = \sqrt{25 - x^2}$ on $[1, 5]$ is

Options:

A. $\sqrt{15}$

B. 5

C. $\sqrt{10}$

D. 1

Answer: A

Solution:

$$f(x) = \sqrt{25 - x^2}$$

$$\therefore f'(x = c) = \frac{-2c}{2\sqrt{25 - c^2}}$$

$$= \frac{-c}{\sqrt{25 - c^2}}$$

Applying Lagrange's mean value theorem, we get

$$f'(c) = \frac{f(1) - f(5)}{1 - 5}$$

$$\therefore \frac{c}{\sqrt{25 - c^2}} = \frac{\sqrt{25 - 1} - \sqrt{25 - 5^2}}{1 - 5}$$

$$\therefore \frac{-c}{\sqrt{25 - c^2}} = \frac{-\sqrt{24}}{4}$$

$$\therefore 4c = \sqrt{24} \cdot \sqrt{25 - c^2}$$

$$\therefore 16c^2 = 24(25 - c^2)$$

$$\therefore c^2 = 15$$

$$\therefore c = \pm\sqrt{15}$$

Since $c = -\sqrt{15}$ does not lie in $[1, 5]$

$$c = \sqrt{15}$$

Question 7

$$\int \frac{x+1}{x(1+xe^x)^2} dx =$$

Options:

A. $\log \left| \frac{xe^x}{1+xe^x} \right| + c$, where c is a constant of integration.

B. $\log \left| \frac{xe^x}{1+xe^x} \right| - \frac{1}{1+xe^x} + c$, where c is a constant of integration.

C. $\log |1 + xe^x| + \frac{1}{1+xe^x} + c$, where c is constant of integration.

D. $\log \left| \frac{xe^x}{1+xe^x} \right| + \frac{1}{1+xe^x} + c$, where c is constant of integration.

Answer: D

Solution:

$$\text{Let } I = \int \frac{x+1}{x(1+xe^x)^2} dx = \int \frac{e'(x+1)}{e^x \cdot x(1+xe^x)^2} dx$$

$$\text{Let } x \cdot e^x = t$$

$$\Rightarrow (x+1)e^x dx = dt$$

$$\therefore I = \int \frac{dt}{t(1+t)^2}$$

$$= \int \frac{1+t-t}{t(1+t)^2} dt$$

$$= \int \frac{1dt}{t(1+t)} - \int \frac{1}{(1+t)^2} dt$$

$$= \int \frac{1+t-t}{t(1+t)} - \int \frac{1}{(1+t)^2} dt$$

$$= \int \frac{1}{t} dt - \int \frac{1}{(t+1)} dt - \int \frac{1}{(1+t)^2} dt$$

$$= \log t - \int \frac{dt}{(t+1)} - \int \frac{1}{(1+t)^2} ddt + c$$

$$\text{Let } y = 1+t$$

$$\Rightarrow dy = dt$$

$$\therefore I = \log t - \int \frac{1}{y} dy - \int \frac{1}{y^2} dy + c$$

$$= \log t - \log y + \frac{1}{y} + c$$

$$= \log t - \log(1+t) + \frac{1}{(1+t)} + c$$

$$= \log xe^x - \log(1+xe^x) + \frac{1}{(1+xe^x)} + c$$

$$\therefore I = \log \left| \frac{xe^x}{1+xe^x} \right| + \frac{1}{1+xe^x} + c$$

Question 8

If $Z_1 = 4i^{40} - 5i^{35} + 6i^{17} + 2$, $Z_2 = -1 + i$, where $i = \sqrt{-1}$, then $|Z_1 + Z_2| =$

Options:

A. 5

B. 13

C. 12

D. 15

Answer: B

Solution:

$$Z_1 = 4i^{40} - 5i^{35} + 6i^{17} + 2$$

$$Z_1 = 6 + 11i$$

$$Z_2 = -1 + i$$

$$\begin{aligned}\therefore Z_1 + Z_2 &= 6 + 11i - 1 + i \\ &= 5 + 12i\end{aligned}$$

$$\therefore |Z_1 + Z_2| = \sqrt{(5)^2 + (12)^2} = \sqrt{169} = 13$$

Question 9

The approximate value of $\log_{10} 998$ is (given that $\log_{10} e = 0.4343$)

Options:

A. 3.0008686

B. 1.9991314

C. 2.0008686

D. 2.9991314

Answer: D

Solution:

$$\begin{aligned}\text{Let } f(x) = \log_{10} x &= \frac{\log_e x}{\log_e 10} = (\log_{10} e) (\log_e x) \\ &= 0.4343 (\log_e x)\end{aligned}$$

On differentiating w.r.t. x , we get

$$f'(x) = \frac{0.4343}{x}$$

$$\text{Let } x = 998$$

$$= 1000 - 2 = a + h$$

$$\therefore a = 1000, h = -2$$

$$f(a) = f(1000)$$

$$= \log_{10}(1000)$$

$$= 3 \log_{10} 10$$

$$\therefore f(a) = 3$$

$$\text{Also, } f'(a) = f'(1000) = \frac{0.4343}{1000} = 0.0004343$$

$$f(a + h) \approx f(a) + hf'(a)$$

$$\therefore \log_{10}(998) \approx 3 - 2(0.0004343)$$

$$\approx 2.9991314$$

Question 10

If $\sin^{-1} x + \cos^{-1} y = \frac{3\pi}{10}$, then the value of $\cos^{-1} x + \sin^{-1} y$ is

Options:

A. $\frac{\pi}{10}$

B. $\frac{7\pi}{10}$

C. $\frac{9\pi}{10}$

D. $\frac{3\pi}{10}$

Answer: B

Solution:

$$\sin^{-1} x + \cos^{-1} y = \frac{3\pi}{10}$$

$$\therefore \frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \sin^{-1} y = \frac{3\pi}{10}$$

$$\therefore \pi - \cos^{-1} x - \sin^{-1} y = \frac{3\pi}{10}$$

$$\therefore \cos^{-1} x + \sin^{-1} y = \pi - \frac{3\pi}{10} = \frac{7\pi}{10}$$

Question 11

The area (in sq. units) of the region $A = \left\{ (x, y) / \frac{y^2}{2} \leq x \leq y + 4 \right\}$ is

Options:

A. 30

B. $\frac{53}{3}$

C. 16

D. 18

Answer: D

Solution:

Given that $\frac{y^2}{2} \leq x \leq y + 4$

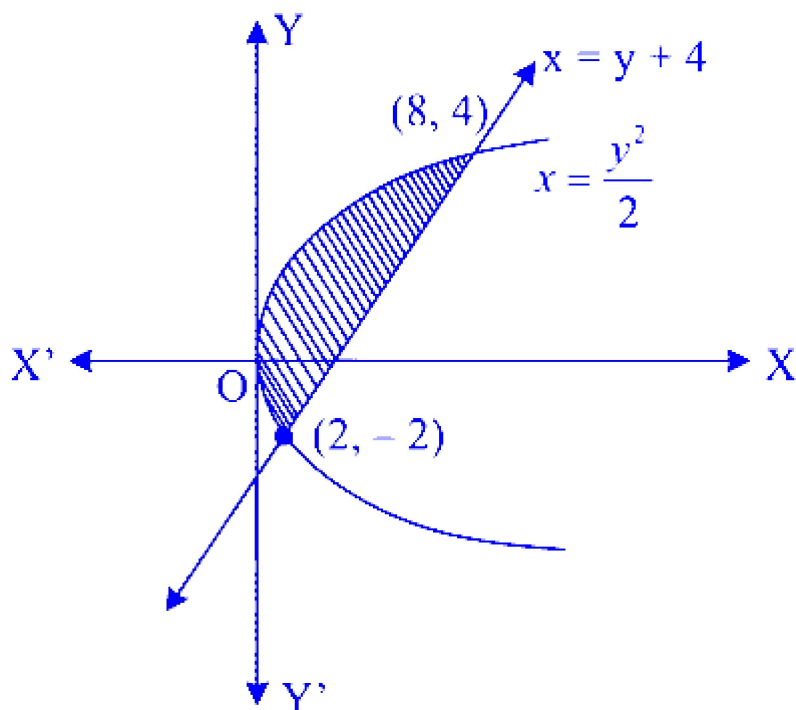
$\therefore x = \frac{y^2}{2}$ and $x = y + 4$

$$\frac{y^2}{2} = y + 4$$

$\therefore y^2 - 2y - 8 = 0$

$\therefore y = 4$ or -2

$\Rightarrow x = 8$ or 2



$$\therefore A = \int_{-2}^4 \left(y + 4 - \frac{y^2}{2} \right) dy$$

$$\therefore A = \left[\frac{y^2}{2} + 4y - \frac{y^3}{6} \right]_{-2}^4$$

$$\therefore A = \left(8 + 16 - \frac{64}{6} \right) - \left(2 - 8 + \frac{8}{6} \right)$$

$$\therefore A = 18$$

Question 12

A, B, C are three events, one of which must and only one can happen. The odds in favor of A are 4 : 6, the odds against B are 7 : 3. Thus, odds against C are

Options:

A. 7 : 3

B. 4 : 6

C. 6 : 4

D. 3 : 7

Answer: A

Solution:

Odd in favor of A is 4 : 6.

$$\therefore P(A) = \frac{4}{10}$$

Odd against B is 7 : 3

$$\therefore P(B) = \frac{3}{10}$$

Since only one of the events A, B and C can happen, we get

$$P(A) + P(B) + P(C) = 1$$

$$\therefore \frac{4}{10} + \frac{3}{10} + P(C) = 1$$

$$\therefore P(C) = \frac{3}{10}$$

$$\therefore P(C') = \frac{7}{10}$$

∴ odds against the event C are $P(C') : P(C)$

$$\begin{aligned} &= \frac{7}{10} : \frac{3}{10} \\ &= 7 : 3 \end{aligned}$$

Question 13

The value of α , so that the volume of the parallelopiped formed by $\hat{i} + \alpha\hat{j} + \hat{k}, \hat{j} + \alpha\hat{k}$ and $\alpha\hat{i} + \hat{k}$ becomes maximum, is

Options:

A. $\frac{-1}{\sqrt{3}}$

B. $\frac{1}{\sqrt{3}}$

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: A

Solution:

Volume of parallelopiped is $[\vec{a}\vec{b}\vec{c}]$

$$\begin{aligned} \therefore V &= \begin{vmatrix} 1 & \alpha & 1 \\ 0 & 1 & \alpha \\ \alpha & 0 & 1 \end{vmatrix} \\ &= 1 - \alpha(-\alpha^2) - \alpha \\ &= 1 + \alpha^3 - \alpha \end{aligned}$$

Differentiating w.r.t. α , we get

$$\frac{dV}{d\alpha} = 3\alpha^2 - 1$$

$$\therefore \frac{d^2V}{d\alpha^2} = 6\alpha$$

$$\text{Let } \frac{dV}{d\alpha} = 0$$

$$\therefore 3\alpha^2 - 1 = 0$$

$$\therefore \alpha = \pm \frac{1}{\sqrt{3}}$$

$$\text{at } \alpha = \frac{-1}{\sqrt{3}},$$

$$\frac{d^2V}{d\alpha^2} = \frac{-6}{\sqrt{3}} < 0$$

$$\therefore V \text{ is maximum at } \alpha = \frac{-1}{\sqrt{3}}$$

Question 14

Two sides of a triangle are $\sqrt{3} + 1$ and $\sqrt{3} - 1$ and the included angle is 60° , then the difference of the remaining angles is

Options:

A. 30°

B. 45°

C. 60°

D. 90°

Answer: D

Solution:

$$\text{Let } a = \sqrt{3} + 1, b = \sqrt{3} - 1, C = 60^\circ$$

Using cosine Rule,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = (\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2 - 2(\sqrt{3} + 1)(\sqrt{3} - 1) \frac{1}{2}$$

$$c^2 = 6$$

$$\therefore c^2 = 6$$

$$c = \sqrt{6}$$

Using sine Rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Consider

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\therefore \frac{\sqrt{3}-1}{\sin B} = \frac{\frac{\sqrt{6}}{2}}{\frac{\sqrt{3}}{2}}$$

$$\therefore \sin B = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\therefore \angle B = 15^\circ$$

$$\therefore \angle A = 105^\circ$$

.... [Remaining angle of a Triangle]

$$\therefore \text{Difference} = 90^\circ$$

Question 15

The standard deviation of the following distribution

C. I.	0 - 6	6 - 12	12 - 18
f_i	2	4	6

is

Options:

A. $5\sqrt{2}$

B. $\sqrt{5}$

C. $2\sqrt{5}$

D. 20

Answer: C

Solution:

C. I.	f_i	x_i	x_i^2	$f_i x_i$	$f_i x_i^2$
0 - 6	2	3	9	6	18
6 - 12	4	9	81	36	324
12 - 18	6	15	225	90	1350
Total	12			132	1692

Here $\sum f_i = 12$, $\sum f_i x_i = 132$, $\sum f_i x_i^2 = 1692$

$$\begin{aligned}\therefore V(X) &= \frac{1692}{12} - \left(\frac{132}{12} \right)^2 \\ &= 141 - 121 \\ &= 20\end{aligned}$$

$$\therefore \text{Standard deviation} = \sqrt{20} = 2\sqrt{5}$$

Question 16

The maximum value of xy when $x + 2y = 8$ is

Options:

- A. 20
- B. 16
- C. 24
- D. 8

Answer: D

Solution:

$$\begin{aligned}x + 2y &= 8 \\ \therefore 2y &= 8 - x \\ \therefore y &= \frac{8 - x}{2}\end{aligned}$$

Let $f(x) = xy$

$$\therefore f(x) = x \cdot \frac{(8-x)}{2}$$

Differentiating w.r.t x , we get

$$f'(x) = \frac{(8-x) - x}{2}$$

$$f'(x) = 4 - x$$

To find critical points,

$$f'(x) = 0$$

$$\therefore 4 - x = 0$$

$$\therefore x = 4$$

critical point at $x = 4$

$$\therefore f(4) = \frac{4(8-4)}{2} = 8$$

$\therefore$ Maximum value of the given function is 8.

Question 17

If truth values of statements p , q are true, and r , s are false, then the truth values of the following statement patterns are respectively

$$a : \sim (p \wedge \sim r) \vee (\sim q \vee s)$$

$$b : (\sim q \wedge \sim r) \leftrightarrow (p \vee s)$$

$$c : (\sim p \vee q) \rightarrow (r \wedge \sim s)$$

Options:

A. T, F, F

B. F, F, F

C. F, T, T

D. T, F, T

Answer: B

Solution:

$$\begin{aligned}
 \text{a. } & \sim (p \wedge \sim r) \vee (\sim q \vee s) \\
 & \equiv \sim (T \wedge \sim F) \vee (\sim T \vee F) \\
 & \equiv (F \vee F) \vee (F \vee F) \\
 & \equiv F \vee F \\
 & \equiv F
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } & (\sim q \wedge \sim r) \leftrightarrow (p \vee s) \\
 & \equiv (\sim T \wedge \sim F) \leftrightarrow (T \vee F) \\
 & \equiv F \leftrightarrow T \\
 & \equiv F
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } & (\sim p \vee q) \rightarrow (r \wedge \sim s) \\
 & \equiv (\sim T \vee T) \rightarrow (F \wedge \sim F) \\
 & \equiv (F \vee T) \rightarrow (F \wedge T) \\
 & \equiv T \rightarrow F \\
 & \equiv F
 \end{aligned}$$

Question 18

The rate of change of $\sqrt{x^2 + 16}$ with respect to $\frac{x}{x-1}$ at $x = 5$ is

Options:

A. $\frac{-80}{\sqrt{41}}$

B. $\frac{80}{\sqrt{41}}$

C. $\frac{12}{5}$

D. $\frac{-12}{5}$

Answer: A

Solution:

$$y = \sqrt{x^2 + 16}$$

$$\therefore \frac{dy}{dx} = \frac{2x}{2\sqrt{x^2+16}}$$

$$\therefore \frac{dy}{dx} = \frac{x}{\sqrt{x^2+16}} \quad \dots (i)$$

$$\text{Let } z = \frac{x}{x-1}$$

$$\therefore \frac{dz}{dx} = \frac{(x-1)-x}{(x-1)^2}$$

$$\therefore \frac{dz}{dx} = \frac{-1}{(x-1)^2} \quad \dots (ii)$$

$$\begin{aligned} \therefore \left(\frac{dy}{dz} \right)_{x=5} &= \frac{\frac{x}{\sqrt{x^2+16}}}{\frac{-1}{(x-1)^2}} \\ &= \frac{-5}{\sqrt{25+16}} \times 16 = \frac{-80}{\sqrt{41}} \end{aligned}$$

Question 19

If $x^2 + y^2 = t + \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $\frac{dy}{dx}$ is equal to

Options:

A. $\frac{y}{x}$

B. $\frac{-y}{x}$

C. $\frac{x}{y}$

D. $\frac{-x}{y}$

Answer: B

Solution:

$$x^2 + y^2 = t + \frac{1}{t}$$

Squaring on both sides, we get

$$x^4 + y^4 + 2x^2y^2 = t^2 + \frac{1}{t^2} + 2$$

$$\therefore t^2 + \frac{1}{t^2} + 2x^2y^2 = t^2 + \frac{1}{t^2} + 2 \quad \dots [\because x^4 + y^4 = t^2 + \frac{1}{t^2}, \text{given}]$$

$$2x^2y^2 = 2$$

$$x^2y^2 = 1$$

differentiating w.r.t. x , we get

$$x^2 2y \frac{dy}{dx} + 2xy^2 = 0$$

$$x^2 2y \frac{dy}{dx} = -2xy^2$$

$$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y} = \frac{-y}{x}$$

Question 20

Two tangents to the circle $x^2 + y^2 = 4$ at the points A and B meet at the point P(−4, 0). Then the area of the quadrilateral PAOB, O being the origin, is

