

Question 1

With usual notations, in any $\triangle ABC$, if $a \cos B = b \cos A$, then the triangle is

Options:

- A. an isosceles triangle
- B. an equilateral triangle
- C. a right angled triangle
- D. a scalene triangle

Answer: A

Solution:

Solution:

Using sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = K$$

$$\text{We have, } \frac{\cos A}{a} = \frac{\cos B}{b}$$

$$\Rightarrow \frac{\cos A}{K \sin A} = \frac{\cos B}{K \sin B}$$

$$\Rightarrow K \sin B \cos A = K \sin A \cos B$$

$$\Rightarrow \cos A \sin B - \cos B \sin A = 0$$

$$\Rightarrow \sin(A - B) = 0$$

$$A = B$$

$\therefore$ Triangle is an isosceles triangle.

Question 2

If θ lies in the first quadrant and $5 \tan \theta = 4$, then $\frac{5 \sin \theta - 3 \cos \theta}{\sin \theta + 2 \cos \theta}$ is equal to

Options:

A. $\frac{5}{14}$

B. $\frac{3}{14}$

C. $\frac{1}{14}$

D. 0

Answer: A

Solution:

Solution:

Given, $\tan \theta = \frac{4}{5}$

$\therefore \sin \theta = \frac{4}{\sqrt{41}}$ and $\cos \theta = \frac{5}{\sqrt{41}}$

Now,
$$\frac{5 \sin \theta - 3 \cos \theta}{\sin \theta + 2 \cos \theta} = \frac{5 \times \frac{4}{\sqrt{41}} - 3 \times \frac{5}{\sqrt{41}}}{\frac{4}{\sqrt{41}} + 2 \times \frac{5}{\sqrt{41}}} = \frac{5}{14}$$

Question 3

If $\cos^{-1}x > \sin^{-1}x$, then

Options:

A. $\frac{1}{\sqrt{2}} < x \leq 1$

B. $0 \leq x \leq \frac{1}{\sqrt{2}}$

C. $-1 \leq x < \frac{1}{\sqrt{2}}$

D. $x > 0$

Answer: C

Solution:

Solution:

$\cos^{-1}x > \sin^{-1}x$ [where $x \in [-1, 1]$]

$\therefore \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$\therefore \frac{\pi}{2} - \sin^{-1}x > \sin^{-1}x \Rightarrow 2\sin^{-1}x < \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x < \frac{\pi}{4} \Rightarrow x < \frac{1}{\sqrt{2}}$

$\therefore -1 \leq x \leq 1$

$-1 \leq x < \frac{1}{\sqrt{2}}$

Question 4

The value of $\tan 3A - \tan 2A - \tan A$ is

Options:

A. $\tan 3A \tan 2A \tan A$

B. $-\tan 3A \tan 2A \tan A$

C. $\tan A \tan 2A - \tan 2A \tan 3A$

D. $\tan 3A \tan A - \tan 2A \tan 3A$

Answer: A**Solution:****Solution:**

Let $3A = A + 2A$

$\tan 3A = \tan(A + 2A)$

$$\tan 3A = \frac{\tan A + \tan 2A}{1 - \tan A \cdot \tan 2A}$$

$$\Rightarrow \tan A + \tan 2A = \tan 3A - \tan 3A \tan 2A \cdot \tan A$$

$$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$$

Question 5

The three straight lines $ax + by = c$, $bx + cy = a$ and $cx + ay = b$ are collinear, if

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Options:

A. $b + c = a$

B. $c + a = b$

C. $a + b + c = 0$

D. $a + b = c$

Answer: C**Solution:****Solution:**Given lines are $ax + by = c$, $bx + cy = a$ and $cx + ay = b$

On adding the given three equations, we get

$$ax + by + bx + cy + cx + ay = a + b + c$$

$$\Rightarrow (a + b + c)x + (a + b + c)y = (a + b + c)$$

On comparing with $0x + 0y = 0$ for collinearity, we get

$$a + b + c = 0$$

Question 6

A line through the point $A(2, 0)$ which makes an angle of 30° with the positive direction of X-axis is rotated about A in clockwise direction

through an angle of 15° . Then, the equation of the straight line in the new position is

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Options:

A. $(2 - \sqrt{3})x + y - 4 + 2\sqrt{3} = 0$

B. $(2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$

C. $(2 - \sqrt{3})x - y + 4 + 2\sqrt{3} = 0$

D. $(2 - \sqrt{3})x + y + 4 + 2\sqrt{3} = 0$

Answer: B

Solution:

Solution:

The equation of line in new position is

$$y - 0 = \tan 15^\circ (x - 2)$$

$$\Rightarrow y = (2 - \sqrt{3})(x - 2)$$

$$\Rightarrow (2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$$

Question 7

Find the value of $\cos\left(\frac{x}{2}\right)$, if $\tan x = \frac{5}{12}$ and x lies in third quadrant.

Options:

A. $\frac{5}{\sqrt{13}}$

B. $\frac{5}{\sqrt{26}}$

C. $\frac{5}{13}$

D. $\sqrt{\frac{1}{26}}$

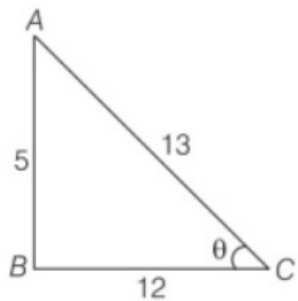
Answer: D

Solution:

Solution:

Given, $\tan x = \frac{5}{12}$ and x lies in III quadrant.

$$\therefore \sin x = \frac{-5}{13} \text{ and } \cos x = \frac{-12}{13}$$



Now, $\cos x = 2\cos^2 \frac{x}{2} - 1$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{2}(\cos x + 1)$$

$$= \frac{1}{2} \left(\frac{-12}{13} + 1 \right) = \frac{1}{2} \left(\frac{1}{13} \right) = \frac{1}{26}$$

$$\therefore \cos \frac{x}{2} = \sqrt{\frac{1}{26}}$$

Question 8

The locus of a point which moves, so that the ratio of the length of the tangents to the circles $x^2 + y^2 + 4x + 3 = 0$ and $x^2 + y^2 - 6x + 5 = 0$ is **2 : 3**, is

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Options:

A. $5x^2 + 5y^2 - 60x + 7 = 0$

B. $5x^2 + 5y^2 + 60x - 7 = 0$

C. $5x^2 + 5y^2 - 60x - 7 = 0$

D. $5x^2 + 5y^2 + 60x + 7 = 0$

Answer: D

Solution:

Solution:

Since, $\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{2}{3}$

$$\therefore \frac{\sqrt{x_1^2 + y_1^2 + 4x_1 + 3}}{\sqrt{x_1^2 + y_1^2 - 6x_1 + 5}} = \frac{2}{3}$$

$$\Rightarrow 9x_1^2 + 9y_1^2 + 36x_1 + 27 - 4x_1^2 - 4y_1^2 + 24x_1 - 20 = 0$$

$$\Rightarrow 5x_1^2 + 5y_1^2 + 60x_1 + 7 = 0$$

$\therefore$ Locus of point (x, y) is

$$5x^2 + 5y^2 + 60x + 7 = 0$$

Question 9

The circles $x^2 + y^2 + 6x + 6y = 0$ and $x^2 + y^2 - 12x - 12y = 0$

Options:

- A. cut orthogonally
- B. touch each other internally
- C. intersect two points
- D. touch each other externally

Answer: D**Solution:****Solution:**

The centres of given circles are $C_1(-3, -3)$, $C_2(6, 6)$ and radii are

$r_1 = \sqrt{9 + 9 + 0} = 3\sqrt{2}$, $r_2 = \sqrt{36 + 36 + 0} = 6\sqrt{2}$, respectively.

Now, $C_1C_2 = \sqrt{(6 + 3)^2 + (6 + 3)^2} = 9\sqrt{2}$

and $r_1 + r_2 = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}$

Here, $C_1C_2 = r_1 + r_2$

So, both circles touch each other externally.

Question 10

The mean and variance of n observations $x_1, x_2, x_3, \dots, x_n$ are 5 and 0, respectively. If $\sum_{i=1}^n x_i^2 = 400$, then the value of n is equal to

Options:

- A. 80
- B. 25
- C. 20
- D. 16

Answer: D**Solution:****Solution:**

$\bar{x} = 5$, variance = $\frac{1}{n} \sum x_i^2 - (\bar{x})^2$

$$\Rightarrow 0 = \frac{1}{n} \cdot 400 - 25$$

$$\Rightarrow n = \frac{400}{25} = 16$$

Question 11

The variance of first n natural numbers is

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Options:

A. $\frac{n(n+1)}{2}$

B. $\frac{(n+1)(n+5)}{12}$

C. $\frac{(n+1)(n-5)}{12}$

D. $\frac{(n^2-1)}{12}$

Answer: D

Solution:

Solution:

Variance of first n natural numbers

$$\begin{aligned} &= \frac{\sum n^2}{n} - \left(\frac{\sum n}{n} \right)^2 \\ &= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n} \right)^2 \\ &= (n+1) \left[\frac{2n+1}{6} - \frac{(n+1)}{4} \right] \\ &= \frac{(n+1)}{12} [4n+2-3n-3] \\ &= \frac{(n+1)}{12} \times (n-1) = \frac{n^2-1}{12} \end{aligned}$$

Question 12

Out of 50 tickets numbered 00, 01, 02, ..., 49, one ticket is drawn randomly, the probability of the ticket having the product of its digits 7 , given that the sum of the digits is 8 , is

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Options:

A. $\frac{1}{14}$

B. $\frac{3}{14}$

C. $\frac{1}{5}$

D. None of these

Answer: C

Solution:

Solution:

Total number of cases = ${}^{50}C_1 = 50$

Let A be the event of selecting ticket with sum of digits ' 8 '.

Favourable cases to A are {08, 17, 26, 35, 44}.

Let B be the event of selecting ticket with product of its digits ' 7 '.

Favourable cases to B is only {17}.

$$\text{Now, } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1 / 50}{5 / 50} = \frac{1}{5}$$

Question 13

The probability of India winning a test match against South Africa is 1/2 assuming independence form match to match played. The probability that in a match series India's second win occurs at the third day is

Options:

A. $\frac{1}{8}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{2}{3}$

Answer: C

Solution:**Solution:**

Given, probability of winning a test match, $P(W) = \frac{1}{2}$

Probability of lossing a match, $P(L) = 1 - \frac{1}{2} = \frac{1}{2}$

Probability that India's second win occurs at the third day

$$= P(L) \cdot P(W) \cdot P(W) + P(W) \cdot P(L) \cdot P(W)$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

Question 14

Consider the following statements

r : If a number is a multiple of 9 , then it is multiple of 3.

Let p and q denote the statements

p: A number is a multiple of 9.

q : A number is a multiple of 3 .

Then, if p then q is the same as

Options:

- A. p only if q
- B. q is a necessary condition for p
- C. $\sim q$ implies $\sim p$
- D. All of the above

Answer: D

Solution:**Solution:**

I. p only if q . This says that number is a multiple of 9 only, if it is a multiple of 3.

II. q is a necessary condition for p .

This says that when a number is a multiple of 9, it is necessarily a multiple of 3.

III. $\sim q$ implies $\sim p$. This says that if a number is not a multiple of 3, then it is not a multiple of 9.

Question 15

The negation of the statement 7 is greater than 4 or 6 is less than 7 .

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Options:

- A. 7 is not greater than 4 and 6 is not less than 7
- B. 7 is not greater than 4 or 6 is not less than 7
- C. 7 is greater than 4 and 6 is less than 7
- D. None of the above

Answer: A

Solution:**Solution:**

Let p : 7 is greater than 4

and q : 6 is less than 7 .

Then, the given statement is disjunction $p \vee q$.

Here, $\sim p$: 7 is not greater than 4 .

and $\sim q$: 6 is not less than 7 .

$\therefore \sim(p \vee q)$: 7 is not greater than 4 and 6 is not less than 7 .

Question 16

For an invertible matrix A, if $A(\text{adj} A) = \begin{vmatrix} 20 & 0 \\ 0 & 20 \end{vmatrix}$, then $|A| =$

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Options:

- A. 20
- B. -200
- C. 200
- D. -20

Answer: A**Solution:****Solution:**

$$\text{Given, } A(\text{adj}A) = \begin{vmatrix} 20 & 0 \\ 0 & 20 \end{vmatrix}$$

$$20 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 20I$$

We know that

$$A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\Rightarrow A(\text{adj}A) = |A| I$$

$$\Rightarrow 20I = |A| I$$

$$\Rightarrow |A| = 20$$

Question 17

If matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $A^2 - 4A + 5I$ is where I is a unit matrix.

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Options:

- A. Null matrix
- B. Skew symmetric matrix
- C. symmetric Matrix
- D. None of the above

Answer: A**Solution:****Solution:**

$$\text{Given, } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\begin{aligned} \therefore A^2 &= A \cdot A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + (-1) \times 2 & 1 \times (-1) + (-1) \times 3 \\ 2 \times 1 + 3 \times 2 & 2 \times (-1) + 3 \times 3 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

Now, $A^2 - 4A + 5I$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - 4 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Question 18

If $f(x) = \begin{cases} \frac{3 \sin \pi x}{5x} & x \neq 0 \\ 2k & x = 0. \end{cases}$ is continuous at $x = 0$, then the value of k is

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Options:

A. $\frac{\pi}{10}$

B. $\frac{3\pi}{10}$

C. $\frac{3\pi}{2}$

D. $\frac{3\pi}{5}$

Answer: D

Solution:

Solution:

Given, $f(x) = \begin{cases} \frac{3 \sin \pi x}{5x} & x \neq 0 \\ 2k & x = 0. \end{cases}$

Now, $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left(\frac{3 \sin \pi x}{5x} \right)$

$= \frac{3}{5} \lim_{x \rightarrow 0} \left(\sin \frac{\pi x}{\pi x} \right) \times \pi = \frac{3}{5} \times 1 \times \pi = \frac{3}{5} \pi$

Also, $f(0) = 2k$

Since, $f(x)$ is continuous at $x = 0$.

$\therefore f(0) = \lim_{x \rightarrow 0} f(x)$

$\Rightarrow 2k = \frac{3}{5} \pi \Rightarrow k = \frac{3\pi}{10}$

Question 19

If $f(x) = \begin{cases} \frac{\sin^3(\sqrt{3}) \cdot \log(1+3x)}{(\tan^{-1}\sqrt{x})^2(e^{5\sqrt{3}}-1)x} & x \neq 0 \\ a & x = 0. \end{cases}$ is continuous in $[0, 1]$ then a equals

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Options:

A. 0

B. $\frac{3}{5}$

C. 2

D. $\frac{5}{3}$

Answer: B

Solution:

Solution:

We have, $f(x) = \begin{cases} \frac{\sin^3(\sqrt{3}) \cdot \log(1+3x)}{(\tan^{-1}\sqrt{x})^2(e^{5\sqrt{3}}-1)x} & x \neq 0 \\ a & x = 0. \end{cases}$

For continuity in $[0, 1]$, $f(0) = \lim_{x \rightarrow 0} f(x)$ otherwise it is discontinuous.

$$\begin{aligned} \therefore a &= \lim_{x \rightarrow 0} \frac{\sin^3(\sqrt{x}) \cdot \log(1+3x)}{x(\tan^{-1}\sqrt{x})^2 \cdot (e^{5\sqrt{x}}-1)} \\ &= \lim_{x \rightarrow 0} \left[\frac{3}{5} \cdot \frac{\sin^3\sqrt{x}}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^3}{(\tan^{-1}\sqrt{x})^3} \right. \\ &\quad \left. \& \times \frac{\log(1+3x)}{3x} \cdot \frac{5\sqrt{x}}{e^{5\sqrt{x}}-1} \right] \\ &= \frac{3}{5} \lim_{x \rightarrow 0} \frac{\sin^3}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^3}{\tan^{-1}\sqrt{x}} \\ \therefore a &= \frac{3}{5} \end{aligned}$$