Options:

A. $2\sqrt{3}$ sq. units

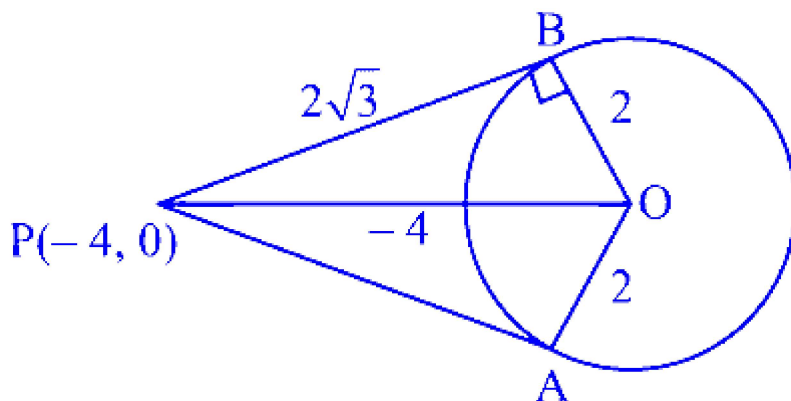
B. $8\sqrt{3}$ sq. units

C. $4\sqrt{3}$ sq. units

D. $6\sqrt{3}$ sq. units

Answer: C

Solution:



Required area = $2 \times$ Area of $\triangle PBO$

$$= 2 \times \frac{1}{2} \times 2 \times 2\sqrt{3}$$

$$= 4\sqrt{3} \text{ sq. units}$$

Question 21

$f : \mathbb{R} - \left(-\frac{3}{5}\right) \rightarrow \mathbb{R}$ is defined by $f(x) = \frac{3x-2}{5x+3}$, then $f \circ f(1)$ is

Options:

A. 1

B. $\frac{-13}{29}$

C. $\frac{13}{29}$

D. -1

Answer: B

Solution:

$$f(x) = \frac{3x-2}{5x+3}$$

$$f(f(x)) = \frac{3\left(\frac{3x-2}{5x+3}\right) - 2}{5\left(\frac{3x-2}{5x+3}\right) + 3}$$

$$= \frac{3(3x-2) - 2(5x+3)}{5(3x-2) + 3(5x+3)}$$

$$= \frac{9x-6-10x-6}{15x-10+15x+9}$$

$$= \frac{-x-12}{30x-1}$$

$$f \circ f(1) = \frac{-1-12}{30-1} = \frac{-13}{29}$$

Question 22

The value of $\cot \left(\sum_{n=1}^{23} \cot^{-1} \left(1 + \sum_{k=1}^n 2k \right) \right)$ is

Options:

A. $\frac{23}{25}$

B. $\frac{25}{23}$

C. $\frac{23}{24}$

D. $\frac{24}{23}$

Answer: B

Solution:

$$\begin{aligned} & \cot \left(\sum_{n=1}^{23} \cot^{-1} \left(1 + \sum_{k=1}^n 2k \right) \right) \\ &= \cot \left(\sum_{n=1}^{23} \cot^{-1} \left(1 + 2 \times \frac{n(n+1)}{2} \right) \right) \\ &= \cot \left(\sum_{n=1}^{23} \cot^{-1}(1 + n(n+1)) \right) \\ &= \cot \left(\sum_{n=1}^{23} \tan^{-1} \left(\frac{1}{1 + n(n+1)} \right) \right) \\ &= \cot \left(\sum_{n=1}^{23} \tan^{-1} \left(\frac{n+1-n}{1 + n(n+1)} \right) \right) \\ &= \cot \left(\sum_{n=1}^{23} \tan^{-1}(n+1) - \sum_{n=1}^{23} \tan^{-1} n \right) \\ &= \cot [(\tan^{-1}(2) + \tan^{-1}(3) + \dots + \tan^{-1}(24)) - (\tan^{-1}(1) + \tan^{-1}(2) + \dots + \tan^{-1}(23))] \\ &= \cot (\tan^{-1}(24) - \tan^{-1}(1)) \\ &= \cot \left(\tan^{-1} \left(\frac{24-1}{1+24(1)} \right) \right) \\ &= \cot \left(\tan^{-1} \left(\frac{23}{25} \right) \right) \\ &= \cot \left(\cot^{-1} \left(\frac{25}{23} \right) \right) \\ &= \frac{25}{23} \end{aligned}$$

Question 23

An object is moving in the clockwise direction around the unit circle $x^2 + y^2 = 1$. As it passes through the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, its y -co-ordinate is decreasing at the rate of 3 units per sec. The rate at which the x -co-ordinate changes at this point is

Options:

- A. 2 units/sec
- B. $3\sqrt{3}$ units/sec
- C. $\sqrt{3}$ units /sec
- D. $2\sqrt{3}$ units /sec

Answer: B

Solution:

Given equation is $x^2 + y^2 = 1$

Differentiating w.r.t. t , we get

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2x \frac{dx}{dt} + 2y(-3) = 0$$

$$\frac{dx}{dt} = \frac{6y}{2x}$$

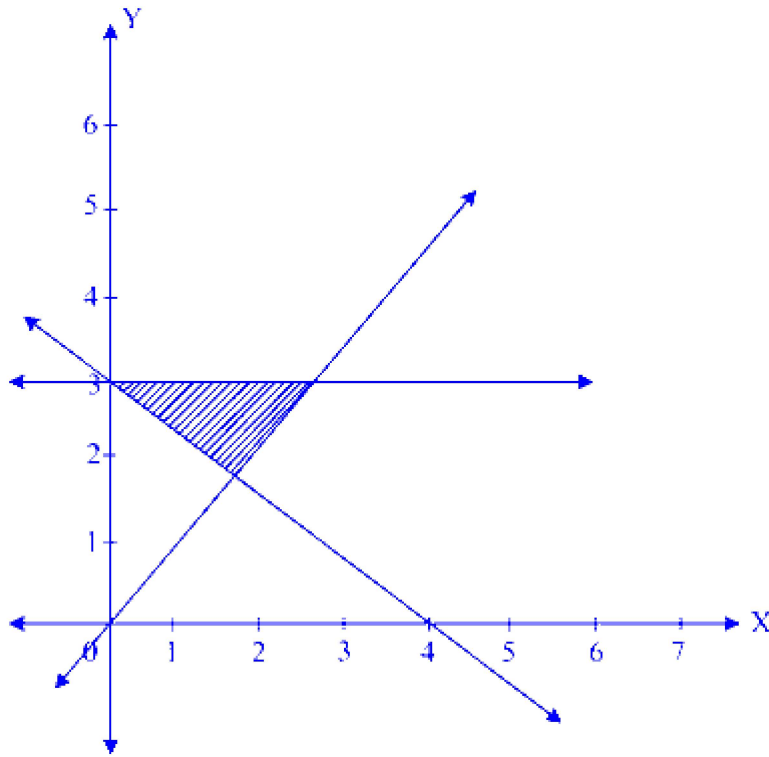
$$\frac{dx}{dt} = \frac{3y}{x}$$

$$\left. \frac{dx}{dt} \right|_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \frac{3 \times \frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= 3\sqrt{3} \text{ units /sec}$$

Question 24

If feasible region is as shown in the figure, then the related inequalities are



Options:

- A. $3x + 4y \geq 12, y - x \geq 0, y \leq 3, x, y \geq 0$
- B. $3x + 4y \leq 12, y - x \leq 0, y \geq 3, x, y \geq 0$
- C. $3x + 4y \leq 12, y - x \geq 0, y \leq 3, x, y \geq 0$
- D. $3x + 4y \geq 12, y - x \leq 0, y \geq 3, x, y \geq 0$

Answer: A

Solution:

The shaded region lies:

on non-origin side of line $3x + 4y = 12$ i.e., $3x + 4y \geq 12$,

on the side of the line $y - x = 0$, where $y \geq x$ i.e., $y - x \geq 0$,

on origin side of line $y = 3$ i.e., $y \leq 3$,

and in first quadrant i.e., $x \geq 0, y \geq 0$.

Question 25

$$\int \frac{e^{\tan^{-1} x}}{1+x^2} \left[\left(\sec^{-1} \sqrt{1+x^2} \right)^2 + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right] dx, x > 0 =$$

Options:

- A. $(\tan^{-1} x)^2 e^{\tan^{-1} x} + c$, where c is a constant of integration.
- B. $(\tan^{-1} x) e^{\tan^{-1} x} + c$, where c is a constant of integration.
- C. $(\tan^{-1} x) e^{2 \tan^{-1} x} + c$, where c is a constant of integration.
- D. $(\tan^{-1} x)^2 e^{2 \tan^{-1} x} + c$, where c is a constant of integration.

Answer: A

Solution:

$$\int \frac{e^{\tan^{-1} x}}{1+x^2} \left[\left(\sec^{-1} \sqrt{1+x^2} \right)^2 + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right] dx,$$

Put $x = \tan t$

$$\therefore dx = \sec^2 t dt$$

$$\begin{aligned} \therefore I &= \int \frac{e^{\sec^{-1}(\tan t)}}{1+\tan^2 t} \left[\left(\sec^{-1} \sqrt{1+\tan^2 t} \right)^2 + \cos^{-1} \left(\frac{1-\tan^2 t}{1+\tan^2 t} \right) \right] \sec^2 t dt \\ &= \int \frac{e^t}{\sec^2 t} \left[\left(\sec^{-1}(\sec t) \right)^2 + \cos^{-1}(\cos 2t) \right] \sec^2 t dt \\ &= \int e^t [t^2 + 2t] dt \\ &= e^t \cdot t^2 + c \quad \dots \left[\int e^x f(x) \cdot f'(x) = e^x f(x) + c \right] \\ &= t^2 \cdot e^t + c \\ &= (\tan^{-1} x)^2 e^{\tan^{-1} x} + c \end{aligned}$$

Question 26

If $f(x)$ is a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ is a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$ is

Options:

A. $e - \frac{e^2}{2} - \frac{5}{2}$

B. $e + \frac{e^2}{2} - \frac{3}{2}$

C. $e - \frac{e^2}{2} - \frac{3}{2}$

D. $e + \frac{e^2}{2} + \frac{5}{2}$

Answer: C

Solution:

As $f'(x) = f(x)$

$$\frac{f'(x)}{f(x)} = 1$$

Integrating on both sides, we get

$$\log f(x) = x + c \dots (i)$$

As $f(0) = 1$

$$\therefore (i) \Rightarrow c = 0$$

$$\therefore \log f(x) = x$$

$$\therefore f(x) = e^x$$

As $f(x) + g(x) = x^2$

$$g(x) = x^2 - e^x$$

$$\therefore f(x)g(x) = e^x (x^2 - e^x)$$

$$= \int_0^1 (e^x x^2 - e^{2x}) dx$$

$$= \left[(x^2 - 2x + 2)e^x \right]_0^1 - \frac{1}{2}e^2 + \frac{1}{2}$$

$$= e - \frac{1}{2}e^2 - \frac{3}{2}$$

Question 27

The foot of the perpendicular drawn from the origin to the plane is $(4, -2, 5)$, then the Cartesian equation of the plane is

Options:

A. $4x - 2y + 5z = 45$

B. $-4x + 2y + 5z = 45$

C. $4x - 2y + 5z + 45 = 0$

D. $4x + 2y - 5z + 45 = 0$

Answer: A

Solution:

Substitute $x = 4, y = -2, z = 5$ in all options.

∴ For option A

$$\begin{aligned} 4x - 2y + 5z &= 4(4) - 2(-2) + 5(5) \\ &= 16 + 4 + 25 \\ &= 45 \end{aligned}$$

∴ Only option A is satisfied by $(4, -2, 5)$

Question 28

$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

Options:

A. $\frac{1}{24}$

B. $\frac{1}{16}$

C. $\frac{1}{8}$

D. $\frac{1}{4}$

Answer: B

Solution:

$$\text{Let } I = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x(1 - \sin x)}{\sin x(\pi - 2x)^3}$$

$$\text{Put } x = \frac{\pi}{2} - h$$

$$\therefore \pi - 2x = 2h$$

$$\text{As } x \rightarrow \frac{\pi}{2}, h \rightarrow 0$$

$$\therefore I = \lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} - h\right)(1 - \sin\left(\frac{\pi}{2} - h\right))}{\sin\left(\frac{\pi}{2} - h\right)(2h)^3}$$

$$= \lim_{h \rightarrow 0} \frac{\sinh \cdot 2 \sin^2\left(\frac{h}{2}\right)}{\cosh \cdot 8h^3}$$

$$= \frac{2}{8} \lim_{h \rightarrow 0} \frac{\sinh}{h} \times \lim_{h \rightarrow 0} \frac{\sin^2 \frac{h}{2}}{\frac{h^2}{4} \cdot 4}$$

$$= \frac{2}{8}(1) \times (1) \times \frac{1}{4}$$

$$= \frac{1}{16}$$

Question 29

If the distance between the parallel lines given by the equation $x^2 + 4xy + py^2 + 3x + qy - 4 = 0$ is λ , then $\lambda^2 =$

Options:

A. 5

B. $\sqrt{5}$

C. 25

D. $\frac{9}{5}$

Answer: A

Solution:

Given equation is

$$x^2 + 4xy + 4y^2 + 3x + 6y - 4 = 0$$

$$(x + 2y)^2 + 3(x + 2y) - 4 = 0$$

$$(x + 2y + 4)(x + 2y - 1) = 0$$

$\therefore$ The lines are: $x + 2y + 4 = 0$ and $x + 2y - 1 = 0$

$$\begin{aligned}\therefore \text{ Required distance} &= \frac{|4 - (-1)|}{\sqrt{1 + 4}} \\ &= \frac{5}{\sqrt{5}} = \sqrt{5} \text{ units}\end{aligned}$$

$$\lambda = \sqrt{5} \text{ units}$$

$$\lambda^2 = 5$$

Question 30

The particular solution of the differential equation $(1 + y^2)dx - xy dy = 0$ at $x = 1, y = 0$, represents

Options:

A. circle

B. pair of straight lines

C. hyperbola

D. ellipse

Answer: C

Solution:

$$(1 + y^2)dx - xy dy = 0$$

$$\therefore (1 + y^2)dx = xy dy$$

$$\therefore \frac{1}{x} dx = \frac{y dy}{1+y^2}$$

Integrating both sides, we get

$$\int \frac{1}{x} dx = \int \frac{y}{1+y^2} dy$$

$$\log x = \frac{1}{2} \log (1 + y^2) + c$$

$$\text{At } x = 1, y = 0 \dots [\text{Given}]$$

$$\therefore 0 = \frac{1}{2} \log(1 + 0) + c$$

$$\therefore c = 0$$

$$\therefore \log x = \frac{1}{2} \log (1 + y^2)$$

$$\therefore x^2 = 1 + y^2$$

$$\therefore x^2 - y^2 = 1,$$

Which is a rectangular hyperbola.

Question 31

If at the end of certain meeting, everyone had shaken hands with everyone else, it was found that 45 handshakes were exchanged, then the number of members present at the meeting, are

Options:

A. 10

B. 15

C. 20

D. 21

Answer: A

Solution:

Let 'n' be the number of members in the meeting

$$\therefore \text{Total number of handshakes} = {}^nC_2$$

$$\therefore {}^nC_2 = 45$$

$$\frac{n!}{2!(n-2)!} = 45$$

$$\frac{n(n-1)(n-2)!}{2 \times (n-2)!} = 45$$

$$n(n-1) = 90$$

$$\therefore n^2 - n - 90 = 0$$

$$n = 10 \text{ or } n = -9 \text{ (not possible)}$$

$$\therefore n = 10$$

Question 32

If $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$, then $\cos^2 48^\circ - \sin^2 12^\circ$ has the value

Options:

A. $\frac{-\sqrt{5}+1}{8}$

B. $\frac{\sqrt{5}-1}{8}$

C. $\frac{\sqrt{5}+1}{8}$

D. $\frac{-1-\sqrt{5}}{8}$

Answer: C

Solution:

$$\cos^2 A - \sin^2 B = \cos(A+B) \cdot \cos(A-B)$$

$$\therefore \cos^2 48^\circ - \sin^2 12^\circ = \cos(60^\circ) \cdot \cos(36^\circ)$$

$$= \frac{1}{2} \cdot \left(1 - 2 \sin^2 \frac{36^\circ}{2}\right)$$

$$= \frac{1}{2} (1 - 2 \sin^2 18^\circ)$$

$$= \frac{1}{2} \left[1 - 2 \left(\frac{\sqrt{5}-1}{4}\right)^2\right]$$

$$= \frac{\sqrt{5}+1}{8}$$

Question 33

If $A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$, then the inverse of $(2A^2 + 5A)$ is

Options:

A. $\frac{1}{95} \begin{bmatrix} 7 & 3 \\ 3 & 4 \end{bmatrix}$

B. $\frac{1}{95} \begin{bmatrix} -7 & 3 \\ 3 & -4 \end{bmatrix}$

C. $\frac{1}{95} \begin{bmatrix} -7 & -3 \\ 3 & 4 \end{bmatrix}$

D. $\frac{1}{95} \begin{bmatrix} 4 & 3 \\ 3 & 7 \end{bmatrix}$

Answer: A

Solution:

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix}$$

$$\text{Now, } 2A^2 + 5A = 2 \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix} + 5 \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -15 \\ -15 & 35 \end{bmatrix}$$

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\therefore (2A^2 + 5A)^{-1} = \frac{1}{475} \begin{bmatrix} 35 & 15 \\ 15 & 20 \end{bmatrix}$$

$$= \frac{1}{95} \begin{bmatrix} 7 & 3 \\ 3 & 4 \end{bmatrix}$$

Question 34

If $I = \int \frac{\sin x + \sin^3 x}{\cos 2x} dx = P \cos x + Q \log \left| \frac{\sqrt{2} \cos x - 1}{\sqrt{2} \cos x + 1} \right|$ (where c is a constant of integration), then values of P and Q are respectively

Options:

A. $\frac{1}{2}, \frac{3}{4\sqrt{2}}$

B. $\frac{1}{2}, \frac{-3}{4\sqrt{2}}$

C. $\frac{1}{2}, \frac{3}{2\sqrt{2}}$

D. $\frac{1}{2}, \frac{-3}{2\sqrt{2}}$

Answer: B

Solution:

$$\begin{aligned} I &= \int \frac{\sin x + \sin^3 x}{\cos 2x} dx \\ &= \int \frac{\sin x (1 + \sin^2 x)}{2 \cos^2 x - 1} dx \\ &= \int \frac{\sin x (2 - \cos^2 x)}{2 \cos^2 x - 1} dx \end{aligned}$$

Put $\cos x = t$

$-\sin x dx = dt$

$$\begin{aligned} I &= \int \frac{t^2 - 2}{2t^2 - 1} dt \\ &= \frac{1}{2} \int \frac{2t^2 - 4}{2t^2 - 1} dt \\ &= \frac{1}{2} \left[\int \frac{2t^2 - 1}{2t^2 - 1} dt - \int \frac{3}{2t^2 - 1} dt \right] \\ I &= \frac{1}{2} t - \frac{1}{2} \times \frac{3}{2\sqrt{2}} \log \left| \frac{\sqrt{2}t - 1}{\sqrt{2}t + 1} \right| + c \\ &= \frac{1}{2} \cos x - \frac{3}{4\sqrt{2}} \log \left| \frac{\sqrt{2} \cos x - 1}{\sqrt{2} \cos x + 1} \right| + c \\ \therefore P &= \frac{1}{2} \text{ and } Q = \frac{-3}{4\sqrt{2}} \end{aligned}$$

Question 35

The negation of the statement $(p \wedge q) \rightarrow (\sim p \vee r)$ is

Options:

A. $p \vee q \vee \sim r$

B. $p \wedge q \wedge \sim r$

C. $\sim p \vee q \wedge r$

D. $\sim p \vee \sim q \vee \sim r$

Answer: B

Solution:

$$\begin{aligned} & \sim [(p \wedge q) \rightarrow (\sim p \vee r)] \\ & \equiv (p \wedge q) \wedge \sim (\sim p \vee r) \dots [\because \sim (p \rightarrow q) \equiv p \wedge \sim q] \\ & \equiv p \wedge q \wedge p \wedge \sim r \dots [\text{Associative Law}] \\ & \equiv p \wedge q \wedge \sim r \dots [\text{Idempotent Law}] \end{aligned}$$

Question 36

The probability mass function of random variable X is given by

$$P[X = r] = \begin{cases} \frac{{}^nC_r}{32}, & n, r \in \mathbb{N} \\ 0, & \text{otherwise} \end{cases}, \text{ then } P[X \leq 2] =$$

Options:

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{5}$

Answer: B

Solution:

$$\text{Since } \sum_{x=0}^n P(X = x) = 1$$

$$\frac{{}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n}{32} = 1$$

$$2^n = 32$$

$$\therefore n = 5$$

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \frac{{}^5C_0}{32} + \frac{{}^5C_1}{32} + \frac{{}^5C_2}{32} = \frac{1}{2}$$

Question 37

The distance of a point $(2, 5)$ from the line $3x + y + 4 = 0$ measured along the line L_1 and L_2 are same. If slope of line L_1 is $\frac{3}{4}$, then slope of the line L_2 is

Options:

A. $-\frac{3}{4}$

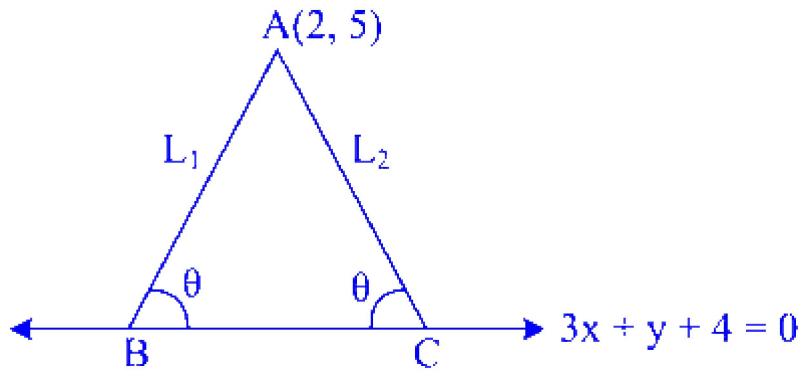
B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. 0

Answer: D

Solution:



According to the given condition, in $\triangle ABC$, $AB = AC$.

$\therefore \triangle ABC$ is an isosceles triangle.

Let m, m_1, m_2 be the slopes of given line, L_1 and L_2 respectively.

$$\therefore m = -3, m_1 = \frac{3}{4}$$

$$\therefore \left| \frac{m-m_1}{1+mm_1} \right| = \left| \frac{m-m_2}{1+mm_2} \right|$$

$$\therefore 3 = \left| \frac{-3-m_2}{1+3m_2} \right|$$

$$\Rightarrow m_2 = 0$$

Question 38

The distance of the point having position vector $\hat{i} - 2\hat{j} - 6\hat{k}$, from the straight line passing through the point $(2, -3, -4)$ and parallel to the vector $6\hat{i} + 3\hat{j} - 4\hat{k}$ is units.

Options:

A. $\sqrt{\frac{340}{61}}$

B. $\frac{341}{61}$

C. $\frac{\sqrt{341}}{61}$

D. $\sqrt{\frac{341}{61}}$

Answer: D

Solution:

Given equation of the line is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\therefore \vec{r} = 2\hat{i} - 3\hat{j} - 4\hat{k} + \lambda(6\hat{i} + 3\hat{j} - 4\hat{k})$$

To find: Its distance from point $\vec{\alpha} = \hat{i} - 2\hat{j} - 6\hat{k}$,

$$\therefore \text{ Required distance} = \sqrt{|\vec{\alpha} - \vec{a}|^2 - \left[\frac{(\vec{\alpha} - \vec{a}) \cdot \vec{b}}{|\vec{b}|} \right]^2}$$

$$\text{Here, } |\vec{\alpha} - \vec{a}|^2 = 6 \text{ and } \left[\frac{(\vec{\alpha} - \vec{a}) \cdot \vec{b}}{|\vec{b}|} \right]^2 = \frac{25}{61}$$

$$\therefore \text{ Required distance} = \sqrt{\frac{341}{61}}$$

Question 39

A vector $\vec{n}$ is inclined to X-axis at 45° , Y-axis at 60° and at an acute angle to Z-axis. If $\vec{n}$ is normal to a plane passing through the point $(-\sqrt{2}, 1, 1)$, then equation of the plane is

Options:

A. $\sqrt{2}x + y + z = 0$

B. $x + \sqrt{2}y + z = 1$

C. $-\sqrt{2}x + y + 2z = 5$

D. $x + y + \sqrt{2}z = 1$

Answer: A

Solution:

Let $\vec{n}$ be inclined at angles α, β, γ to X, Y, Z axes respectively.

$$\alpha = 45^\circ, \beta = 60^\circ, \gamma = ?$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\therefore \frac{1}{2} + \frac{1}{4} + \cos^2 \gamma = 1$$

$$\therefore \cos^2 \gamma = \frac{1}{4}$$

$$\therefore \gamma = 60^\circ$$

$$\vec{n} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}$$

$$\therefore \vec{n} = \frac{1}{\sqrt{2}} \hat{i} + \frac{1}{2} \hat{j} + \frac{1}{2} \hat{k}$$

$\therefore$ Equation of the required plane is

$$\frac{1}{\sqrt{2}}(x + \sqrt{2}) + \frac{1}{2}(y - 1) + \frac{1}{2}(z - 1) = 0$$

$$\text{i.e., } \sqrt{2}x + y + z = 0$$

Question 40

If $\cos^{-1} x - \cos^{-1} \frac{y}{3} = \alpha$, where $-1 \leq x \leq 1, -3 \leq y \leq 3, x \leq \frac{y}{3}$, then for all x, y $9x^2 - 6xy \cos \alpha + y^2$ is equal to

Options:

A. $\sin^2 \alpha$

B. $3 \sin^2 \alpha$

C. $9 \sin^2 \alpha$

D. $\frac{4}{9} \sin^2 \alpha$

Answer: C

Solution:

$$\cos^{-1} a - \cos^{-1} b = \cos^{-1} \left(ab + \sqrt{1-a^2} \cdot \sqrt{1-b^2} \right)$$

$$\therefore \cos^{-1} x - \cos^{-1} \frac{y}{3}$$

$$= \cos^{-1} \left(\frac{xy}{3} + \sqrt{1-x^2} \cdot \sqrt{1-\frac{y^2}{9}} \right) = \alpha$$

$$\therefore \frac{xy}{3} + \frac{\sqrt{1-x^2} \cdot \sqrt{9-y^2}}{3} = \cos \alpha$$

$$xy + \sqrt{1-x^2} \cdot \sqrt{9-y^2} = 3 \cos \alpha$$

$$xy - 3 \cos \alpha = -\sqrt{1-x^2} \cdot \sqrt{9-y^2}$$

squaring on both sides, we get

$$x^2 y^2 - 6xy \cos \alpha + 9 \cos^2 \alpha = (1-x^2)(9-y^2)$$

$$x^2 y^2 - 6xy \cos \alpha + 9 \cos^2 \alpha = 9 - y^2 - 9x^2 + x^2 y^2$$

$$\text{i.e., } 9x^2 - 6xy \cos \alpha + y^2 = 9 \sin^2 \alpha$$

Question 41

If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

Options:

A. 12

B. 19

C. 23

D. 33

Answer: D

Solution:

$$\begin{aligned}
 &\text{Let } y = f(f(f(x))) + (f(x))^2 \\
 \therefore \frac{dy}{dx} &= f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) + 2f(x)f'(x) \\
 \left. \frac{dy}{dx} \right|_{x=1} &= f'(f(f(1))) \cdot f'(f(1)) \cdot f'(1) + 2f(1)f'(1) \\
 &= 3 \cdot 3 \cdot 3 + 2 \cdot 1 \cdot 3 \\
 &= 33
 \end{aligned}$$

Question 42

Three fair coins numbered 1 and 0 are tossed simultaneously. Then variance $\text{Var}(X)$ of the probability distribution of random variable X , where X is the sum of numbers on the uppermost faces, is

Options:

- A. 0.7
- B. 0.75
- C. 0.65
- D. 0.62

Answer: B

Solution:

Three fair coins numbered 1,0 are tossed.

$$\therefore \text{Sample space} = \{111, 110, 101, 011, 100, 010, 001, 000\}$$

$$\therefore n(S) = 8$$

X represents the sum of numbers on upper most face

$$\begin{aligned}
 P(X = 0) &= \frac{1}{8} \\
 P(X = 1) &= \frac{3}{8} \\
 P(X = 2) &= \frac{3}{8}, \\
 P(X = 3) &= \frac{1}{8}
 \end{aligned}$$

$\therefore$ Probability distribution of X is

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$E(X) = \sum_{x=0}^3 x_i P(x_i)$$

$$\therefore E(X) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$= \frac{12}{8} = \frac{3}{2}$$

$$E(X^2) = \sum_{x=0}^3 x_i^2 P(x_i) = 3$$

$$\text{Variance of } X = E(X^2) - [E(X)]^2$$

$$= 3 - \left(\frac{3}{2}\right)^2$$

$$= 3 - \frac{9}{4}$$

$$= \frac{3}{4} = 0.75$$

Question 43

$$\int \frac{1}{\sin(x-a) \sin x} dx =$$

Options:

- A. $\sin a(\log(\sin(x-a) \cdot \operatorname{cosec} x)) + c$, where c is a constant of integration.
- B. $\operatorname{cosec} a(\log(\sin(x-a) \cdot \operatorname{cosec} x)) + c$, where c is a constant of integration.
- C. $-\sin a(\log(\sin(x-a) \cdot \sin x)) + c$, where c is a constant of integration.
- D. $-\operatorname{cosec} a(\log(\sin(x-a) \cdot \sin x)) + c$, where c is a constant of integration.

Answer: B

Solution:

$$\text{Let } I = \int \frac{1}{\sin(x-a) \sin x} dx$$

$$\text{Put } x - a = t \rightarrow x = a + t$$

$$dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{1}{\sin t \cdot \sin(a+t)} dt \\ &= \frac{1}{\sin a} \int \frac{\sin a}{\sin t \cdot \sin(a+t)} dt \\ &= \frac{1}{\sin a} \int \frac{\sin((a+t)-t)}{\sin(a+t) \cdot \sin t} dt \\ &= \frac{1}{\sin a} \left[\int \frac{\sin(a+t) \cos t}{\sin(a+t) \sin t} dt - \int \frac{\sin t \cos(a+t)}{\sin(a+t) \sin t} dt \right] \\ &= \frac{1}{\sin a} \left[\int \cot t dt - \int \cot(a+t) dt \right] \\ &= \operatorname{cosec} a [\log |\sin t| - \log |\sin(a+t)|] + c \\ &= \operatorname{cosec} a \left[\log \left| \frac{\sin t}{\sin(a+t)} \right| \right] + c \\ &= \operatorname{cosec} a [\log(\sin(x-a) \cdot \operatorname{cosec} x)] + c \end{aligned}$$

Question 44

The derivative of $f(\sec x)$ with respect to $g(\tan x)$ at $x = \frac{\pi}{4}$, where $f'(\sqrt{2}) = 4$ and $g'(1) = 2$, is

Options:

A. 2

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. $\frac{1}{2\sqrt{2}}$

Answer: C

Solution:

Let $y = f(\sec x)$ and $z = g(\tan x)$

$$\frac{dy}{dx} = f'(\sec x) \cdot \sec x \tan x$$

$$\frac{dz}{dx} = g'(\tan x) \cdot \sec^2 x$$

$$\text{Now, } \frac{dy}{dz} = \frac{f'(\sec x) \sec x \tan x}{g'(\tan x) \sec^2 x}$$

$$\frac{dy}{dz} = \frac{f'(\sec x) \tan x}{g'(\tan x) \cdot \sec x}$$

$$\left. \frac{dy}{dz} \right|_{x=\frac{\pi}{4}} = \frac{f'(\sec \frac{\pi}{4}) \tan \frac{\pi}{4}}{g'(\tan \frac{\pi}{4}) \sec \frac{\pi}{4}}$$

$$= \frac{f'(\sqrt{2}) \cdot (1)}{g'(1) \cdot \sqrt{2}} \Rightarrow \frac{4 \times 1}{2\sqrt{2}}$$

$$= \sqrt{2}$$

Question 45

The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1, then value of λ is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

$$(2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k}) = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

Unit vector along the above vector is

$$\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

Scalar product of $(\hat{i} + \hat{j} + \hat{k})$ with this unit vector is 1 .

$$\therefore (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}} = 1$$

$$\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}} = 1$$

$$\sqrt{\lambda^2 + 4\lambda + 44} = \lambda + 6$$

$$\therefore \lambda^2 + 4\lambda + 44 = (\lambda + 6)^2$$

$$8\lambda = 8$$

$$\lambda = 1$$

Question 46

The number of discontinuities of the greatest integer function

$$f(x) = [x], x \in \left(-\frac{7}{2}, 100\right)$$

Options:

A. 104

B. 100

C. 102

D. 103

Answer: D

Solution:

$$f(x) = [x]$$

$$x \in \left(-\frac{7}{2}, 100\right)$$

$$x \in (-3.5, 100)$$

As we know greatest integer is discontinuous on integer values in given interval, the integer values are $\{-3, -2, -1, 0 \dots 99\}$

$\therefore$ Total number of discontinuities are 103 .