Question 20

The differential equation of all parabolas having vertex at the origin and axis along positive Y-axis is

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Options:

A. $x^2 \frac{dy}{dx} - y = 0$

B. $x \frac{dy}{dx} + 2y = 0$

C. $x \frac{dy}{dx} = 2y$

D. $x^3 \frac{dy}{dx} = 3y$

Answer: C

Solution:

Solution:

The general equation of an parabola having vertex at the origin and axis along positive Y -axis is

$$x^2 = 4ay. \dots (i)$$

On differentiating Eq. (i), we get

$$2x = 4a \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2a} \Rightarrow 2a = \frac{x}{dy/dx}$$

Putting value of 2a in Eq. (i), we get

$$x^2 = 2 \left(\frac{x}{dy/dx} \right) y \Rightarrow x \frac{dy}{dx} = 2y$$

Question 21

The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4 \frac{dy}{dx} - 7x = 0$ are

Options:

A. 1 and 1/2

B. 2 and 1

C. 1 and 1

D. 1 and 2

Answer: D

Solution:

Solution:

Given equation is $\sqrt{\frac{dy}{dx}} - 4 \frac{dy}{dx} - 7x = 0$

$$\Rightarrow \frac{dy}{dx} = 16 \left(\frac{dy}{dx} \right)^2 + 49x^2 + 56x \frac{dy}{dx}$$

Obviously, it is of first order and second degree differential equation.

Question 22

The solution of the differential equation

$e^{-x}(y + 1)dy + (\cos^2 x - \sin 2x)ydx = 0$ subjected to the condition that $y = 1$, when $x = 0$ is

Options:

A. $y + \log y + e^x \cos^2 x = 2$

B. $\log(y + 1) + e^x \cos^2 x = 1$

C. $y + \log y = e^x \cos^2 x$

D. $(y + 1) + e^x \cos^2 x = 2$

Answer: A**Solution:****Solution:**

Given equation can be rewritten as

$$\left(1 + \frac{1}{y}\right) dy = -e^x (\cos^2 x - \sin 2x) dx$$

On integrating both sides, we get

$$y + \log y = -e^x \cos^2 x + \int e^x \sin 2x dx$$

$$- \int e^x \sin 2x dx + C$$

$$\Rightarrow y + \log y = -e^x \cos^2 x + C$$

At $x = 0$ and $y = 1$,

$$1 + 0 = -e^0 \cos 0 + C$$

$$C = 2 \text{ [given]}$$

 $\therefore$ Required solutions is

$$y + \log y = -e^x \cos^2 x + 2$$

$$\Rightarrow y + \log y + e^x \cos^2 x = 2$$

Question 23

$$\int_{-1}^1 \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2 + 1} dx \text{ is equal to}$$

Options:

A. $\frac{4}{5}$

B. $\frac{5}{4}$

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer: C**Solution:****Solution:**

$$\int_{-1}^1 \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2 + 1} dx$$

$$\begin{aligned}
 &= \int_{-1}^1 \frac{17x^5 + 29x^3 - 31x}{x^2 + 1} dx - \int_{-1}^1 \frac{x^4 - 1}{x^2 + 1} dx \\
 &\quad \text{Odd function} \qquad \qquad \qquad \text{Even function} \\
 &= 0 - 2 \int_0^1 \frac{(x^2 - 1)(x^2 + 1)}{(x^2 + 1)} dx = -2 \left[\left(\frac{x^3}{3} - x \right) \right]_0^1 = \frac{4}{3}
 \end{aligned}$$

Question 24

The number of ways of arranging letters of the 'HAVANA', so that V and N do not appear together, is

Options:

- A. 60
- B. 80
- C. 100
- D. 120

Answer: B

Solution:

Solution:
 Given word is 'HAVANA' (3A, 1H, 1N, 1V)
 Total number of ways of arranging the given word
 $= \frac{6!}{3!} = 120$
 Total number of words in which N, V together
 $= \frac{5!}{3!} \times 2! = 40$
 $\therefore$ Required number of ways = $120 - 40 = 80$

Question 25

Out of 7 consonants and 4 vowels, the number of words (not necessarily meaningful) that can be made, each consisting of 3 consonants and 2 vowels, is

Options:

- A. 24800
- B. 25100
- C. 25200
- D. 25400

Answer: C

Solution:

Solution:

3 consonants can be selected from 7 consonants = 7C_3 ways

2 vowels can be selected from 4 vowels = 4C_2 ways

∴ Required number of words = ${}^7C_3 \times {}^4C_2 \times 5!$

[selected 5 letters can be arranged in 5! ways, to get a different word]
= $35 \times 6 \times 120 = 25200$

Question 26

If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then $g(f(x))$ is equal to

Options:

- A. 1
- B. -1
- C. 2
- D. -2

Answer: A

Solution:

Solution:

Given $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x$

$\cos\left(x + \frac{\pi}{3}\right)$

$$= \sin^2 x + \left[\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} \right]^3$$

$$+ \cos x \left[\cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3} \right]$$

$$= \sin^2 x + \left[\frac{\sin x}{2} + \cos x \cdot \frac{\sqrt{3}}{2} \right]^2$$

$$\therefore = \cos x \left[\frac{\cos x}{2} - \sin x \cdot \frac{\sqrt{3}}{2} \right]$$

$$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3 \cos 2}{4} + \sin x \cos x \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{5 \sin^2 x}{4} + \frac{5 \cos^2 x}{4} = \frac{5}{4}$$

$$\Rightarrow f(x) = g[f(x)] = g\left(\frac{5}{4}\right) = 1$$

Question 27

Range of the function $f(x) = \frac{x}{1+x^2}$ is

Options:

A. $(-\infty, \infty)$

B. $[-1, 1]$

C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $[-\sqrt{2}, \sqrt{2}]$

Answer: C

Solution:

Solution:

$$\text{Let } y = \frac{x}{1+x^2} \Rightarrow x^2y - x + y = 0$$

$$\text{For } x \text{ to be real, } 1 - 4y^2 \geq 0 \{ \text{Discriminant} = 1 - 4y^2 \}$$

$$\Rightarrow 4y^2 - 1 \leq 0$$

$$\Rightarrow (2y - 1)(2y + 1) \leq 0$$

$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

$$\therefore y = f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Question 28

The domain of $f(x) = \sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$ is

Options:

A. $0 \leq x \leq 1$

B. $0 \leq x \leq 4$

C. $1 \leq x \leq 4$

D. $4 \leq x \leq 6$

Answer: C

Solution:

Solution:

$$\text{Given, } f(x) = \sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$$

$$\Rightarrow -1 \leq \log_2\left(\frac{x}{2}\right) \leq 1 \Rightarrow 2^{-1} \leq \frac{x}{2} \leq 2^1$$

$$\Rightarrow \frac{1}{2} \leq \frac{x}{2} \leq 2 \Rightarrow 1 \leq x \leq 4$$

Question 29

If $f(x) = \frac{k}{2^x}$ is a probability distribution of a random variable X that can take on the values $x = 0, 1, 2, 3, 4$. Then, k is equal to

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Options:

- A. $\frac{16}{15}$
- B. $\frac{15}{16}$
- C. $\frac{31}{16}$
- D. None of these

Answer: D

Solution:

Solution:

We have, $f(x) = \frac{k}{2^x}$, $x = 0, 1, 2, 3, 4$

Since, $f(x)$ is a probability distribution of a random variable X , therefore we have

$$\sum_{x=0}^4 f(x) = 1 \Rightarrow \sum_{x=0}^4 \left(\frac{k}{2^x} \right) = 1$$

$$\Rightarrow k \sum_{x=0}^4 \frac{1}{2^x} = 1$$

$$\Rightarrow k \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} \right) = 1$$

$$\Rightarrow k \left(\frac{16 + 8 + 4 + 2 + 1}{2^4} \right) = 1$$

$$\Rightarrow k \times \left(\frac{31}{16} \right) = 1$$

$$\therefore k = \frac{16}{31}$$

Question 30

$\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$ is equal to

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Options:

- A. 0
- B. $\frac{1}{2}$
- C. 1

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2} & \left[\frac{0}{0} \text{ form} \right] \text{ [using L'Hospital's rule]} \\ &= \lim_{x \rightarrow 0} \frac{2xe^{x^2} + \sin x}{2x} \left[\frac{0}{0} \text{ form} \right] \\ &= \lim_{x \rightarrow 0} \frac{2e^{x^2} + 4x^2e^{x^2} + \cos x}{2} \\ &= \frac{2 + 0 + 1}{2} = \frac{3}{2} \text{ [using L'Hospital's rule]} \end{aligned}$$

Question 31

Evaluate $\lim_{x \rightarrow \sqrt{2}^-} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$.