Question 47

If $[(\bar{a} + 2\bar{b} + 3\bar{c}) \times (\bar{b} + 2\bar{c} + 3\bar{a})] \cdot (\bar{c} + 2\bar{a} + 3\bar{b}) = 54$ then the value of $[\bar{a} \quad \bar{b} \quad \bar{c}]$ is

Options:

A. 0

B. 1

C. 3

D. 2

Answer: C

Solution:

R.H.S. of the given equality can be written as

$$\begin{aligned}
 & (2\bar{a} + 3\bar{b} + \bar{c}) \cdot [(\bar{a} + 2\bar{b} + 3\bar{c}) \times (3\bar{a} + \bar{b} + 2\bar{c})] \\
 &= (2\bar{a} + 3\bar{b} + \bar{c}) \cdot [3(\bar{a} \times \bar{a}) + (\bar{a} \times \bar{b}) + 2(\bar{a} \times \bar{c}) \\
 &+ 6(\bar{b} \times \bar{a}) + 2(\bar{b} \times \bar{b}) + 4(\bar{b} \times \bar{c}) \\
 &+ 9(\bar{c} \times \bar{a}) + 3(\bar{c} \times \bar{b}) + 6(\bar{c} \times \bar{c})] \\
 &= (2\bar{a} + 3\bar{b} + \bar{c})[0 + (\bar{a} \times \bar{b}) + 2(\bar{a} \times \bar{c}) \\
 &- 6(\bar{a} \times \bar{b}) + 0 + 4(\bar{b} \times \bar{c}) \\
 &- 9(\bar{a} \times \bar{c}) - 3(\bar{b} \times \bar{c}) + 0] \\
 &= (2\bar{a} + 3\bar{b} + \bar{c})[-5(\bar{a} \times \bar{b}) + (\bar{b} \times \bar{c}) - 7(\bar{a} \times \bar{c})] \\
 &= -10[\bar{a} \cdot (\bar{a} \times \bar{b})] + 2[\bar{a} \cdot (\bar{b} \times \bar{c})] - 14[\bar{a} \cdot (\bar{a} \times \bar{c})] \\
 &- 15[\bar{b} \cdot (\bar{a} \times \bar{b})] + 3[\bar{b} \cdot (\bar{b} \times \bar{c})] - 21[\bar{b} \cdot (\bar{a} \times \bar{c})] \\
 &- 5[\bar{c} \cdot (\bar{a} \times \bar{b})] + [\bar{c} \cdot (\bar{b} \times \bar{c})] - 7[\bar{c} \cdot (\bar{a} \times \bar{c})] \\
 &= 0 + 2[\bar{a}\bar{b}\bar{c}] + 0 \\
 &+ 0 + 0 + 21[\bar{a}\bar{b}\bar{c}] \\
 &- 5[\bar{a}\bar{b}\bar{c}] + 0 + 0 \\
 &= 18[\bar{a}\bar{b}\bar{c}]
 \end{aligned}$$

$$\therefore 18[\bar{a}\bar{b}\bar{c}] = 54$$

$$\Rightarrow [\bar{a}\bar{b}\bar{c}] = 3$$

Question 48

A spherical raindrop evaporates at a rate proportional to its surface area. If originally its radius is 3 mm and 1 hour later it reduces to 2 mm, then the expression for the radius R of the raindrop at any time t is

Options:

A. $6R = t + 2$

B. $R(t + 2) = 6$

C. $R = 6(t + 2)$

D. $6R = t$

Answer: B

Solution:

According to the given conditions, when $t = 0$, $R = 3$ and when $t = 1$, $R = 2$

This condition is satisfied by only option (B)

Question 49

If the Cartesian equation of a line is $6x - 2 = 3y + 1 = 2z - 2$, then the vector equation of the line is

Options:

A. $\bar{r} = \left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k} \right) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

B. $\bar{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

C. $\bar{r} = \left(\frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k} \right) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

D. $\bar{r} = \left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} - \hat{k} \right) + \lambda(\hat{i} - \hat{j} + \hat{k})$

Answer: A

Solution:

Given Cartesian equation of the line is

$$6x - 2 = 3y + 1 = 2z - 2$$

$$\Rightarrow 6\left(x - \frac{1}{3}\right) = 3\left(y + \frac{1}{3}\right) = 2(z - 1)$$

$$\Rightarrow \frac{x - \frac{1}{3}}{\frac{1}{6}} = \frac{y + \frac{1}{3}}{\frac{1}{3}} = \frac{z - 1}{\frac{1}{2}}$$

$$\Rightarrow \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{3}}{2} = \frac{z - 1}{3}$$

$\therefore$ The given line passes through $\left(\frac{1}{3}, \frac{-1}{3}, 1\right)$ and has direction ratios proportional to 1, 2, 3.

$\therefore$ Vector equation is

$$\vec{r} = \left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

Question 50

The volume of parallelopiped, whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$, $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ is 1 cu. units. If θ is the angle between $\vec{u}$ and $\vec{w}$, then the value of $\cos \theta$ is

Options:

A. $\frac{3}{4}$

B. $\frac{5}{6}$

C. $\frac{1}{5}$

D. $\frac{1}{6}$

Answer: B

Solution:

Volume of parallelepiped = $\left[\vec{u} \ \vec{v} \ \vec{w} \right]$

$$\begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = 1$$

$$\Rightarrow \lambda = 2$$

$$\therefore \cos \theta = \frac{2 + 1 + 2}{\sqrt{6} \cdot \sqrt{6}} = \frac{5}{6}$$

Chemistry

Question 51

Which of the following molecules has no lone pair of electrons on central atom?

Options:

A. SO_2

B. SF_6

C. NH_3

D. SF_4

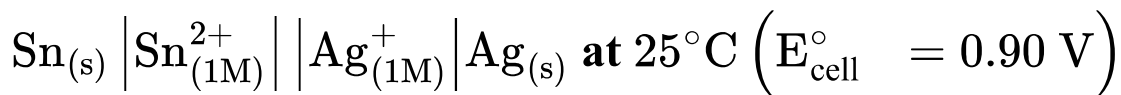
Answer: B

Solution:

Molecule	No. of lone pairs on the central atom
SO_2	1
SF_6	0
NH_3	1
SF_4	1

Question 52

Calculate ΔG° for the cell:



Options:

A. -173.7 kJ

B. -225.3 kJ

C. -100.2 kJ

D. -290.8 kJ

Answer: A

Solution:

$$\begin{aligned}\Delta G^\circ &= -nFE_{\text{cell}}^\circ \\ &= -2 \times 96500 \times 0.90 \\ &= -173700 \text{ J} \\ &= -173.7 \text{ kJ}\end{aligned}$$

Question 53

Which from following statements is NOT correct?

Options:

A. All alkali metals are silvery white.

B. Density of potassium is less than sodium.

C. Compounds of group-1 elements are diamagnetic.

D. Melting point of group-1 elements increase down the group.

Answer: D

Solution:

Melting point of group-1 elements decreases down the group.

Question 54

A compound made of elements A and B form fcc structure. Atoms of A are at the corners and atoms of B are present at the centres of faces of cube. What is the formula of the compound?

Options:

A. AB

B. AB₂

C. AB₃

D. A₂B

Answer: C

Solution:

Atom	Location	Contribution to a unit cell
A	Corners of cube	$\frac{1}{8} \times 8 = 1$
B	Centres of faces	$\frac{1}{2} \times 6 = 3$
Ratio	A : B = 1 : 3	
Formula	AB ₃	

Question 55

What are different possible oxidation states exhibited by scandium?

Options:

- A. +4
- B. +5
- C. +4, +5
- D. +2, +3

Answer: D

Solution:

Scandium typically exhibits a +3 oxidation state. This is because it has an electronic configuration of $[\text{Ar}] 3d^1 4s^2$, and it typically loses the three outer electrons to achieve a noble gas configuration. While a +2 oxidation state is theoretically possible, it is not commonly observed. Scandium does not commonly exhibit +4 or +5 oxidation states.

Thus, the correct answer to the question is :

Option D : +2, +3

Question 56

Which from following is the slope of the graph of rate versus concentration of the reactant for first order reaction?

Options:

- A. $-k$
- B. k
- C. $\frac{k}{2.303}$
- D. $\frac{-k}{2.303}$

Answer: B

Solution:

For a first-order reaction, the rate of the reaction is directly proportional to the concentration of the reactant. The rate law for a first-order reaction can be written as :

$$\text{Rate} = k[\text{Reactant}]$$

where k is the rate constant.

The graph of rate versus concentration of the reactant for a first-order reaction would be a straight line with a slope equal to the rate constant k . This is because the rate is directly proportional to the concentration, making the slope of the line the rate constant itself.

Therefore, the correct answer is :

Option B : k

Question 57

What is the packing efficiency of silver metal in its unit cell?

Options:

A. 52.4%

B. 68.0%

C. 32.0%

D. 74.0%

Answer: D

Solution:

Silver has fcc crystal structure. Packing efficiency of fcc unit cell is 74.0%.

Question 58

Which from following polymers is obtained from C_2F_4 ?

Options:

- A. PVC
- B. Polyisobutylene
- C. Polyacrylonitrile
- D. Teflon

Answer: D

Solution:

The monomer used in preparation of teflon is tetrafluoroethylene, $(CF_2 = CF_2)$.

Question 59

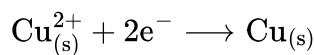
Calculate current in ampere required to deposit 4.8 g Cu from it's salt solution in 30 minutes. [Molar mass of Cu = 63.5 g mol^{-1}]

Options:

- A. 8.1 ampere
- B. 6.4 ampere
- C. 10.5 ampere
- D. 12.3 ampere

Answer: A

Solution:



$$\text{Mole ratio} = \frac{1 \text{ mol}}{2 \text{ mole}^{-}}$$

$$W = \frac{I(\text{A}) \times t(\text{s})}{96500 (\text{C}/\text{mole}^{-})} \times \text{mole ratio} \times \text{molar mass}$$

$$4.8 \text{ g} = \frac{I(\text{A}) \times 30 \times 60}{96500 (\text{C}/\text{mole}^{-})} \times \frac{1 \text{ mol}}{2 \text{ mol e}} \times 63.5 \text{ g mol}^{-1}$$

$$I(\text{A}) = \frac{4.8 \times 96500 \times 2}{63.5 \times 30 \times 60} = 8.1 \text{ A}$$

Question 60

Identify number of moles of donor atoms in 2n mole of SCN^{-} .

Options:

A. 3n

B. 6n

C. 4n

D. n

Answer: C

Solution:

SCN^{-} has two donor atoms nitrogen and sulfur either of which links to metal atom/ion when in forms a coordinate bond. Therefore, 1 mole of SCN^{-} will have 2 moles of donor atoms. Hence, 2n moles of SCN^{-} will have 4n moles of donor atoms.

Question 61

0.2 M aqueous solution of glucose has osmotic pressure 4.9 atm at 300 K. What is the concentration of glucose if it has osmotic pressure 1.5 atm at same temperature?

Options:

A. 0.03 M

B. 0.04 M

C. 0.05 M

D. 0.06 M

Answer: D

Solution:

$$\pi = M \times R \times T$$

$$\therefore M = \frac{\pi}{RT} = \frac{1.5}{0.082 \times 300} = 0.06 \text{ M}$$

Question 62

Identify the product when phenol is heated with zinc dust.

Options:

A. Benzoquinone

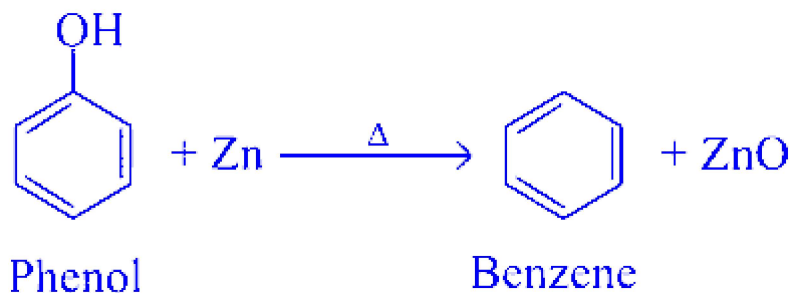
B. Cyclohexane

C. Benzene

D. Cyclohexanol

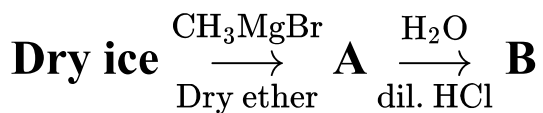
Answer: C

Solution:



Question 63

Identify the product 'B' in the following reaction.



Options:

A. Methanoic acid

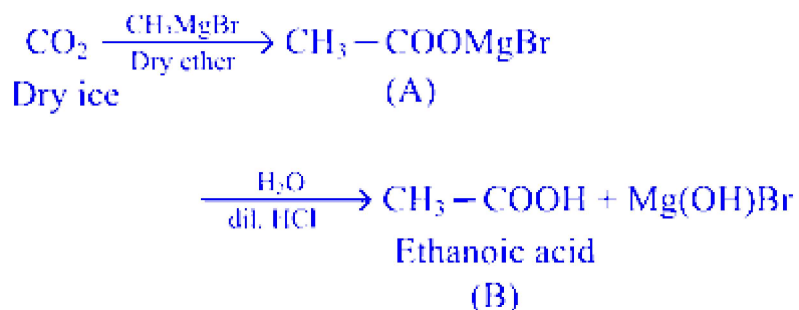
B. Ethanoic acid

C. Methanol

D. Ethanol

Answer: B

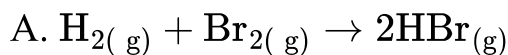
Solution:

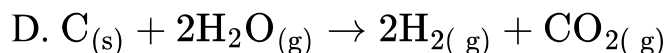
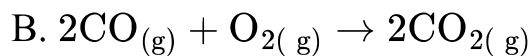


Question 64

Which among the following reactions exhibits $\Delta H = \Delta U$?

Options:





Answer: A

Solution:

When $\Delta n_g = 0$, $\Delta H = \Delta U$.

For reaction (A), $\Delta n_g = 0$.

Question 65

A buffer solution is prepared by mixing 0.2 M sodium acetate and 0.1 M acetic acid. If pK_a for acetic acid is 4.7, find the pH.

Options:

A. 3.0

B. 4.0

C. 5.0

D. 2.0

Answer: C

Solution:

Using Henderson's equation,

$$\text{pH} = \text{pK}_a + \log_{10} \frac{[\text{Salt}]}{[\text{Acid}]}$$

$$\text{pH} = 4.7 + \log_{10} \frac{0.2}{0.1}$$

$$\text{pH} = 4.7 + \log_{10} 2 = 4.7 + 0.3010 = 5.001$$

For acidic buffer, if $[\text{Salt}] > [\text{Acid}]$, then $\text{pH} > \text{pK}_a$ of acid. Hence, only option (C) is valid.

Question 66

A solution of nonvolatile solute is obtained by dissolving 3.5 g in 100 g solvent has boiling point elevation 0.35 K. Calculate the molar mass of solute.

(Molal elevation constant = $2.5 \text{ K kg mol}^{-1}$)

Options:

A. 270 g mol^{-1}

B. 260 g mol^{-1}

C. 250 g mol^{-1}

D. 240 g mol^{-1}

Answer: C

Solution:

$$M_2 = \frac{K_b \times W_2 \times 1000}{\Delta T_b \times W_1} = \frac{2.5 \times 3.5 \times 1000}{0.35 \times 100} = 250 \text{ g mol}^{-1}$$

Question 67

At 0°C a gas occupies 22.4 liters. What is the temperature in Kelvin to reach the volume of 224 liters?

Options:

A. 546 K

B. 273 K

C. 2730 K

D. 5460 K

Answer: C

Solution:

According to Charles' law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (\text{at constant } n \text{ and } P)$$

$$\therefore T_2 = \frac{V_2 \times T_1}{V_1} = \frac{224 \times 273}{22.4} = 2730 \text{ K}$$

Question 68

Which from following rule / principle states that "No two electrons in an atom can have the same set of four quantum numbers"?

Options:

- A. Pauli's exclusion principle
- B. Hund's rule
- C. Aufbau rule
- D. Heisenberg uncertainty principle

Answer: A

Question 69

Which from following molecules exhibits highest acidic nature?

Options:

- A. H_2O
- B. H_2S

C. H_2Se

D. H_2Te

Answer: D

Solution:

The acidic character of the hydrides of group 16 increases from H_2O to H_2Te .

Question 70

What is the number of moles of sp^2 hybrid carbon atoms in one mole of hexa-1,4-diyne?

Options:

A. 5

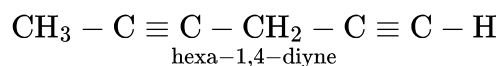
B. 3

C. 4

D. Zero

Answer: D

Solution:



It does not contain sp^2 hybridized carbon atom. Hence, the number of moles of sp^2 hybrid carbon atoms in one mole of hexa-1,4-diyne is zero.

Question 71

Which of the following is NOT obtained when a mixture of methyl chloride and n-propyl chloride is treated with sodium metal in dry

ether?

Options:

- A. Ethane
- B. Butane
- C. Propane
- D. Hexane

Answer: C

Solution:

When a mixture of methyl chloride and n-propyl chloride is treated with sodium metal in dry ether, three possible alkanes formed are: ethane, butane and hexane.

Question 72

Which carbon atoms (numbered from C_1 to C_6) are involved in the formation of ring structure of glucose?

Options:

- A. C_2 and C_5
- B. C_1 and C_3
- C. C_1 and C_4
- D. C_1 and C_5

Answer: D

Solution:

The ring structure of glucose is formed by reaction between the formyl ($-\text{CHO}$) group (C_1) and the alcoholic ($-\text{OH}$) group at C_5 .

Question 73

Calculate the amount of reactant in percent that remains after 60 minutes involved in first order reaction. ($k = 0.02303 \text{ minute}^{-1}$)

Options:

- A. 25%
- B. 75%
- C. 50%
- D. 12.5%

Answer: A

Solution:

For a first order reaction,

$$k = \frac{0.693}{t_{1/2}}$$

$$t_{1/2} = \frac{0.693}{0.02303} = 30 \text{ min}$$

Percent of reactant that remains after $t_{1/2} = 50\%$

$$2 \times t_{1/2} = 60 \text{ min}$$

Therefore, percent of reactant that remains after $2t_{1/2} = 25\%$

Question 74

Which salt from following forms aqueous solution having pH less than 7 ?

Options:

- A. CH_3COONa
- B. Na_2SO_4

C. CuSO_4

D. Na_2CO_3

Answer: C

Solution:

A salt of strong acid and weak base forms aqueous solution having pH less than 7 (that is, it forms an acidic solution). CuSO_4 is salt of strong acid H_2SO_4 and weak base $\text{Cu}(\text{OH})_2$.

Question 75

Identify neutral ligand from following

Options:

A. Ammine

B. Nitrato

C. Cyano

D. Chloro

Answer: A

Solution:

The neutral ligand from the options provided is :

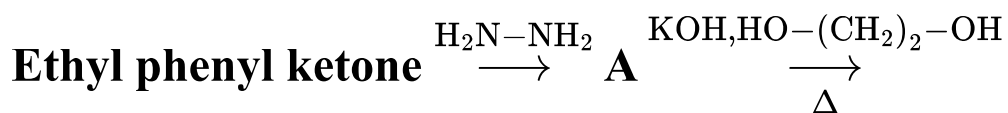
Option A

Ammine

An ammine in coordination chemistry refers to an ammonia molecule acting as a ligand, attached to the central metal atom/ion through a lone pair of electrons. It is neutral because the ammonia molecule itself does not carry any net charge.

Question 76

Identify the product 'B' in following reaction.

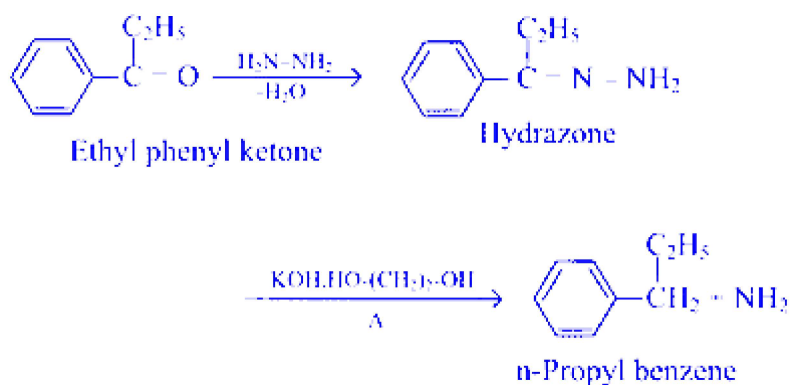


Options:

- A. Phenylhydrazone
- B. Ethyl benzene
- C. n-Propyl benzene
- D. Isopropyl benzene

Answer: C

Solution:



Question 77

Which of the following amines undergoes acylation reaction?

Options:

- A. Ethyldimethylamine
- B. N-Methylaniline
- C. N,N-Dimethylmethanamine

D. N,N-Dimethylaniline

Answer: B

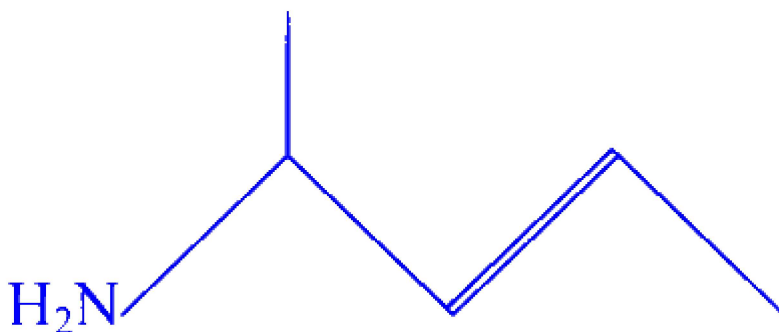
Solution:

Aliphatic and aromatic primary and secondary amines undergo acylation reaction.

Among the given options, N-Methylaniline is a secondary amine and hence, it undergoes acylation reaction. All other amines given are tertiary amines.

Question 78

What is the IUPAC name of following compound?



Options:

A. 1-Methylbut-2-en-1-amine

B. Pent-3-en-2-amine

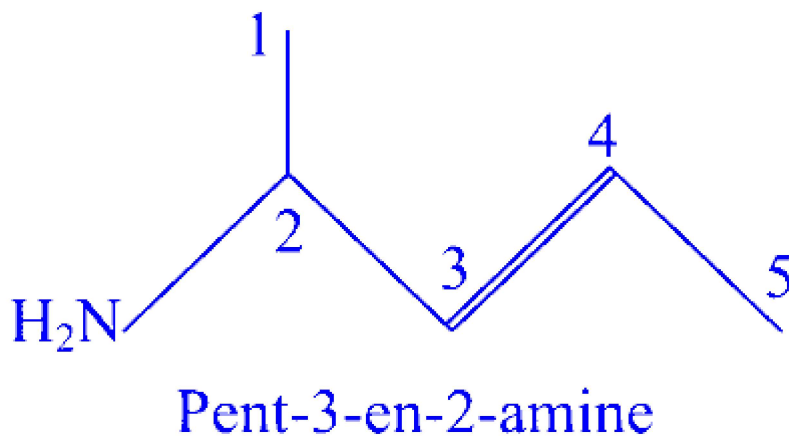
C. Pent-2-en-4-amine

D. 4-Methylbut-2-en-4-amine

Answer: B

Solution:

In IUPAC nomenclature of organic compounds, —NH_2 group has a higher priority than a carbon-carbon double bond.



Question 79

What is the number of unit cells when one mole atom of a metal that forms simple cubic structure?

Options:

A. 6.022×10^{23}

B. 1.204×10^{24}

C. 9.033×10^{23}

D. 3.011×10^{23}

Answer: A

Solution:

Number of atoms in one mole of atom of a metal = 6.022×10^{23}

Number of atoms in a simple cubic unit cell = 1

$\therefore$ Number of unit cells = 6.022×10^{23}

Question 80

Identify the gas produced due to reduction of NH_4^+ ions at cathode during working of dry cell.

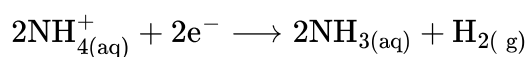
Options:

- A. Ammonia
- B. Hydrogen
- C. Hydrogen chloride
- D. Chlorine

Answer: B

Solution:

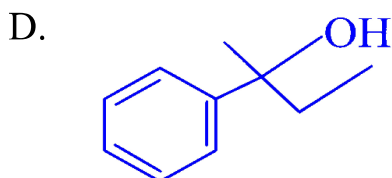
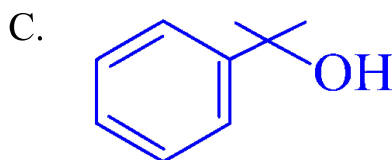
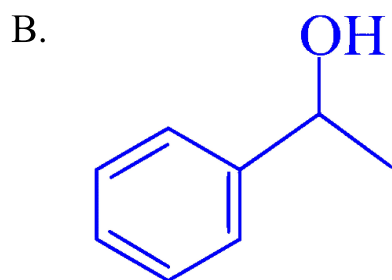
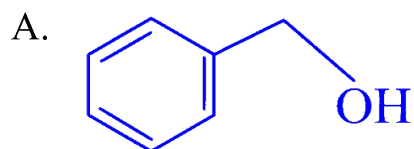
In a dry cell, at cathode, NH_4^+ ions are reduced and hydrogen gas is produced.



Question 81

Which of the following is secondary benzylic alcohol?

Options:



Answer: B

Question 82

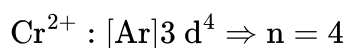
Find the value of spin only magnetic moment for chromium cation in +2 state.

Options:

- A. 3.87 BM
- B. 4.90 BM
- C. 2.84 BM
- D. 1.73 BM

Answer: B

Solution:



$\therefore$ Spin-only magnetic moment,

$$\begin{aligned}\mu &= \sqrt{n(n+2)} \text{ BM} \\ &= \sqrt{4(4+2)} = \sqrt{24} \text{ BM} \\ &= 4.90 \text{ BM}\end{aligned}$$

Since the number of unpaired electrons (n) = 4, $\mu \approx 5$. Hence, option (B) is the correct answer.

Question 83

An ideal gas expands by performing 200 J of work, during this internal energy increases by 432 J. What is enthalpy change?

Options:

- A. 200 J
- B. 232 J
- C. 432 J

D. 632 J

Answer: D

Solution:

Assuming constant pressure,

$$\Delta H = \Delta U + P_{\text{ext}} \Delta V = \Delta U - W$$

Work done (W) = -200 J (for expansion of a gas)

$$\Delta U = +432 \text{ J}$$

$$\Delta H = \Delta U - W = +432 - (-200) = +632 \text{ J}$$

Question 84

Calculate the wavenumber of the photon emitted during transition from the orbit of $n = 2$ to $n = 1$ in hydrogen atom.

$$\left[R_H = 109677 \text{ cm}^{-1} \right]$$

Options:

A. 27419.3 cm^{-1}

B. 109677.0 cm^{-1}

C. 12064.5 cm^{-1}

D. 82257.8 cm^{-1}

Answer: D

Solution:

For hydrogen atom,

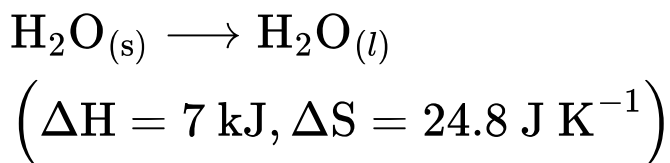
$$\bar{\nu} = 109677 \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right] \text{ cm}^{-1}$$

Here, $n_i = 2$, $n_f = 1$

$$\begin{aligned}
 \therefore \bar{\nu} &= 109677 \left[\frac{1}{1^2} - \frac{1}{2^2} \right] \text{cm}^{-1} \\
 &= 109677 \left[\frac{1}{1} - \frac{1}{4} \right] \text{cm}^{-1} \\
 &= 109677 \left[\frac{3}{4} \right] \text{cm}^{-1} \\
 &= 82257.8 \text{ cm}^{-1}
 \end{aligned}$$

Question 85

Calculate the value of ΔG for following reaction at 300 K.



Options:

- A. 0.74 kJ mol^{-1}
- B. $-0.82 \text{ kJ mol}^{-1}$
- C. 0.21 kJ mol^{-1}
- D. $-0.44 \text{ kJ mol}^{-1}$

Answer: D

Solution:

$$\begin{aligned}
 \Delta H &= 7 \text{ kJ} \\
 \Delta S &= 24.8 \text{ J K}^{-1} = 24.8 \times 10^{-3} \text{ kJ K}^{-1} \\
 T &= 300 \text{ K} \\
 \Delta G &= \Delta H - T\Delta S \\
 \therefore \Delta G &= 7 \text{ kJ} - \left(300 \text{ K} \times 24.8 \times 10^{-3} \text{ kJ K}^{-1} \right) \\
 &= 7 \text{ kJ} - 7.44 \text{ kJ} = -0.44 \text{ kJ}
 \end{aligned}$$

Question 86

What is the oxidation state of carbon in CaC_2 and $\text{K}_2\text{C}_2\text{O}_4$ respectively?

Options:

A. -2 and $+6$

B. -1 and $+3$

C. $+2$ and $+2$

D. -2 and $+3$

Answer: B

Solution:

$$\begin{array}{l} \text{CaC}_2 \\ \therefore +2 + (x \times 2) = 0 \text{ or } x = -1 \end{array}$$

$$\begin{array}{l} \text{K}_2\text{C}_2\text{O}_4 \\ \therefore (+1 \times 2) + (x \times 2) + (-2 \times 4) = 0 \text{ or } x = +3 \end{array}$$

Question 87

What is the number of moles of water molecules required to prepare n moles of methane from n moles of methyl magnesium iodide?

Options:

A. n

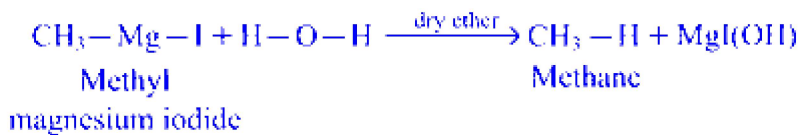
B. $2n$

C. $0.5n$

D. $0.1n$

Answer: A

Solution:



From the reaction, n moles water molecules are required to prepare n moles of methane from n moles of methyl magnesium iodide.

Question 88

Which of the following metals is used as catalyst in manufacture of sulphuric acid by contact process?

Options:

- A. Iron
- B. Platinum
- C. Nickel
- D. Cobalt

Answer: B

Solution:

In the contact process of industrial production of sulfuric acid; sulphur dioxide and oxygen from the air react reversibly over a solid catalyst of platinised asbestos.

Question 89

The rate for reaction $A + B \rightarrow \text{product}$, is $1.8 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$. Calculate the rate constant if the reaction is second order in A and first order in B. ($[A] = 0.2\text{M}$; $[B] = 0.1\text{M}$)

Options:

- A. $9.0 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

B. $18.0 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

C. $4.5 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

D. $16.0 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

Answer: C

Solution:

$$\text{Rate} = k[A]^2[B]$$

$$\therefore k = \frac{\text{rate}}{[A]^2[B]}$$

$$k = \frac{1.8 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}}{(0.2 \text{ mol dm}^{-3})^2 \times 0.1 \text{ mol dm}^{-3}}$$
$$= 4.5 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$$

Question 90

Which of the following concentration terms depends on temperature?

Options:

A. Molality

B. Molarity

C. Mole fraction

D. Percent by mass

Answer: B

Solution:

Among the given options, the concentration term that depends on temperature is :

- **Option B - Molarity**

Molarity is defined as the number of moles of solute per liter of solution. Because the volume of a solution changes with temperature due to thermal expansion or contraction, molarity is temperature-dependent. As

temperature increases, the volume of the solution can increase, causing the molarity to decrease, and vice versa.

The other options are not temperature-dependent :

- **Option A - Molality** : Molality is the number of moles of solute per kilogram of solvent. Since it's based on mass, which does not change with temperature, molality is independent of temperature.
- **Option C - Mole Fraction** : Mole fraction is the ratio of the number of moles of a component to the total number of moles in the mixture. Since it is a ratio of moles and does not involve volume, it is not affected by temperature changes.
- **Option D - Percent by Mass** : Percent by mass (or weight percent) is the ratio of the mass of the solute to the total mass of the solution, multiplied by 100. Since mass is not affected by temperature, this concentration term is also temperature-independent.

Therefore, molarity is the term among the given options that is affected by changes in temperature.

Question 91

What type of reaction is the formation of aldol from aldehyde?

Options:

- A. Condensation reaction
- B. Addition reaction
- C. Substitution reaction
- D. Elimination reaction

Answer: A

Solution:

The formation of aldol from an aldehyde is a :

Option A : Condensation reaction

This is because aldol formation involves the combination of two aldehyde molecules (or one aldehyde and one ketone) to form a β -hydroxyaldehyde (aldol) with the loss of a water molecule, which is characteristic of a condensation reaction.

Question 92

Identify homopolymer from following.

Options:

- A. Polyacrylonitrile
- B. Glyptal
- C. Polycarbonate
- D. Buna-S

Answer: A

Solution:

The homopolymer from the options given is :

Option A : Polyacrylonitrile

This polymer is a homopolymer because it is made up of only one type of monomer, acrylonitrile.

Question 93

Identify the chiral molecule from following.

Options:

- A. 2-Bromopropane
- B. 2-Bromo-2-methylbutane
- C. 2-Bromo-3-methylbutane
- D. 3-Bromopentane

Answer: C

Solution:

Question 95

Solubility of a salt $A_2 B_3$ is $1 \times 10^{-3} \text{ mol dm}^{-3}$. What is the value of its solubility product?

Options:

A. 1.08×10^{-13}

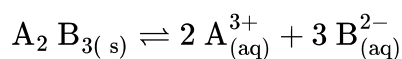
B. 8.1×10^{-15}

C. 2.7×10^{-15}

D. 2.0×10^{-13}

Answer: A

Solution:



Here, $x = 2, y = 3$

$$\begin{aligned} K_{sp} &= x^x y^y S^{x+y} \\ &= (2)^2 (3)^3 S^{2+3} \\ &= 108 S^5 \end{aligned}$$

$$\begin{aligned} K_{sp} &= 108 \times (1 \times 10^{-3})^5 \\ &= 108 \times 10^{-15} \\ &= 1.08 \times 10^{-13} \end{aligned}$$

Question 96

Identify the element having general electronic configuration $ns^2 np^4$ from following.

Options:

A. Se

B. Br

C. Xe

D. Kr

Answer: A

Solution:

The general electronic configuration of the group 16 elements is ns^2np^4 . Among the given options, Se is a group 16 element.

Question 97

Identify the product A obtained in the following reaction.



Options:

A. o-Nitrophenol

B. p-Nitrophenol

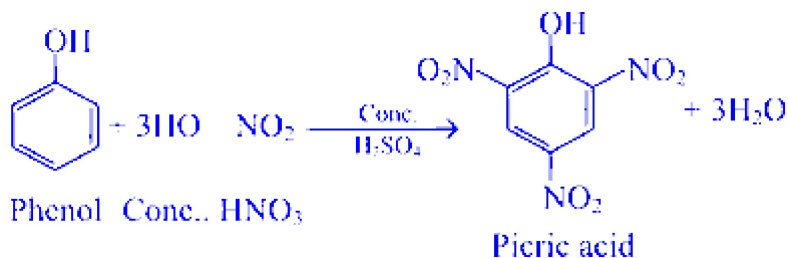
C. 2,4,6-Trinitrophenol

D. Mixture of ortho and para-nitrophenol

Answer: C

Solution:

Phenol reacts with concentrated nitric acid in presence of concentrated H_2SO_4 to form 2,4,6-trinitrophenol (picric acid).



Question 98

What is the value of specific rotation exhibited by glucose molecule?

Options:

- A. $+52.7^\circ$
- B. -92.4°
- C. $+66.5^\circ$
- D. $+40.3^\circ$

Answer: A

Solution:

The specific rotation of a glucose molecule is a measure of the degree to which it rotates the plane of polarized light. The specific rotation for glucose is :

Option A : $+52.7^\circ$

This value is typical for the D-form of glucose (dextrose) in solution at a standard temperature and wavelength.

Question 99

What type of information is collected using scanning electron microscopy?