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Options:

A. $\frac{8}{5}$

B. $\frac{8}{3}$

C. $\frac{5}{8}$

D. $\frac{3}{8}$

Answer: A

Solution:

Solution:

$$\begin{aligned} \lim_{x \rightarrow \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8} \\ &= \lim_{x \rightarrow \sqrt{2}} \frac{(x + \sqrt{2})(x - \sqrt{2})(x^2 + 2)}{(x - \sqrt{2})(x + 4\sqrt{2})} \\ &= \lim_{x \rightarrow \sqrt{2}} \frac{(x + \sqrt{2})(x^2 + 2)}{(x + 4\sqrt{2})} \\ &= \frac{(\sqrt{2} + \sqrt{2})(2 + 2)}{(\sqrt{2} + 4\sqrt{2})} = \frac{(2\sqrt{2})(4)}{5\sqrt{2}} = \frac{8\sqrt{2}}{5\sqrt{2}} = \frac{8}{5} \end{aligned}$$

Question 32

What will be projection of the vector $4\hat{i} - 3\hat{j} + \hat{k}$ on the line joining the

points $(2, 3, -1)$ and $(-2, -4, 3)$?

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Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

Solution:

Let point A = $(2, 3, -1)$ and point B = $(-2, -4, 3)$.

Now the position vector of line joining A and B,

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= -2\hat{i} - 4\hat{j} + 3\hat{k} - 2\hat{i} - 3\hat{j} + \hat{k}$$

$$= -4\hat{i} - 7\hat{j} + 4\hat{k}$$

$$\text{Again let } \vec{a} = \vec{AB} = -4\hat{i} - 7\hat{j} + 4\hat{k}$$

$$\text{and } \vec{b} = 4\hat{i} - 3\hat{j} + \hat{k}$$

$$\text{Then, } \vec{a} \cdot \vec{b} = (-4\hat{i} - 7\hat{j} + 4\hat{k}) \cdot (4\hat{i} - 3\hat{j} + \hat{k})$$

$$= -16 + 21 + 4 = 9$$

$$|\vec{a}| = \sqrt{(-4)^2 + (-7)^2 + (4)^2}$$

$$= \sqrt{16 + 49 + 16} = 9$$

Now projection of b on a

$$= \frac{(\vec{a} \cdot \vec{b})}{|\vec{a}|} = \frac{9}{9} = 1$$

Question 33

If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k}$, then $|\vec{a} - 2\vec{b}|$ will be

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Options:

- A. 9
- B. $\sqrt{86}$
- C. $\sqrt{94}$
- D. 10

Answer: B

Solution:

Solution:

$$a = 3\hat{i} - 2\hat{j} + \hat{k} \dots (i)$$

$$b = 2\hat{i} - 4\hat{j} - 3\hat{k} \dots (ii)$$

Multiplying by 2 both sides, we get

$$2b = 4\hat{i} - 8\hat{j} - 6\hat{k} \dots (iii)$$

Subtracting Eq. (iii) from Eq. (i), we get

$$\begin{aligned} a - 2b &= (3\hat{i} - 2\hat{j} + \hat{k}) - (4\hat{i} - 8\hat{j} - 6\hat{k}) \\ &= -\hat{i} + 6\hat{j} + 7\hat{k} \\ &= \sqrt{(-1)^2 + (6)^2 + (7)^2} = \sqrt{86} \end{aligned}$$

Question 34

What will be the equation of plane passing through a point (1, 4, -2) and parallel to the given plane $-2x + y - 3z = 9$?

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Options:

A. $2x - y + 3z + 8 = 0$

B. $2x = y + 3z + 8 = 0$

C. $2x - 2y + 2z + 10 = 0$

D. $2x + 2y - 3z + 8 = 0$

Answer: A

Solution:

Solution:

$$r \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + 2 = 0$$

Equation of plane parallel to the given plane is

$$-2x + y - 3z = \lambda \dots (i)$$

Since, plane (i) passes through the point (1, 4, -2).

Therefore this point satisfy the given plane. Put $x = 1$, $y = 4$ and $z = -2$ in Eq. (i)

$$-2(1) + 4 - 3(-2) = \lambda$$

$$-2 + 4 + 6 = \lambda$$

$$\lambda = 8$$

Put $\lambda = 8$ in Eq. (i), we get

$$-2x + y - 3z = 8$$

$$2x - y + 3z + 8 = 0$$

Question 35

If the line joining two points A(2, 0) and B(3, 1) is rotated about A in anticlockwise direction through an angle of 15° , then the equation of the line in new position is

Options:

A. $y = -\sqrt{3}x + 2\sqrt{3}$

B. $y = 3x - 6$

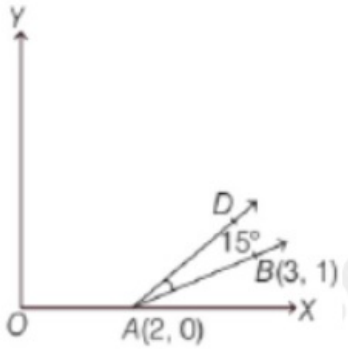
C. $y = \frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$

D. $y = \sqrt{3}x - 2\sqrt{3}$

Answer: D

Solution:

Solution:



The slope of the line is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{3 - 2} = 1$ or $\tan 45^\circ$ rotated anti-clockwise direction the line through 15° the slope of line AD in new position. New position will be $\tan 60^\circ = \sqrt{3}$.
Therefore, the equation of the new line AD is
 $y - 0 = \sqrt{3}(x - 2)$
 $y = \sqrt{3}x - 2\sqrt{3}$

Question 36

$$\int \frac{(\log x)^2}{x} dx$$

Options:

A. $\left(\frac{\log x}{x} \right)^3 + C$

B. $\left(\frac{\log x}{3} \right)^3 + C$

C. $\frac{(\log x)^3}{2} + C$

D. $\frac{(\log x)^3}{3} + C$

Answer: D

Solution:

Solution:

$$\text{Let } I = \int \frac{(\log x)^2}{x} dx$$

$$\text{Put } \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$\begin{aligned}\therefore I &= \int \frac{(\log x)^2}{x} dx = \int t^2 dt \\ &= \frac{t^3}{3} + C = \frac{(\log x)^3}{3} + C \quad [\text{put } t = \log x]\end{aligned}$$

Question 37

$$\int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx =$$

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Options:

A. $\frac{-5}{2}[\tan \sqrt{x}]^5 + C$

B. $[\tan \sqrt{x}]^5 + C$

C. $\frac{2}{5}[\tan \sqrt{x}]^5 + C$

D. $\frac{5}{2}[\tan \sqrt{x}]^5 + C$

Answer: C

Solution:

Solution:

$$\text{Let } I = \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx$$

$$\text{Put } \tan \sqrt{x} = t$$

$$\Rightarrow \frac{\sec^2 \sqrt{x}}{2\sqrt{x}} dx = dt$$

$$\therefore I = \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx = \int 2t^4 dt$$

$$= \frac{2t^5}{5} + C = \frac{2(\tan \sqrt{x})^5}{5} + C$$

Question 38

The slant height of a right circular cone is 3cm. The height of the cone for maximum volume is

Options:

A. 5cm

B. $\sqrt{3}$ cm

C. 3cm

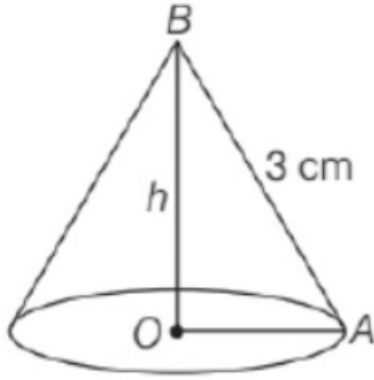
D. $\sqrt{5}$ cm

Answer: B

Solution:

Solution:

Let height of a right circular cone = h cm and $OA = r$ cm



Given, slant height of a right circular cone = 3 cm In $\triangle OAB$,

$$\angle BOA = 90^\circ$$

$$(OB)^2 + (OA)^2 = (AB)^2$$

[apply pythagoras theorem]

$$(h)^2 + (r)^2 = (3)^2$$

$$r = \sqrt{9 - h^2} \dots (i)$$

We know that, Volume of cone

$$= \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3} (9 - h^2) \times h$$

$$[\text{ from Eq. (i), } r = \sqrt{9 - h^2}]$$

$$V = \frac{\pi}{3} (9h - h^3)$$

$$\frac{dV}{dh} = \frac{\pi}{3} (9 - 3h^2)$$

$$\therefore \frac{dV}{dh} = 0 \Rightarrow \frac{\pi}{3} (9 - 3h^2) = 0$$

$$\Rightarrow \frac{h^2}{3} = \frac{9}{3} = 3 \Rightarrow h = \sqrt{3}$$

$$\frac{d^2V}{dh^2} = \frac{-6 \times h}{3} < 0$$

So, $h = \sqrt{3}$ of the cone for maximum volume

Question 39

If $y = p[x(x - 2)]^2$ is an increasing function then the value of x is

Options:

A. 1, 2, 3

B. 0, 1, 2

C. 2, 3

D. None of these

Answer: B

Solution:

Solution:

Given, function is $y = [x(x - 2)]^2 = [x^2 - 2x]^2$

On differentiating both sides w.r.t. x, we get

$$\begin{aligned} \frac{d y}{d x} &= 2\left(x^2-2 x\right) \frac{d}{d x}\left(x^2-2 x\right) \\ &= 2\left(x^2-2 x\right)(2 x-2)=4 x(x-2)(x-1) \end{aligned}$$

On putting $\frac{d y}{d x}=0$, we get

$$4 x(x-2)(x-1)=0 \Rightarrow x=0,1 \text { and } 2$$

Now, we find interval in which f(x) is strictly increasing or strictly decreasing.

Interval	$\frac{d y}{d x}=4 x(x-2)(x-1)$	Sign of $f'(x)$
$(-\infty, 0)$	$(-)(-)(-)$	$-ve$
$(0, 1)$	$(+)(-)(-)$	$+ve$
$(1, 2)$	$(+)(-)(+)$	$-ve$
$(2, \infty)$	$(+)(+)(+)$	$+ve$

Hence, y is strictly increasing in (0, 1) and (2, ∞). Also, y is a polynomial function, so it continuous at X = 0, 1 and 2 .

Hence, y is increasing in $[0, 1] \cup [2, \infty]$.

Question 40

If $y = \tan^{-1} \sqrt{\frac{1+\cos x}{1-\cos x}}$, then $\frac{d y}{d x} =$

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Options:

- A. $\frac{3}{2}$
- B. 0
- C. 1
- D. $-\frac{1}{2}$

Answer: D

Solution:

Solution:

$$\begin{aligned} y &= \tan^{-1} \sqrt{\frac{1+\cos x}{1-\cos x}} \\ y &= \tan^{-1} \sqrt{\frac{2 \cos ^2 \frac{x}{2}}{2 \sin ^2 \frac{x}{2}}} \\ y &= \tan^{-1}\left(\cot \frac{x}{2}\right) \\ y &= \tan^{-1}\left[\tan \left(\frac{\pi}{2}-\frac{x}{2}\right)\right] \end{aligned}$$

$$y = \frac{\pi}{2} - \frac{x}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$$

Question 41

If $\left(\frac{\sqrt{1+x^2}-1}{x} \right)$ w.r.t. $\sin^{-1} \left(\frac{2x}{1+x^2} \right)$ is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{2}$

D. $\frac{3}{4}$

Answer: B

Solution:

Solution:

$$\text{Let } u = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$$

Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$, then

$$\begin{aligned} u &= \tan^{-1} \left[\frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right] = \tan^{-1} \left[\frac{\sqrt{\sec^2 \theta}-1}{\tan \theta} \right] \\ &= \tan^{-1} \left[\frac{\sec \theta - 1}{\tan \theta} \right] = \tan^{-1} \left[\frac{1 - \cos \theta}{\sin \theta} \right] \\ &= \tan^{-1} \left[\frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right] = \tan^{-1} \left[\tan \frac{\theta}{2} \right] \end{aligned}$$

$$\Rightarrow u = \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$

$$r \cdot \tan^{-1}(\tan \theta) = \theta$$

On differentiating both sides w.r.t. x , we get

$$\frac{du}{dx} = \frac{1}{2(1+x^2)} \left[\because \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \right] \dots (i)$$

$$\text{Also, let } v = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$, then we get

$$v = \sin^{-1} \left[\frac{2 \tan \theta}{1 + \tan^2 \theta} \right]$$

$$\Rightarrow v = \sin^{-1}[\sin 2\theta]$$

$$\Rightarrow v = 2\theta \Rightarrow v = 2 \tan^{-1} x$$

On differentiating both sides w.r.t. x , we get

$$\frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\text{Now, } \frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv} = \frac{1}{2(1-x^2)} \times \frac{(1+x^2)}{2}$$

[from Eqs. (i) and (ii)]

$$\therefore \frac{du}{dv} = \frac{1}{4}$$

Question 42

The region represented by the inequation system $x, y \geq 0, y \leq 6, x + y \leq 3$ is

Options:

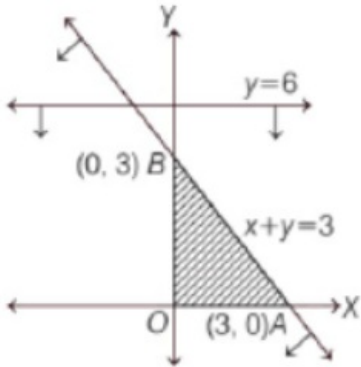
- A. unbounded in first quadrant
- B. unbounded in first and second quadrants
- C. bounded in first quadrant
- D. None of the above

Answer: C

Solution:

Solution:

The given region is bounded in first quadrant.



Question 43

The constraints $-x_1 + x_2 \leq 1, -x_1 + 3x_2 \leq 9$ and $x_1, x_2 \geq 0$ defines on

Options:

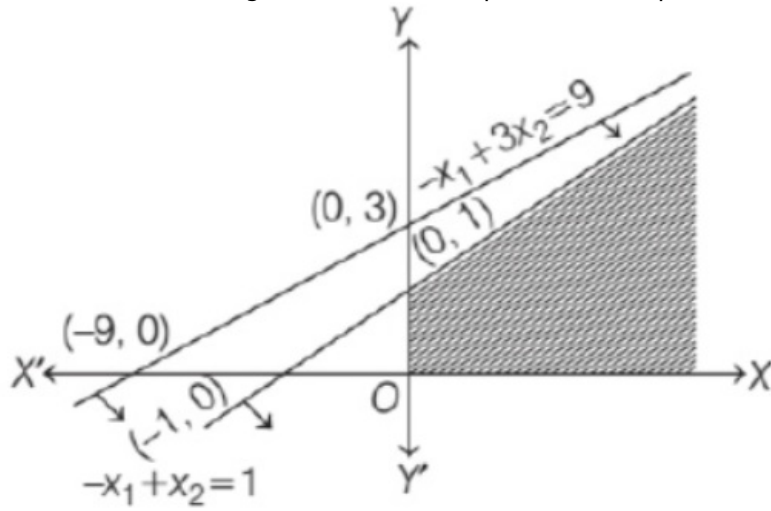
- A. bounded feasible space
- B. unbounded feasible space
- C. both bounded and unbounded feasible space
- D. None of the above

Answer: B

Solution:

Solution:

It is clear from the figure that feasible space (shaded portion) is unbounded.