Options:

- A. Structure of material surface
- B. Crystal structure
- C. Binding nature
- D. Particle size

Answer: A

Solution:

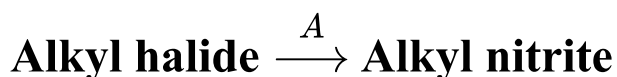
Scanning Electron Microscopy (SEM) is a powerful technique used in various scientific fields for analyzing the surface characteristics of materials.

- **Option A - Structure of Material Surface :** This is a primary use of SEM. It allows for high-resolution imaging of the surface topography and morphology of materials. SEM can provide detailed images of the surface structures, revealing features like cracks, pores, and the arrangement of particles on a surface.
- **Option B - Crystal Structure :** SEM itself is not typically used for determining crystal structure. This aspect is more directly studied using techniques like X-ray diffraction (XRD). However, SEM can be combined with diffraction techniques, such as electron backscatter diffraction (EBSD), to study crystallographic information in materials.
- **Option C - Binding Nature :** SEM does not directly provide information about the binding nature of atoms or molecules in a material. However, when combined with other techniques like Energy-dispersive X-ray spectroscopy (EDX or EDS), it can be used to analyze the elemental composition and infer binding information.
- **Option D - Particle Size :** SEM is highly effective for determining particle size and distribution. It is commonly used in materials science, nanotechnology, and other fields to measure and analyze the size of particles, including nanoparticles.

In summary, the most direct application of SEM among options is the study of the structure of material surfaces (Option A), though it can also contribute to understanding aspects of crystal structure and particle size when combined with other techniques.

Question 100

Identify the reagent A in the following conversion.



Options:

A. KNO_3

B. NaNO_3

C. AgNO_2

D. KNO_2

Answer: D

Solution:

Alkyl halide ($\text{R} - \text{X}$) on treatment with KNO_2 forms alkyl nitrite ($\text{R} - \text{O} - \text{N} = \text{O}$).

Physics

Question 101

A rubber ball filled with water, having a small hole is used as the bob of a simple pendulum. The time period of such a pendulum

Options:

- A. is a constant.
- B. decreases with time.
- C. increases with time.
- D. first increases and then decreases, finally having same value as at the beginning.

Answer: D

Solution:

From $T = 2\pi\sqrt{\frac{L}{g}}$

The effective length increases due to this flow of water. Therefore, T increases. As the water flows out, the length decreases and becomes equal to the original length. Hence, the time period becomes equal to the value at the beginning.

Question 102

The ratio of wavelengths for transition of electrons from 2nd orbit to 1st orbit of Helium (He⁺⁺) and Lithium (Li⁺⁺¹) is (Atomic number of Helium = 2, Atomic number of Lithium = 3)

Options:

- A. 9 : 4
- B. 9 : 36
- C. 4 : 9
- D. 2 : 3

Answer: A

Solution:

To find the ratio of the wavelengths for the transition of electrons from the 2nd orbit to the 1st orbit for Helium ion He⁺⁺ and Lithium ion Li⁺, we can use the formula derived from the Rydberg equation for wavelengths of emitted photons during electron transitions in hydrogen-like atoms.

The Rydberg formula for the wavelength λ is given by:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where,

- R is the Rydberg constant for hydrogen,
- Z is the atomic number of the nucleus,
- n_1 is the lower energy level,
- n_2 is the higher energy level.

For a transition from the 2nd orbit to the 1st orbit ($n_2 = 2$ and $n_1 = 1$), the formula simplifies to:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = RZ^2 \left(\frac{3}{4} \right)$$

$$\text{Therefore, } \lambda = \frac{4}{3RZ^2}.$$

For Helium ion He⁺⁺ ($Z = 2$):

$$\lambda_{\text{He}} = \frac{4}{3R \cdot 2^2} = \frac{4}{12R} = \frac{1}{3R}$$

For Lithium ion Li⁺ ($Z = 3$):

$$\lambda_{\text{Li}} = \frac{4}{3R \cdot 3^2} = \frac{4}{27R}$$

To find the ratio of the wavelengths ($\lambda_{\text{He}} : \lambda_{\text{Li}}$), we divide the expressions obtained for λ_{He} and λ_{Li} :

$$\frac{\lambda_{\text{He}}}{\lambda_{\text{Li}}} = \frac{\frac{1}{3R}}{\frac{4}{27R}} = \frac{27}{12} = \frac{9}{4}$$

Thus, the ratio of the wavelengths for the transition of electrons from the 2nd orbit to the 1st orbit of Helium ion (He^{++}) to Lithium ion (Li^+) is 9 : 4.

Therefore, the correct answer is **Option A**: 9 : 4.

Question 103

For an intrinsic semiconductor (n_h and n_e are the number of holes per unit volume and number of electrons per unit volume respectively)

Options:

A. $n_h < n_e$

B. $n_h = n_e$

C. $n_h = \frac{n_c}{2}$

D. $n_h > n_e$

Answer: B

Question 104

A 5.0 V stabilized power supply is required to be designed using a 12 V DC power supply as input source. The maximum power rating of zener diode is 2.0 W. The minimum value of resistance R_s in Ω connected in series with zener diode will be

Options:

A. 16.5

B. 17.5

C. 18.5

D. 15.5

Answer: B

Solution:

Using the series resistance formula for a zener diode

$$R_S = \frac{(V_S - V_Z)}{I_{Z \max}}$$

$$I_{Z \max} = \frac{P_Z}{V_Z} = \frac{2}{5} = 400 \text{ mA}$$

$$\begin{aligned} \therefore R_S &= \frac{(12 - 5)}{400 \times 10^{-3}} = \frac{7}{400} \times 10^3 \\ &= 17.5 \, \Omega \end{aligned}$$

Question 105

A jar 'P' is filled with gas having pressure, volume and temperature P, V, T respectively. Another gas jar Q filled with a gas having pressure 2P, volume $\frac{V}{4}$ and temperature 2 T. The ratio of the number of molecules in jar P to those in jar Q is

Options:

A. 1 : 1

B. 1 : 2

C. 2 : 1

D. 4 : 1

Answer: D

Solution:

According to the ideal gas equation $PV = Nk_B T$

For jar P, we have

$$PV = N_1 k_B T \dots (i)$$

For jar Q, we have,

$$(2P) \left(\frac{V}{4} \right) = N_2 k_B (2 T)$$

$$\Rightarrow PV = 4 N_2 k_B T \dots (ii)$$

From equations (i) and (ii)

$$N_1 = 4N_2 \Rightarrow \frac{N_1}{N_2} = 4$$

$$\therefore N_1 : N_2 = 4 : 1$$

Question 106

The magnetic flux through a loop of resistance 10Ω varying according to the relation $\phi = 6t^2 + 7t + 1$, where ϕ is in milliweber, time is in second at time $t = 1$ s the induced e.m.f. is

Options:

A. 12 mV

B. 7 mV

C. 19 mV

D. 19 V

Answer: C

Solution:

Given $R = 10 \Omega$, $\phi = 6t^2 + 7t + 1$ mWb, $t = 1$ s

From $e = \frac{d\phi}{dt}$,

$$e = \frac{d}{dt}(6t^2 + 7t + 1) = 12t + 7$$

Put $t = 1$ in the above equation,

$$\therefore e = 19 \text{ mV}$$

Question 107

A thin rod of length ' L ' is bent in the form of a circle. Its mass is ' M '. What force will act on mass ' m ' placed at the centre of this circle?

(G = constant of gravitation)

Options:

A. zero

B. $\frac{GMm}{4L^2\pi^2}$

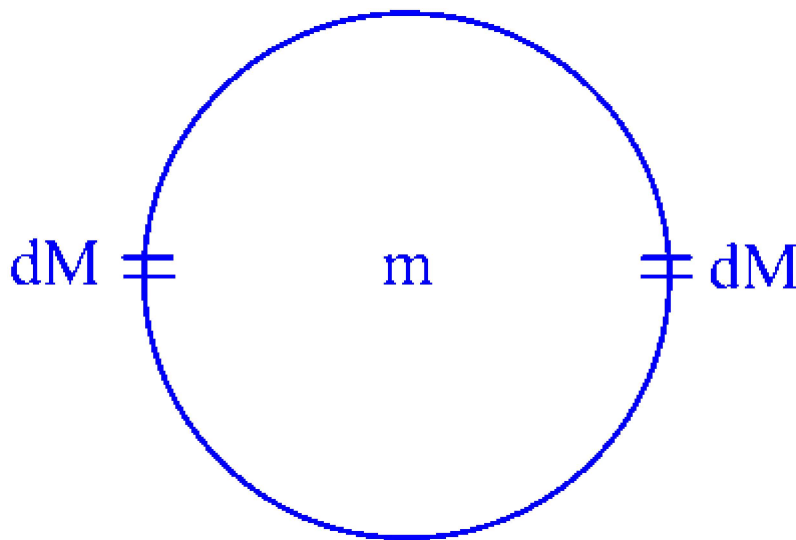
C. $\frac{4\pi^2GMm}{L}$

D. $\frac{2GMm}{L^2}$

Answer: A

Solution:

Consider two diametrically and equal small and equal mass segments dM_1 and dM_2



$\therefore$ Force at the centre due to dM_1 is

$$F_1 = \frac{GmdM_1}{r^2}$$

Similarly,

Force at the centre due to dM_2 is

$$F_2 = \frac{GmdM_2}{r^2}$$

$$\text{But } F_1 = -F_2$$

$$\Rightarrow F_1 + F_2 = 0 \quad (\because \text{the forces cancel each other out as they are equal and opposite})$$

If this process is done for all such dM segments, we find the net force at the centre of the circle to be zero.

Question 108

The coil of an a.c. generator has 100 turns, each of cross-sectional area 2 m^2 . It is rotating at constant angular speed 30 rad/s , in a uniform magnetic field of $2 \times 10^{-2} \text{ T}$. If the total resistance of the circuit is 600Ω then maximum power dissipated in the circuit is

Options:

A. 6 W

B. 9 W

C. 12 W

D. 24 W

Answer: C

Solution:

$$N = 100, A = 2 \text{ m}^2, \omega = 30 \text{ rad/s}$$

$$B = 2 \times 10^{-2} \text{ T}, R = 600 \Omega$$

Maximum power dissipated in the circuit

$$\begin{aligned} P_{\max} &= E_{rms} \times I_{rms} = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \\ &= \frac{E_0 I_0}{2} \quad \dots (i) \end{aligned}$$

$$\text{But } I_0 = \frac{E_0}{R} \quad \dots (ii)$$

Putting (2) into (1) we get,

$$I_0 = \frac{E_0^2}{2R}$$

$$\text{But } E_0 = NAB\omega$$

$$\begin{aligned} E_0 &= 100 \times 2 \times 2 \times 10^{-2} \times 30 \\ &= 120 \text{ V} \end{aligned}$$

$$\therefore P_{\max} = \frac{120 \times 120}{2 \times 600} = 12 \text{ W}$$

Question 109

A beam of unpolarized light passes through a tourmaline crystal A and then it passes through a second tourmaline crystal B oriented so that its principal plane is parallel to that of A. The intensity of emergent light is I_0 . Now B is rotated by 45° about the ray. The emergent light will have intensity $\left(\cos 45^\circ = \frac{1}{\sqrt{2}}\right)$

Options:

A. $\frac{I_0}{2}$

B. $\frac{I_0}{\sqrt{2}}$

C. $\frac{\sqrt{2}}{I_0}$

D. $\frac{2}{I_0}$

Answer: A

Solution:

Using law of Malus,

$$\begin{aligned}
 I &= I_0 \cos^2 \theta \\
 &= I_0 (\cos^2 45) \\
 &= I_0 \left(\frac{1}{\sqrt{2}} \right)^2 \\
 &= \frac{I_0}{2}
 \end{aligned}$$

Question 110

The materials suitable for making electromagnets should have

Options:

- A. high retentivity and high coercivity
- B. low retentivity and low coercivity
- C. high retentivity and low coercivity
- D. low retentivity and high coercivity

Answer: B

Solution:

The correct option for materials suitable for making electromagnets is **Option B: low retentivity and low coercivity**.

To understand why, let's dive into the terms retentivity and coercivity in the context of magnetic materials:

- **Retentivity** (or remanence) is the ability of a magnetic material to retain a certain amount of residual magnetism after an external magnetizing field is removed. High retentivity means the material retains a significant amount of magnetization, while low retentivity means it loses most of the magnetization when the external field is removed.
- **Coercivity** is the measure of the coercive force required to reduce the magnetization of a magnetic material to zero after the magnetization of the sample has been driven to saturation. Essentially, it indicates how difficult it is to demagnetize the material. High coercivity means it is difficult to demagnetize, while low coercivity indicates it is easy to demagnetize.

Electromagnets are designed to be turned on and off (magnetized and demagnetized) easily with the application and removal of an electric current. Therefore, materials used for electromagnets should have **low retentivity** so they can easily lose their magnetism when the current is turned off and **low coercivity** so they can be magnetized and demagnetized easily with small changes in current. This ensures the electromagnet is effective in applications that require rapid switching between the magnetized and demagnetized states.

Materials with high retentivity and high coercivity are more suitable for permanent magnets, which are intended to maintain their magnetic field without a continuous external power source.

Question 111

A body falls on a surface of coefficient of restitution 0.6 from a height of 1 m. Then the body rebounds to a height of

Options:

- A. 1 m
- B. 0.36 m
- C. 0.4 m
- D. 0.6 m

Answer: B

Solution:

As the body falls from a height

$$u = 0$$

$$v^2 = u^2 + 2gh$$

$$\therefore v^2 = 2 \times 9.8 \times 1 = 19.6$$

$$\therefore v = \sqrt{19.6} \text{ m/s}$$

Coefficient of restitution $e = \frac{v}{u}$

$$e = \frac{\text{Velocity after collision } (v_f)}{\text{Velocity before collision } (v_b)}$$

$$\therefore v_f = e \times v_b$$

$$v_f = 0.6 \times \sqrt{19.6} \text{ m/s}$$

After the body rebounds,

$$v^2 = u^2 - 2gh$$

$$\Rightarrow u^2 = 2gh$$

$$\therefore h = u^2/2g$$

Here, $u = v_f$

$$\begin{aligned}\therefore h &= \frac{(0.6 \times \sqrt{19.6})^2}{2 \times 9.8} \\ &= 0.36 \text{ m}\end{aligned}$$

Question 112

In a diffraction pattern due to single slit of width ' a ', the first minimum is observed at an angle of 30° when the light of wavelength $5400\text{\AA}$ is incident on the slit. The first secondary maximum is observed at an angle of $\left(\sin 30^\circ = \frac{1}{2}\right)$

Options:

A. $\sin^{-1}\left(\frac{3}{4}\right)$

B. $\sin^{-1}\left(\frac{2}{3}\right)$

C. $\sin^{-1}\left(\frac{1}{2}\right)$

D. $\sin^{-1}\left(\frac{1}{4}\right)$

Answer: A

Solution:

For n^{th} secondary minimum,

$$\text{path difference} = a \sin \theta_n = n\lambda$$

For n^{th} secondary maximum,

$$\text{path difference} = a \sin \theta_n = (2n + 1)\frac{\lambda}{2}$$

$$\therefore \text{ For } 1^{\text{st}} \text{ minimum, } a \sin 30^\circ = \lambda \dots (i)$$

$$\text{For } 2^{\text{nd}} \text{ maximum, } a \sin \theta_n = (2 + 1)\frac{\lambda}{2} = \frac{3\lambda}{2} \dots (ii)$$

$\therefore$ Dividing equation (i) by equation (ii),

$$\frac{\left(\frac{1}{2}\right)}{\sin \theta_n} = \frac{2}{3} \Rightarrow \theta_n = \sin^{-1}\left(\frac{3}{4}\right)$$

Question 113

A stone is projected vertically upwards with speed ' v '. Another stone of same mass is projected at an angle of 60° with the vertical with the same speed ' v '. The ratio of their potential energies at the highest points of their journey is

$$\left[\sin 30^\circ = \cos 60^\circ = 0.5, \cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2} \right]$$

Options:

A. 4 : 1

B. 2 : 1

C. 3 : 2

D. 1 : 1

Answer: A

Solution:

P.E. of the stone projected vertically is,

$$\text{P.E.} = mgh$$

$$\text{But } h = \frac{v^2}{2g}$$

$$\begin{aligned} \therefore P.E_1 &= mg \left(\frac{v^2}{2g} \right) \\ &= \frac{mv^2}{2} \quad \dots\dots (i) \end{aligned}$$

For the second stone thrown at an angle θ to the horizontal,

$$\begin{aligned} h &= \frac{v^2 \sin^2 \theta}{2g} = \frac{v^2 \sin^2 30^\circ}{2g} = \frac{v^2}{8g} \\ \therefore P.E. 2 &= mg \left(\frac{v^2}{8g} \right) = \frac{mv^2}{8} \quad \dots\dots (ii) \end{aligned}$$

Dividing equation (i) by equation (ii)

$$\frac{P.E_1}{P.E_2} = \frac{\left(\frac{mv^2}{2} \right)}{\left(\frac{mv^2}{8} \right)} = 4 : 1$$

Question 114

An electron (mass m) is accelerated through a potential difference of ' V ' and then it enters in a magnetic field of induction ' B ' normal to the lines. The radius of the circular path is (e = electronic charge)

Options:

A. $\sqrt{\frac{2eV}{m}}$

B. $\sqrt{\frac{2Vm}{eB^2}}$

C. $\sqrt{\frac{2Vm}{eB}}$

D. $\sqrt{\frac{2Vm}{e^2 B}}$

Answer: B

Solution:

Radius of circular path in a cyclotron is given by

$$R = \frac{mV}{qB}$$

Here $q = e$,

$$\therefore R = \frac{mV}{eB} \dots\dots (i)$$

On entering the field,

$$KE = eV = \frac{1}{2}mv^2$$

$$\therefore v = \sqrt{\frac{2eV}{m}} \dots\dots (ii)$$

Putting (ii) into (i),

$$R = \frac{m\sqrt{\frac{2eV}{m}}}{eB} = \sqrt{\frac{2Vm}{eB^2}}$$

Question 115

A capacitor, an inductor and an electric bulb are connected in series to an a.c. supply of variable frequency. As the frequency of the supply is increased gradually, then the electric bulb is found to

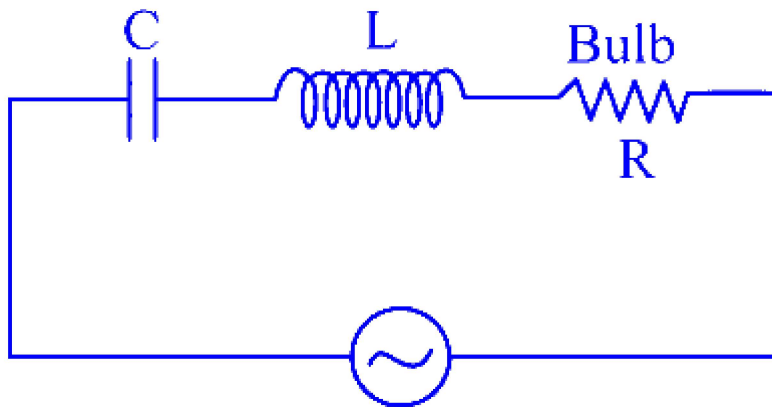
Options:

- A. increase in brightness.
- B. decrease in brightness.
- C. increase, reach a maximum and then decrease in brightness.
- D. show no change in brightness.

Answer: C

Solution:

The given circuit acts like a series LCR circuit.



$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

From the above formula, when $\omega = 0$, $I = \text{maximum}$

When ω is increased, I starts to decrease and when $\omega = \frac{1}{\sqrt{LC}}$ the current will be zero. This cycle will repeat going forward. So brightness will increase, reach a maximum and then decrease.

Question 116

When both source and listener are approaching each other the observed frequency of sound is given by (V_L and V_S is the velocity of listener and source respectively, $n_0 =$ radiated frequency)

Options:

A. $n = n_0 \left[\frac{V+V_L}{V-V_s} \right]$

B. $n = n_0 \left[\frac{V-V_L}{V+V_s} \right]$

C. $n = n_0 \left[\frac{V-V_L}{V-V_s} \right]$

D. $n = n_0 \left[\frac{V+V_L}{V+V_s} \right]$

Answer: A

Solution:

Using Doppler's effect formula for approaching frequency when both source and listener are approaching each other, the observed frequency of sound is given by,

$$n = n_0 \left[\frac{V+V_L}{V-V_s} \right]$$

Question 117

Water is flowing through a horizontal pipe in stream line flow. At the narrowest part of the pipe

Options:

A. velocity is maximum and pressure minimum.

B. pressure is maximum and velocity minimum.

C. both pressure and velocity are minimum.

D. both pressure and velocity are maximum.

Answer: A

Solution:

Using equation of continuity,

$$A_1 v_1 = A_2 v_2$$

In streamlined flow, as the product of Av is a constant, the velocity at the narrowest part will be maximum.

By using Bernoulli's principle,

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant}$$

We see that ρgh is constant because the pipe is horizontal.

As velocity increases, for the equation to remain constant, P has to decrease.

$\therefore$ Velocity will be maximum and pressure is minimum.

Question 118

The angle of prism is A and one of its refracting surface is silvered. Light rays falling at an angle of incidence ' $2A$ ' on the first surface return back through the same path after suffering reflection at the silvered surface. The refractive index of the material of the prism is

Options:

A. $2 \sin \left(\frac{A}{2} \right)$

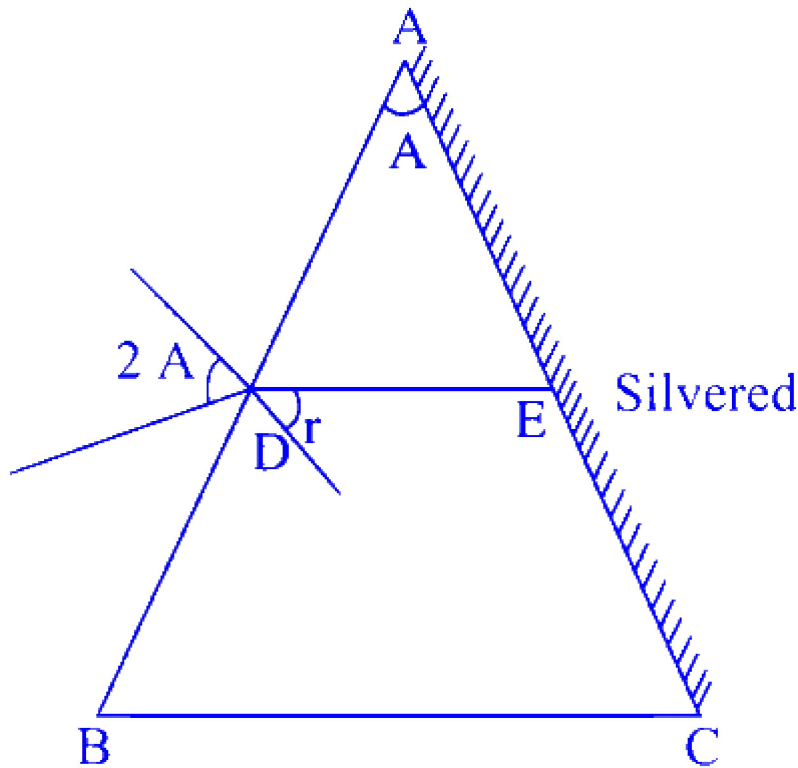
B. $2 \tan A$

C. $2 \cos A$

D. $2 \sin A$

Answer: C

Solution:



Given: Angle of Prism = A , $i = 2A$

As DE is falling normally on the silvered side AC,

$$r = 90 - (90 - A)$$

$$r = A$$

Using Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\sin 2A}{\sin A} = \frac{2 \sin A \cos A}{\sin A} = 2 \cos A$$

Question 119

The maximum velocity of a particle performing S.H.M. is 'V'. If the periodic time is made $\left(\frac{1}{3}\right)^d$ and the amplitude is doubled, then the new maximum velocity of the particle will be

Options:

A. $\frac{V}{6}$

B. $\frac{3V}{2}$

C. 3 V

D. 6 V

Answer: D

Solution:

Given $T' = \frac{1}{3} T$ and $A' = 2 A$

$$\therefore \omega' = \frac{2\pi}{T'} = \frac{2\pi}{(\frac{1}{3}T)} = \frac{6\pi}{T} = 3\omega$$

$\therefore$ The new maximum velocity

$$\begin{aligned} V' &= A'\omega' \\ &= (2 A) \times (3\omega) \\ &= 6 A\omega \\ &= 6 V \end{aligned}$$

Question 120

A conducting wire of length 2500 m is kept in east-west direction, at a height of 10 m from the ground. If it falls freely on the ground then the current induced in the wire is (Resistance of wire = $25\sqrt{2}\Omega$, acceleration due to gravity $g = 10 \text{ m/s}^2$, $B_H = 2 \times 10^{-5} \text{ T}$)

Options:

A. 0.2 A

B. 0.02 A

C. 0.01 A

D. 2 A

Answer: B

Solution:

We know, $e = B/v$

$$v^2 = 2gh$$

$$\therefore v = \sqrt{2gh}$$

$$\therefore e = (2 \times 10^{-5}) \times 2500 \times \sqrt{2 \times 10 \times 10}$$

$$= (2 \times 10^{-5}) \times 25000 \times \sqrt{2}$$

$$= 2\sqrt{2} \times 25 \times 10^3 \times 10^{-5}$$

$$= 50\sqrt{2} \times 10^{-2} \text{ V}$$

$$\therefore I = \frac{e}{R} = \frac{50\sqrt{2} \times 10^{-2}}{25\sqrt{2}} = 2 \times 10^{-2} = 0.02 \text{ A}$$

Question 121

For an electron moving in the n^{th} Bohr orbit the deBroglie wavelength of an electron is

Options:

A. $n\pi r$

B. $\frac{\pi r}{n}$

C. $\frac{nr}{2\pi}$

D. $\frac{2\pi r}{n}$

Answer: D

Solution:

From de Broglie's hypothesis,

$$\lambda = \frac{h}{p_n} = \frac{h}{mv} \quad \dots (i)$$

and from Bohr's atomic model,

$$L = \frac{nh}{2\pi}$$

Also,

$$L = mvr_n$$

Due to quantization of angular momentum, we can write,

$$\frac{nh}{2\pi} = mvr_n$$

$$\therefore v = \frac{nh}{2\pi mr} \dots (ii)$$

putting (ii) into (i), we get,

$$\therefore \lambda = \frac{h}{m(nh/2\pi mr)} = \frac{2\pi r}{n}$$

Question 122

A square lamina of side ' b ' has same mass as a disc of radius ' R ' the moment of inertia of the two objects about an axis perpendicular to the plane and passing through the centre is equal. The ratio $\frac{b}{R}$ is

Options:

A. 1 : 1

B. $\sqrt{3} : 1$

C. $\sqrt{6} : 1$

D. 1 : $\sqrt{3}$

Answer: B

Solution:

$$I_{\text{lamina}} = \frac{Mb^2}{6}$$

$$I_{\text{disc}} = \frac{MR^2}{2}$$

$$\text{Given } \frac{Mb^2}{6} = \frac{MR^2}{2}$$

$$\frac{b^2}{R^2} = 3$$

$$\therefore \frac{b}{R} = \frac{\sqrt{3}}{1}$$

Question 123

A body weighs 300 N on the surface of the earth. How much will it weigh at a distance $\frac{R}{2}$ below the surface of earth? ($R \rightarrow$ Radius of earth)

Options:

- A. 300 N
- B. 250 N
- C. 200 N
- D. 150 N

Answer: D

Solution:

Acceleration due to gravity at depth d,

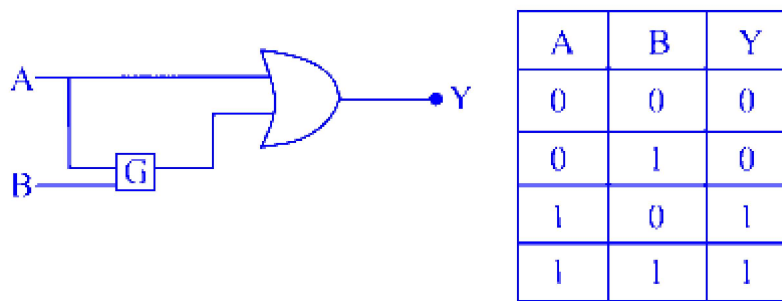
$$\begin{aligned} g_d &= g \left(1 - \frac{d}{R} \right) \\ &= g \left(1 - \frac{1}{2} \right) \quad \because d = \frac{R}{2} \\ &= \frac{g}{2} \end{aligned}$$

$\therefore$ Weight of the body at depth d_1

$$W_d = mg_d = 300 \times \frac{1}{2} = 150 \text{ N}$$

Question 124

To get the truth table shown, from the following logic circuit, the Gate G should be

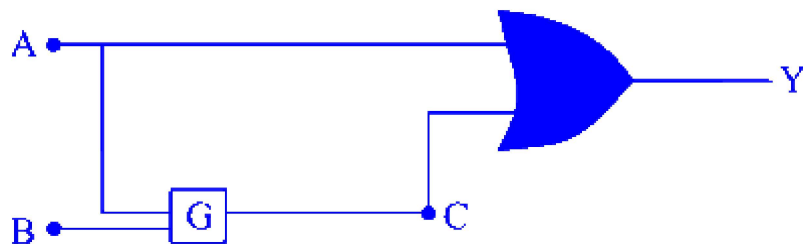


Options:

- A. OR
- B. AND
- C. NOR
- D. NAND

Answer: B

Solution:



Truth table for Y, with the possible values of C is,

A	C	Y
0	0	0
0	0	0
1	0, 1	1
1	0, 1	1

For gate G

	A	B	C
(I)	0	0	0

	A	B	C
(II)	0	1	0
(III)	1	0	0, 1
(IV)	1	1	0, 1

G is not a NOT gate as NOT gate takes only one input. (II) indicates G is not a OR gate as OR gate would give high output for the inputs in (II). Also, (II) indicates it is not a XOR gate as XOR would also give high output for inputs in (II). Hence, the given truth table is satisfied only by AND gate.

Question 125

For a gas having 'X' degrees of freedom, ' γ ' is ($\gamma = \text{ratio of specific heats} = C_P/C_V$)

Options:

A. $\frac{1+X}{2}$

B. $1 + \frac{X}{2}$

C. $1 + \frac{2}{x}$

D. $1 + \frac{1}{x}$

Answer: C

Solution:

γ and degrees of freedom is related by

$$\gamma = \frac{f+2}{f}$$

Where f is the number of degrees of freedoms.

Given $f = X$,

$$\therefore \gamma = \frac{X+2}{X} = 1 + \frac{2}{X}$$

Question 126

A galvanometer of resistance G is shunted with a resistance of 10% of G . The part of the total current that flows through the galvanometer is

Options:

A. $\frac{1}{11}I$

B. $\frac{2}{11}I$

C. $\frac{1}{10}I$

D. $\frac{1}{5}I$

Answer: A

Solution:

$$\frac{I_g}{I} = \frac{S}{S + G} = \frac{0.1G}{0.1G + G} = \frac{1}{11}$$
$$\therefore I_g = \frac{1}{11}I$$

Question 127

If an electron in a hydrogen atom jumps from an orbit of level $n = 3$ to orbit of level $n = 2$, then the emitted radiation frequency is (where R = Rydberg's constant, C = Velocity of light)

Options:

A. $\frac{3RC}{27}$

B. $\frac{RC}{25}$

C. $\frac{8RC}{9}$

D. $\frac{5RC}{36}$

Answer: D

Solution:

$$\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$
$$\therefore \frac{1}{\lambda} = \left[\frac{1}{2^2} - \frac{1}{3^2} \right] = \frac{5}{36} R$$
$$\therefore f = \frac{c}{\lambda} = \frac{5}{36} R c$$

Question 128

Self inductance of solenoid is

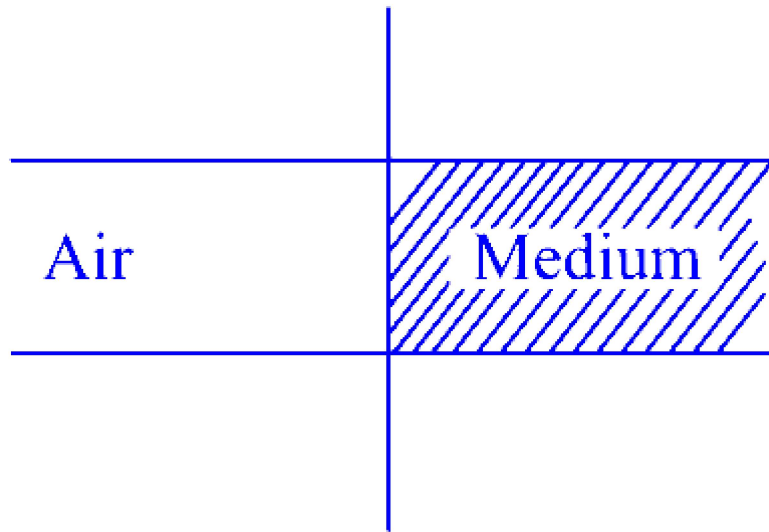
Options:

- A. directly proportional to current flowing through the coil.
- B. directly proportional to the length.
- C. directly proportional to its area of cross-section.
- D. inversely proportional to the area of cross-section.

Answer: C

Question 129

The capacitance of a parallel plate capacitor is $2.5 \mu\text{F}$. When it is half filled with a dielectric as shown in figure, its capacitance becomes $5 \mu\text{F}$. The dielectric constant of the dielectric is



Options:

A. 7.5

B. 3

C. 4

D. 5

Answer: B

Solution:

Given $C = \frac{\epsilon_0 A}{d} = 2.5 \mu F$

When half filled with air,

$$C_1 = \frac{\epsilon_0 (A/2)}{d} = \frac{\epsilon_0 A}{2d} \quad \dots (\because \epsilon_r = 1)$$

When half filled with a dielectric,

$$C_2 = \frac{\epsilon_r \epsilon_0 (A/2)}{d} = \frac{\epsilon_r \epsilon_0 A}{2d}$$

From the figure, it can be seen that C_1 and C_2 are in parallel configuration.

$$\therefore C_{eq} = C_1 + C_2$$

$$\text{Given, } C_{eq} = 5 \mu F$$

$$\Rightarrow 5 \mu F = \frac{\epsilon_0 A}{2d} + \frac{\epsilon_r \epsilon_0 A}{2d}$$

$$5 = \frac{2.5}{2} + \epsilon_r \frac{2.5}{2}$$

$$\frac{3.75}{1.25} = \epsilon_r$$

$$\therefore \epsilon_r = 3$$

Question 130

Equation of simple harmonic progressive wave is given by

$y = \frac{1}{\sqrt{a}} \sin \omega t \pm \frac{1}{\sqrt{b}} \cos \omega t$ then the resultant amplitude of the wave is
($\cos 90^\circ = 0$)

Options:

A. $\frac{a \pm b}{ab}$

B. $\frac{\sqrt{a} \pm \sqrt{b}}{ab}$

C. $\frac{\sqrt{a} \pm \sqrt{b}}{\sqrt{ab}}$

D. $\sqrt{\frac{a+b}{ab}}$

Answer: D

Solution:

$$y = \frac{1}{\sqrt{a}} \sin \omega t \pm \frac{1}{\sqrt{b}} \sin \left(\omega t + \frac{\pi}{2} \right)$$

Here phase difference = $\frac{\pi}{2}$

The resultant amplitude

$$= \sqrt{\left(\frac{1}{\sqrt{a}} \right)^2 + \left(\frac{1}{\sqrt{b}} \right)^2} = \sqrt{\frac{1}{a} + \frac{1}{b}} = \sqrt{\frac{a+b}{ab}}$$

Question 131

Two uniform brass rods A and B of length ' l ' and ' $2l$ ' and their radii ' $2r$ ' and ' r ' respectively are heated to same temperature. The ratio of the increase in the volume of rod A to that of rod B is

Options:

A. 1 : 1

B. 1 : 2

C. 2 : 1

D. 1 : 4

Answer: C

Solution:

To determine the ratio of the increase in the volume of rod A to that of rod B , we need to consider the formula for the volumetric expansion due to heating. The change in volume (ΔV) of a rod due to a change in temperature can be given by:

$$\Delta V = \beta V \Delta T$$

where:

β = coefficient of volumetric expansion (which is the same for both rods since they are made of brass),

V = initial volume of the rod,

ΔT = change in temperature (which is the same for both rods).