Question 44

If $z = \frac{(\sqrt{3} + i)^3(3i + 4)^2}{(8 + 6i)^2}$, then $|z|$ is equal to

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Options:

- A. 8
- B. 2
- C. 5
- D. 4

Answer: B

Solution:**Solution:**

$$\text{Given, } z = \frac{(\sqrt{3} + i)^2(3i + 4)^2}{(8 + 6i)^2}$$

$$\begin{aligned} \text{Now, } |z| &= \left| \frac{(\sqrt{3} + i)^3(3i + 4)^2}{(8 + 6i)^2} \right| \\ &= \frac{|(\sqrt{3} + i)^3| |3i + 4|^2}{|(8 + 6i)^2|} \left[\because \left| \frac{z_1}{z_2} \right| = \left| \frac{z_1}{z_2} \right| \right] \\ &= \frac{|\sqrt{3} + i|^3 |3i + 4|^2}{|8 + 6i|^2} \quad [\because |z^n| = |z|^n] \\ &= \frac{(\sqrt{3} + 1)^3 (\sqrt{9 + 16})^2}{(\sqrt{64 + 36})^2} \\ &= \frac{(2)^3 (5)^2}{(10)^2} = \frac{10^2 \cdot 2}{(10)^2} = 2 \end{aligned}$$

Question 45

If $\frac{3}{2 + \cos \theta + i \sin \theta} = a + ib$, then $[(a - 2)^2 + b^2]$ is equal to

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Options:

- A. 0
- B. 1
- C. -1
- D. 2

Answer: B

Solution:

Solution:

$$\text{Given, } \frac{3}{2 + \cos \theta + i \sin \theta} = a + ib$$

$$\Rightarrow \frac{3[(2 + \cos \theta) - i \sin \theta]}{(2 + \cos \theta)^2 + \sin^2 \theta} = a + ib$$

$$\Rightarrow \frac{3[2 + \cos \theta - i \sin \theta]}{5 + 4 \cos \theta} = a + ib$$

$$\Rightarrow a = \frac{3(2 + \cos \theta)}{5 + 4 \cos \theta} \text{ and } b = -\frac{3 \sin \theta}{5 + 4 \cos \theta}$$

$$\therefore (a - 2)^2 + b^2 = \left(\frac{6 + 3 \cos \theta}{5 + 4 \cos \theta} - 2 \right)^2 + \frac{9 \sin^2 \theta}{(5 + 4 \cos \theta)^2}$$

$$= \frac{(-4 - 5 \cos \theta)^2 + 9 \sin^2 \theta}{(5 + 4 \cos \theta)^2}$$

$$= \frac{16 + 25 \cos^2 \theta + 40 \cos \theta + 9 \sin^2 \theta}{(5 + 4 \cos \theta)^2}$$

$$= \frac{16 + 16 \cos^2 \theta + 40 \cos \theta + 9}{(5 + 4 \cos \theta)^2}$$

$$= \frac{(5 + 4 \cos \theta)^2}{(5 + 4 \cos \theta)^2} = 1$$

Question 46

A box contains 100 bulbs, out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective, is

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Options:

- A. $\frac{9}{10}$
- B. $\left(\frac{1}{10} \right)^5$
- C. $\left(\frac{9}{10} \right)^5$
- D. $\left(\frac{1}{2} \right)^5$

Answer: C

Solution:

Solution:

Let probability of defective bulb,

$$p = \frac{10}{100} = \frac{1}{10} = 0.1$$

and probability of non-defective bulb,

$$q = 1 - 0.1 = 0.9$$

Here, $n = 5$

$$\therefore P(\text{none is defective}) = P(X = 0)$$

$$= {}^5C_0(0.1)^0(0.9)^5$$

$$= 1 \times (0.9)^5 = \left(\frac{9}{10}\right)^5$$

Question 47

A random variable X has the probability distribution given below.

X	1	2	3	4	5
P(x=X)	K	2K	3K	2K	K

Its variance is

Options:

A. $\frac{16}{3}$

B. $\frac{4}{3}$

(C) $\frac{5}{3}$

C. $\frac{10}{3}$

Answer: B

Solution:

Solution:

X	1	2	3	4	5
P(X=x)	K	2K	3K	2K	K

$$\therefore \text{Variance} = \sum x_i^2 p - (\sum x_i p)^2$$

$$= (1k + 8k + 27k + 32k + 25k)$$

$$- (k + 4k + 9k + 8k + 5k)^2$$

$$= (93k) - (27k)^2 = \left(93 \times \frac{1}{9}\right) - \left(27 \times \frac{1}{9}\right)^2$$

$$= \frac{93}{9} - 9 = \frac{93 - 81}{9} = \frac{12}{9} = \frac{4}{3}$$

Question 48

The area (in sq. units) of the region bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$ is

Options:

A. $\frac{16a^2}{3}$

B. $\frac{14a^2}{3}$

C. $\frac{13a^2}{3}$

D. $16a^2$

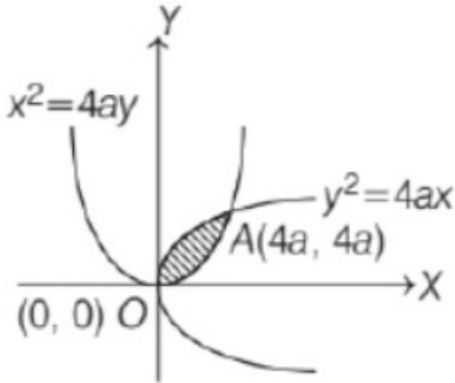
Answer: A

Solution:

Solution:

The equations of given curves are

$$y^2 = 4ax \text{ and } x^2 = 4ay$$



On solving these equations, we get the intersection points, i.e. $(0, 0)$ and $(4a, 4a)$.

$$\begin{aligned}\therefore \text{ Required area} &= \int_0^{4a} \left(2\sqrt{a}\sqrt{x} - \frac{x^2}{4a} \right) dx \\ &= 2\sqrt{a} \left[\frac{x^{3/2}}{3/2} \right]_0^{4a} - \left[\frac{x^3}{12a} \right]_0^{4a} \\ &= \frac{32a^2}{3} - \frac{16a^2}{3} \\ &= \frac{16a^2}{3}\end{aligned}$$

Question 49

The area in the positive quadrant enclosed by the circles $x^2 + y^2 = 4$, the line $x = y\sqrt{3}$ and X-axis is

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

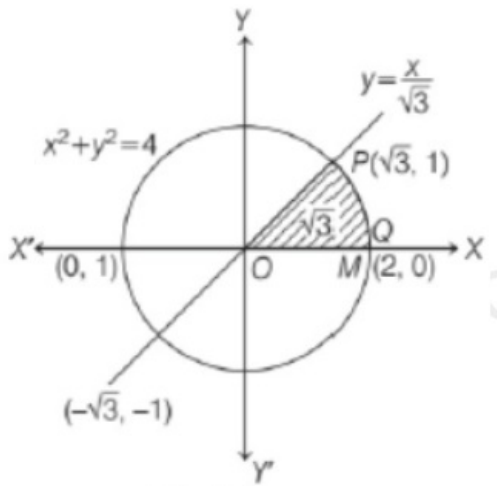
D. π

Answer: C

Solution:

Solution:

The intersection points of curves $x^2 + y^2 = 4$ and $y = \frac{x}{\sqrt{3}}$ are $(0, 0)$ and $P(\sqrt{3}, 1)$.



$$\therefore \text{Area of } \triangle OPM = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$$

$$\text{and area of curve MPQ} = \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx$$

$$= \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \left(\frac{x}{2} \right) \right]_{\sqrt{3}}^2$$

$$= \left[0 + 2 \left(\frac{\pi}{2} \right) - \left(\frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{3} \right) \right]$$

$$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right)$$

Question 50

$$\int_{\pi/6}^{\pi/2} \frac{\operatorname{cosec} x \cdot \cot x}{1 + \operatorname{cosec}^2 x} dx =$$

Options:

A. $\tan^{-1} 2$

B. $\tan^{-1} \left(\frac{1}{3} \right)$

C. $\tan^{-1} 1$

D. $\frac{\pi}{4} - \tan^{-1} 2$

Answer: B

Solution:

Solution:

$$\text{Let } I = \int_{\pi/6}^{\pi/2} \frac{\operatorname{cosec} x \cdot \cot x}{1 + \operatorname{cosec}^2 x} dx$$

$$\text{Let } \operatorname{cosec} x = t$$

$$\Rightarrow -\operatorname{cosec} x \cot x dx = dt$$

$$\text{When } x = \frac{\pi}{6}, \text{ then } t = \operatorname{cosec} \frac{\pi}{6} = 2$$

$$\text{and when } x = \frac{\pi}{2}, \text{ then } t = \operatorname{cosec} \frac{\pi}{2} = 1$$

$$\therefore I = \int_2^1 -\frac{dt}{1+t^2} = -\int_2^1 \frac{dt}{1+t^2}$$

$$= \int_1^2 \frac{dt}{1+t^2} = [\tan^{-1}(t)]_1^2$$

$$= \tan^{-1}(2) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\frac{2-1}{1+2 \times 1}\right) = \tan^{-1}\left(\frac{1}{1+2}\right)$$

$$= \tan^{-1}\left(\frac{1}{3}\right)$$

Question 51

If $\sin A + \cos A = \sqrt{2}$, then the value of $\cos^2 A$ is

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Options:

A. $\sqrt{2}$

B. $\frac{1}{2}$

C. 4

D. -4

Answer: B

Solution:

Solution:

$$\text{Given, } \sin A + \cos A = \sqrt{2}$$

$$\therefore \frac{1}{\sqrt{2}} \sin A + \frac{1}{\sqrt{2}} \cos A = 1$$

$$\Rightarrow \sin \frac{\pi}{4} \sin A + \cos A \cos \frac{\pi}{4} = 0$$

$$\Rightarrow \cos\left(A - \frac{\pi}{4}\right) = 1$$

$$\Rightarrow A - \frac{\pi}{4} = 0$$

$$\Rightarrow A = \frac{\pi}{4}$$

$$\text{Now, } \cos^2 A = \cos^2 \frac{\pi}{4} = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

Question 52

The number of solutions of equation $\tan x + \sec x = 2 \cos x$ lying in the interval $[0, 2\pi]$ is

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Options:

- A. 0
- B. 1
- C. 2
- D. 3

Answer: C

Solution:

Solution:

$$\tan x + \sec x = 2 \cos x$$

$$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2 \cos x$$

$$\Rightarrow 1 + \sin x = 2 \cos^2 x$$

$$\Rightarrow 1 + \sin x = 2(1 - \sin^2 x)$$

$$\Rightarrow 1 + \sin x = 2 - 2 \sin^2 x$$

$$\Rightarrow 2 \sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow 2 \sin^2 x + 2 \sin x - \sin x - 1 = 0$$

$$\Rightarrow 2 \sin x (\sin x + 1) - 1 (\sin x + 1) = 0$$

$$\Rightarrow (\sin x + 1)(2 \sin x - 1) = 0$$

$$\Rightarrow \sin x + 1 = 0 \Rightarrow 2 \sin x - 1 = 0$$

$$\Rightarrow \sin x = -1, \sin x = \frac{1}{2}$$

$$\Rightarrow x = \frac{3\pi}{2}, x = \frac{\pi}{6}$$

Question 53

The value of $\tan 75^\circ - \cot 75^\circ$ is

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Options:

- A. $2\sqrt{3}$
- B. $2 - \sqrt{3}$
- C. $2 + \sqrt{3}$
- D. 0

Answer: A

Solution:

Solution:

$$\tan 75^\circ - \cot 75^\circ$$

$$\begin{aligned}
&= \frac{\sin 75^\circ}{\cos 75^\circ} - \frac{\cos 75^\circ}{\sin 75^\circ} \\
&= \frac{\sin^2 75^\circ - \cos^2 75^\circ}{\sin 75^\circ \cdot \cos 75^\circ} = \frac{-2 \cos 150^\circ}{\sin 150^\circ} \\
&= \frac{-2 \cos(90^\circ + 60^\circ)}{\sin(90^\circ + 60^\circ)} \\
&= \frac{2 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}
\end{aligned}$$

Question 54

$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ equals

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Options:

- A. 8
- B. 1
- C. 6
- D. 12

Answer: C

Solution:

Solution:

$$\begin{aligned}
&\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi \\
&0 \leq \cos^{-1}x \leq \pi \\
&\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi \\
&\cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi \\
&\cos \pi = \alpha = \beta = \gamma \\
&-1 = \alpha = \beta = \gamma \\
&\alpha = \beta = \gamma = -1 \\
&\alpha(\beta + \gamma + \beta(\gamma + \alpha) + \gamma(\alpha + \beta) \\
&-1(-1 - 1) - 1(-1 - 1) - 1(-1 - 1) \\
&= 2 + 2 + 2 = 6
\end{aligned}$$

Question 55

A line has slope m and y-intercept 4. The distance between the origin and the line is equal to

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Options:

- A. $\frac{4}{\sqrt{1 - m^2}}$

B. $\frac{4}{\sqrt{m^2 - 1}}$

C. $\frac{4}{\sqrt{m^2 + 1}}$

D. $\frac{4m}{\sqrt{1 + m^2}}$

Answer: C

Solution:

Solution:

Equation of line is $y = mx + 4$

$$\therefore \text{Required distance} = \frac{4}{\sqrt{1 + m^2}}$$

Question 56

If a line with y-intercept 2 , is perpendicular to the line $3x - 2y = 6$, then its x-intercept is

©

Options:

A. 1

B. 2

C. -4

D. 3

Answer: D

Solution:

Solution:

Let the equation of perpendicular line to the line

$$3x - 2y = 6 \text{ be } 3y + 2x = c$$

Since, it passes through (0, 2).

$$\therefore c = 6$$

On putting the value of c in Eq. (i), we get

$$\Rightarrow 3y + 2x = 6$$

$$\frac{x}{3} + \frac{y}{2} = 1$$

Hence, x-intercept is 3 .

Question 57

A line passes through the point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ and makes equal intercepts with axes. Then, equation

of the line is

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Options:

A. $5x + 5y - 3 = 0$

B. $x + 5y - 3 = 0$

C. $5x - y - 3 = 0$

D. $5x + 5y + 3 = 0$

Answer: A

Solution:

Solution:

The point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ is $\left(-\frac{4}{5}, \frac{7}{5}\right)$

The equation of line, which makes equal intercepts with axes, is $x + y = a$.

$$\therefore -\frac{4}{5} + \frac{7}{5} = a \Rightarrow a = \frac{3}{5}$$

Now, equation of line is $x + y - \frac{3}{5} = 0$

$$\Rightarrow 5x + 5y - 3 = 0$$

Question 58

If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other internally, then α is equal to

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Options:

A. $\pm \frac{4}{3}$

B. 1

C. $\frac{4}{3}$

D. $-\frac{4}{3}$

Answer: A

Solution:

Solution:

Centres and radii of the given circles are $C_1(0, 0)$, $r_1 = 3$ and $C_2(-\alpha, -1)$

$$r_2 = \sqrt{\alpha^2 + 1 - 1} = |\alpha|$$

Since, two circles touch internally.

$$\therefore C_1C_2 = r_1 - r_2$$

$$\Rightarrow \sqrt{\alpha^2 + 1^2} = 3 - |\alpha|$$

$$\Rightarrow \alpha^2 + 1 = 9 + \alpha^2 - 6|\alpha|$$

$$\Rightarrow 6|\alpha| = 8 \Rightarrow |\alpha| = \frac{4}{3}$$

$$\therefore \alpha = \pm \frac{4}{3}$$

Question 59

The length of the common chord of the two circles $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 8x - 4y + 11 = 0$ is

Options:

A. $\frac{\sqrt{145}}{4}$ cm

B. $\frac{\sqrt{11}}{2}$ cm

C. $\sqrt{135}$ cm

D. $\frac{\sqrt{135}}{4}$ cm

Answer: D

Solution:

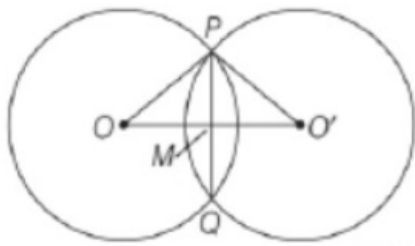
Solution:

Given, equation of circles are $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 8x - 4y + 11 = 0$

$\therefore$ Equation of chord is

$$x^2 + y^2 - 4y - (x^2 + y^2 - 8x - 4y + 11) = 0$$

$$\Rightarrow 8x - 11 = 0$$



So, centre and radius of first circle are $O(0, 2)$ and $OP = r = 2$.