First, let's calculate the volume of each rod. The volume V of a cylindrical rod is:

$$V = \pi r^2 l$$

For rod A :

Length = l ,

Radius = $2r$.

Therefore, the initial volume of rod A (V_A) is:

$$V_A = \pi(2r)^2 l = \pi \cdot 4r^2 \cdot l = 4\pi r^2 l$$

For rod B :

Length = $2l$,

Radius = r .

Therefore, the initial volume of rod B (V_B) is:

$$V_B = \pi r^2 (2l) = 2\pi r^2 l$$

Now, the increase in volume (ΔV) of each rod due to heating is given by:

For rod A :

$$\Delta V_A = \beta V_A \Delta T = \beta(4\pi r^2 l) \Delta T = 4\beta\pi r^2 l \Delta T$$

For rod B :

$$\Delta V_B = \beta V_B \Delta T = \beta(2\pi r^2 l) \Delta T = 2\beta\pi r^2 l \Delta T$$

To find the ratio of the increase in the volume of rod A to that of rod B , we take:

$$\frac{\Delta V_A}{\Delta V_B} = \frac{4\beta\pi r^2 l \Delta T}{2\beta\pi r^2 l \Delta T} = \frac{4}{2} = 2$$

Thus, the ratio of the increase in volume of rod A to that of rod B is: **2:1**.

Therefore, the correct option is **Option C: 2:1**.

Question 132

In a single slit experiment, the width of the slit is doubled. Which one of the following statements is correct?

Options:

- A. The intensity and width of the central maximum are unaffected.
- B. The intensity remains same and angular width becomes half.
- C. The intensity and angular width both are doubled.
- D. The intensity increases by a factor 4 and the angular width decreases by a factor of $\frac{1}{2}$.

Answer: D

Solution:

For single slit diffraction,

$$\text{Fringe width } W = \frac{\lambda D}{d}$$

$$\text{Also, } I \propto D^2$$

$I \propto D^2$ This means I will be 4 times the original value of I . As only 1 option mentions this, the correct answer is D.

Question 133

A gas at N.T.P. is suddenly compressed to $\left(\frac{1}{4}\right)^{\text{th}}$ of its original volume. The final pressure in (Given $\gamma = \text{ratio of sp. heats} = \frac{3}{2}$) atmosphere is (P = original pressure)

Options:

A. 4P

B. $\frac{3}{2}P$

C. 8P

D. $\frac{1}{4}P$

Answer: C

Solution:

In Adiabatic compression,

$$PV^\gamma = \text{constant}$$

$$\text{Given } V_{\text{new}} = \frac{1}{4}V \text{ and } \gamma = \frac{3}{2}$$

$$\therefore \frac{P_{\text{new}}}{P} = \left(\frac{V}{V_{\text{new}}} \right)^\gamma = \left(\frac{V}{\frac{1}{4}V} \right)^{3/2}$$

$$\therefore P_{\text{new}} = 8P$$

Question 134

The excess of pressure in a first soap bubble is three times that of other soap bubble. Then the ratio of the volume of first bubble to other is

Options:

A. 1 : 3

B. 27 : 1

C. 1 : 9

D. 1 : 27

Answer: D

Solution:

$$\Delta P \propto \frac{1}{r} \Rightarrow \frac{r_1}{r_2} = \frac{\Delta P_2}{\Delta P_1} = \frac{1}{3}$$
$$\therefore \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{27}$$

Question 135

In a meter bridge experiment null point is obtained at l cm from the left end. If the meter bridge wire is replaced by a wire of same material but twice the area of across-section, then the null point is obtained at a distance

Options:

- A. $2l$ cm from left end.
- B. l cm from left end.
- C. $l/2$ cm from left end.
- D. $l/4$ cm from left end.

Answer: B

Solution:

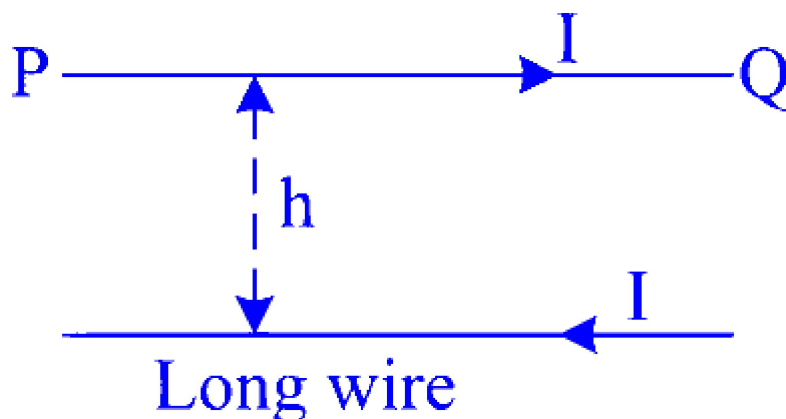
At null condition, $\frac{R_1}{R_2} = \frac{X}{100-X}$

This condition is independent of the cross-sectional area of the wire used.

$\therefore$ The null point will remain the same. (i.e., l cm from left end)

Question 136

A long straight wire carrying a current of 25 A rests on the table. Another wire PQ of length 1 m and mass 2.5 g carries the same current but in the opposite direction. The wire PQ is free to slide up and down. To what height will wire PQ rise? ($\mu_0 = 4\pi \times 10^{-7}$ SI unit)



Options:

- A. 3 mm
- B. 4 mm
- C. 5 mm
- D. 8 mm

Answer: C

Solution:

Given $I_1 = I_2 = 25$ A, $l = 1$ m,

$$B = \frac{\mu_0 I}{2\pi h}$$

$$F = BIl \sin \theta = BIl$$

Force applied on PQ = Weight of the smaller current carrying wire

$$\text{i.e., } mg = \frac{\mu_0 I^2 l}{2\pi h}$$

$$\therefore h = \frac{4\pi \times 10^{-7} \times 250 \times 25 \times 1}{2\pi \times 2.5 \times 10^{-3} \times 9.8} = 5 \text{ mm}$$

In a thermodynamic process, there is no exchange of heat between the system and surroundings. Then the thermodynamic process is

Options:

- A. isothermal
- B. isobaric
- C. isochoric
- D. adiabatic

Answer: D

Question 138

According to kinetic theory of gases, which one of the following statements is wrong?

Options:

- A. All the molecules of a gas are identical.
- B. Collisions between the molecules of a gas and that of the molecules with the walls of the container are perfectly elastic.
- C. The molecules do not exert appreciable force on one another except during collision.
- D. The pressure exerted by a gas is due to the collision between the molecules of the gas.

Answer: D

Question 139

The radii of two soap bubbles are r_1 and r_2 . In isothermal condition they combine with each other to form a single bubble. The radius of resultant bubble is

Options:

A. $R = \frac{r_1 + r_2}{2}$

B. $R = r_1 (r_1 r_2 + r_2)$

C. $R = \sqrt{r_1^2 + r_2^2}$

D. $R = r_1 + r_2$

Answer: C

Solution:

Under isothermal condition, T is constant.

This means the surface energy of the bubbles before combining will be equal to the surface energy after combining.

$$\text{i.e. } 4\pi r_1^2 T + 4\pi r_2^2 T = 4\pi R^2 T$$

$$\Rightarrow r_1^2 + r_2^2 = R^2$$

$$\therefore R = \sqrt{r_1^2 + r_2^2}$$

Question 140

The rays of different colours fail to converge at a point after passing through a thick converging lens. This defect is called

Options:

A. spherical aberration

B. distortion

C. coma

D. chromatic aberration

Answer: D

Question 141

The ratio of potential difference that must be applied across parallel and series combination of two capacitors C_1 and C_2 with their capacitance in the ratio 1 : 2 so that energy stored in these two cases becomes same is

Options:

A. $3 : \sqrt{2}$

B. $\sqrt{2} : 3$

C. $2 : 9$

D. $9 : 2$

Answer: B

Solution:

Given: $C_1 : C_2 = 1 : 2$

$$\therefore C_2 = 2C_1$$

$$C_P = C_1 + C_2 = C_1 + 2C_1 = 3C_1$$

$$C_S = \frac{C_1 C_2}{C_1 + C_2} = \frac{2C_1^2}{3C_1} = \frac{2}{3}C_1$$

Let V_P and V_S be the potentials applied across the parallel and series combinations respectively, then using $E = \frac{1}{2}CV^2$ we can write,

$$\frac{1}{2}C_P V_P^2 = \frac{1}{2}C_S V_S^2$$

$$\therefore \frac{V_P^2}{V_S^2} = \frac{C_S}{C_P}$$

$$\frac{V_P}{V_S} = \sqrt{\frac{\frac{2}{3}C_1}{3C_1}} = \frac{\sqrt{2}}{3}$$

Question 142

The potential energy of charged parallel plate capacitor is v_0 . If a slab of dielectric constant K is inserted between the plates, then the new

potential energy will be

Options:

A. $\frac{v_0}{K}$

B. $v_0 K^2$

C. $\frac{v_0}{K^2}$

D. v_0^2

Answer: A

Solution:

We know, $v_0 = \frac{Q^2}{2C}$

On inserting the slab of dielectric constant k , the new capacitance $C' = KC$

$\therefore$ New potential energy $v'_0 = \frac{Q^2}{2C'}$

$$v_0^1 = \frac{Q^2}{2KC} = \frac{v_0}{K}$$

Question 143

A solid sphere rolls without slipping on an inclined plane at an angle θ . The ratio of total kinetic energy to its rotational kinetic energy is

Options:

A. $\frac{7}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{3}$

D. $\frac{5}{4}$

Answer: A

Solution:

Moment of Inertia of a solid sphere, $I = \frac{2}{5}MR^2$

Since there is no slipping,

$$v = R\omega$$

$\therefore$ Rotational kinetic energy

$$\begin{aligned} E_{\text{rot}} &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2} \times \frac{2}{5} \times M \times R^2 \times \omega^2 \\ &= \frac{MR^2\omega^2}{5} \\ &= \frac{MV^2}{5} \quad \dots (i) \end{aligned}$$

Total kinetic energy

$$\begin{aligned} E_K &= \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2 \\ &= \frac{MV^2}{5} + \frac{MV^2}{2} \\ &= \frac{7MV^2}{10} \quad \dots (ii) \end{aligned}$$

Dividing (ii) by (i), we get,

$$\frac{E_K}{E_{\text{rot}}} = \frac{\left(\frac{7MV^2}{10}\right)}{\left(\frac{MV^2}{5}\right)} = \frac{7}{2}$$

Question 144

Two discs of same mass and same thickness (t) are made from two different materials of densities ' d_1 ' and ' d_2 ' respectively. The ratio of the moment of inertia I_1 to I_2 of two discs about an axis passing through the centre and perpendicular to the plane of disc is

Options:

A. $d_1 : d_2$

B. $d_2 : d_1$

C. $1 : d_1 d_2$

D. $1 : d_1^2 d_2$

Answer: B

Solution:

The ratio of moments of inertia of two discs of the same mass and same thickness but of different densities is given by $\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2} = \frac{d_2}{d_1}$

Question 145

When a string of length ' l ' is divided into three segments of length l_1, l_2 and l_3 . The fundamental frequencies of three segments are n_1, n_2 and n_3 respectively. The original fundamental frequency ' n ' of the string is

Options:

A. $n = n_1 + n_2 + n_3$

B. $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$

C. $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

D. $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$

Answer: C

Solution:

The fundamental frequency of a string is given by

Given: $l = l_1 + l_2 + l_3 \dots (i)$

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

$$\Rightarrow n \propto \frac{1}{l} \text{ or } nl = k$$

$$\therefore l_1 = \frac{k}{n_1}, l_2 = \frac{k}{n_2} \text{ and } l_3 = \frac{k}{n_3} \dots (ii)$$

∴ Original length $l = \frac{k}{n}$ (iii)

Putting eq (ii) and (iii) into eq (i)

$$\frac{k}{n} = \frac{k}{n_1} + \frac{k}{n_2} + \frac{k}{n_3}$$
$$\therefore \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$$

Question 146

A seconds pendulum is placed in a space laboratory orbiting round the earth at a height '3R' from the earth's surface. The time period of the pendulum will be (R = radius of earth)

Options:

A. zero

B. $\frac{2}{3}$ s

C. 4 s

D. infinite

Answer: D

Solution:

In outer space, $g = 0$. Therefore, $T = \infty$.

Question 147

A solid metallic sphere has a charge $+3Q$. Concentric with this sphere is a conducting spherical shell having charge $-Q$. The radius of the sphere is 'A' and that of the spherical shell is 'B'. ($B > A$). The electric field at a distance 'R' ($A < R < B$) from the centre is (ϵ_0 = permittivity of vacuum)

Options:

A. $\frac{Q}{2\pi\epsilon_0 R}$

B. $\frac{3Q}{2\pi\epsilon_0 R}$

C. $\frac{3Q}{4\pi\epsilon_0 R^2}$

D. $\frac{4Q}{2\pi\epsilon_0 R^2}$

Answer: C

Solution:

We know the electric field inside a shell is zero. So, we only consider the electric field due to the solid sphere.

$\therefore$ Using $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$, we get

$$E = \frac{1}{4\pi\epsilon_0} \frac{(3Q)}{R^2}$$

Question 148

A closed organ pipe of length ' L_1 ' and an open organ pipe contain diatomic gases of densities ' ρ_1 ' and ' ρ_2 ' respectively. The compressibilities of the gases are same in both pipes, which are vibrating in their first overtone with same frequency. The length of the open organ pipe is (Neglect end correction)

Options:

A. $\frac{4}{3} L_1$

B. $\frac{4L_1}{3} \sqrt{\frac{\rho_1}{\rho_2}}$

C. $\frac{4L_1}{3} \sqrt{\frac{\rho_2}{\rho_1}}$

D. $\frac{3}{4L_1} \sqrt{\frac{\rho_1}{\rho_2}}$

Answer: B

Solution:

Given both gases are vibrating in the first overtone with same frequency, we get

$$f_{\text{closed}} = f_{\text{open}}$$

$$\Rightarrow \frac{3v}{4L_1} = \frac{v}{L_2}$$

According to Laplace's correction

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\frac{3}{4L_1} \times \sqrt{\frac{\gamma P}{\rho_1}} = \frac{1}{L_2} \times \sqrt{\frac{\gamma P}{\rho_2}}$$

$$\therefore L_2 = \frac{4L_1}{3} \sqrt{\frac{\rho_1}{\rho_2}}$$

Question 149

From a metallic surface photoelectric emission is observed for frequencies v_1 and v_2 ($v_1 > v_2$) of the incident light. The maximum values of the kinetic energy of the photoelectrons emitted in the two cases are in the ratio 1 : x. Hence the threshold frequency of the metallic surface is

Options:

A. $\frac{v_1 - v_2}{x}$

B. $\frac{v_1 - v_2}{x - 1}$

C. $\frac{xv_1 - v_2}{x - 1}$

D. $\frac{xv_2 - v_1}{x - 1}$

Answer: C

Solution:

Using Einstein's photoelectric equation,

$$E_k = h\nu - \phi_0$$

$$E_k = h\nu - h\nu_0$$

$$\therefore E_{K_1} = h(v_1 - v_0) \text{ and } E_{K_2} = h(v_2 - v_0)$$

$$\text{Given } \frac{E_{K_1}}{E_{K_2}} = \frac{1}{x}$$

$$\Rightarrow \frac{v_1 - v_0}{v_2 - v_0} = \frac{1}{x}$$

$$(v_1 - v_0)x = v_2 - v_0$$

$$v_1x - v_0x = v_2 - v_0$$

$$\therefore v_1x - v_2 = v_0x - v_0$$

$$v_1x - v_2 = v_0(x - 1)$$

$$\therefore v_0 = \frac{v_1x - v_2}{x - 1}$$

Question 150

In an AC circuit, the current is $i = 5 \sin(100t - \frac{\pi}{2})$ A and voltage is $e = 200 \sin(100t)$ volt. Power consumption in the circuit is $(\cos 90^\circ = 0)$

Options:

A. 200 W

B. 0 W

C. 40 W

D. 1000 W

Answer: B

Solution:

$$P = I_{\text{rms}} V_{\text{rms}} \cos \phi$$

$$\phi = 90^\circ$$

$$\therefore P = 0 \quad (\because \cos 90^\circ = 0)$$