Now, perpendicular distance from $O(0, 2)$ to the line $8x - 11 = 0$ is

$$d = OM = \frac{|8 \times 0 - 11|}{\sqrt{8^2}} = \frac{11}{8}$$

$$\text{In } \triangle OMP, PM = \sqrt{OP^2 - OM^2}$$

$$= \sqrt{2^2 - \left(\frac{11}{8}\right)^2} = \sqrt{4 - \frac{121}{64}}$$

$$= \sqrt{\frac{256 - 121}{64}} = \frac{\sqrt{135}}{8}$$

$\therefore$ Length of chord $PQ = 2PM$

$$= 2 \times \frac{\sqrt{135}}{8} = \frac{\sqrt{135}}{4} \text{ cm}$$

Question 60

If $x_1, x_2, \dots, x_{18}$ are observations such that $\sum_{j=1}^{18} (x_j - 8) = 9$ and $\sum_{j=1}^{18} (x_j - 8)^2 = 45$, then standard deviation of these observations is

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Options:

A. $\sqrt{\frac{81}{34}}$

B. 5

C. $\sqrt{5}$

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:

Standard deviation

$$\begin{aligned} & \sqrt{\frac{\sum_{j=1}^{18} (x_j - 8)^2}{n} - \left(\frac{\sum_{j=1}^{18} (x_j - 8)}{n} \right)^2} \\ &= \sqrt{\frac{45}{18} - \left(\frac{9}{18} \right)^2} = \sqrt{\frac{45}{18} - \frac{1}{4}} \\ &= \sqrt{\frac{81}{36}} = \frac{9}{6} = \frac{3}{2} \end{aligned}$$

Question 61

The sum of the deviations of the variates from the arithmetic mean is always

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Options:

A. +1

B. 0

C. -1

D. real number

Answer: B

Solution:

Solution:

Let $x_1, x_2, \dots, x_n$ be n variates. Then, their arithmetic mean will be

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \dots (i)$$

Now, the sum of deviation of the variates from the AM, i.e. $\sum (x_i - \bar{x})$

$$\begin{aligned} &= \sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n x_i - \sum_{i=1}^n \bar{x} \\ &= \sum_{i=1}^n x_i - n\bar{x} = \sum_{i=1}^n x_i - \sum_{i=1}^n x_i \quad [\text{from Eq. (i)}] \\ &= 0 \end{aligned}$$

Question 62

A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p , $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

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Options:

A. $\frac{3p}{4p+3}$

B. $\frac{5p}{3p+2}$

C. $\frac{5p}{4p+1}$

D. $\frac{4p}{3p+1}$

Answer: C

Solution:

Solution:

Let E_1 = Student does not know the answer E_2 = Student knows the answer

and E = student answer correctly

$$\therefore P(E_1) = 1 - p \Rightarrow P(E_2) = p$$

$$\Rightarrow P\left(\frac{E}{E_2}\right) = 1 \text{ and } P\left(\frac{E}{E_1}\right) = \frac{1}{5}$$

Note that, the probability that student did not know the answer randomly = The probability that student know the answer.

$$\begin{aligned} \therefore P\left(\frac{E_2}{E}\right) &= \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)} \\ &= \frac{p(1)}{(1-p)\frac{1}{5} + p(1)} = \frac{p}{\frac{1-p+5p}{5}} \\ &= \frac{5p}{1+4p} \end{aligned}$$

Question 63

Box A contains 2 black and 3 red balls, while box B contains 3 black and 4 red balls. Out of these two boxes one is selected at random and the probability of choosing box A is double that of box B. If a red ball is drawn from the selected box, then the probability that it has come from box B, is

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Options:

A. $\frac{21}{41}$

B. $\frac{10}{31}$

C. $\frac{12}{31}$

D. $\frac{13}{41}$

Answer: B

Solution:

Solution:

Let probability of choosing box B, $P(B) = p$

According to the given condition,

$$P(A) = 2P(B) = 2p$$

$$\text{Now, } P\left(\frac{R}{A}\right) = \frac{{}^3C_1}{{}^5C_1} = \frac{3}{5}$$

$$\text{and } P\left(\frac{R}{B}\right) = \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$$

$$\therefore P\left(\frac{B}{R}\right) = \frac{P(B) \cdot P\left(\frac{R}{B}\right)}{P(A) \cdot P\left(\frac{R}{A}\right) + P(B) \cdot P\left(\frac{R}{B}\right)}$$

$$= \frac{p \cdot \frac{4}{7}}{2p \cdot \frac{3}{5} + p \cdot \frac{4}{7}} = \frac{10}{31}$$

Question 64

The contrapositive of the statement. 'If $2^2 = 5$, then I get first class' is

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Options:

A. If I do not get a first class, then $2^2 = 5$

B. If I do not get a first class, then $2^2 \neq 5$

C. If I get a first class, then $2^2 = 5$

D. None of the above

Answer: B

Solution:

Solution:

Let p and q be two propositions given by $p : 2^2 = 5$, q : I get first class.

Then, given statement is $p \rightarrow q$.

The contrapositive of this statement is

$\sim q \rightarrow \sim p$, i.e. if I do not get first class, then $2^2 \neq 5$.

Question 65

The truth value of the statement 'Patna is in Bihar or $5 + 6 = 11$ ' is

Options:

A. true

B. false

C. Cannot say anything

D. None of these

Answer: A

Solution:

Solution:

Let p : Patna is in Bihar and q : $5 + 6 = 11$

Then, the given statement is disjunction $p \vee q$.

Since, p is true and q is false.

$\therefore$ The disjunction $p \vee q$ is true.

Hence, truth value of given statement is true.

Question 66

If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^3 is

Options:

A. $3A$

B. $2A$

C. $4A$

D. A

Answer: C

Solution:

Solution:

$$\text{We have, } A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\text{Now, } A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\text{and } A^3 = A^2 \cdot A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & -2-2 \\ -2-2 & 2+2 \end{bmatrix} \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

$$\therefore A^3 = 4A$$

Question 67

If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \operatorname{adj} A = AA^T$, then $5a + b$ is equal to

Options:

A. 5

B. 4

C. 13

D. -1

Answer: A

Solution:

Solution:

$$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \text{ and } A \operatorname{adj} A = AA^T$$

$$\therefore \operatorname{adj} A = \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$$

$$\text{Now, } AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$$

$$\text{and } A \cdot \operatorname{adj} A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$$

$$= \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

$\therefore A \cdot (\text{adj} A) = AA^T$ is given, so equating the two expression, we get

$$\begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

We have, $10a + 3b = 13$ and $15a - 2b = 0$

On solving, we get

$$a = \frac{2}{5} \text{ and } b = 3$$

$$\Rightarrow 5a + b = 5 \times \frac{2}{5} + 3$$

$$\Rightarrow 5a + b = 2 + 3$$

$$\Rightarrow 5a + b = 5$$

Question 68

The value of f at $x = 0$, so that function $f(x) = \frac{2^x - 2^{-x}}{x}$, $x \neq 0$ is continuous at $x = 0$, is

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Options:

A. 0

B. $\log 4$

C. 4

D. e^4

Answer: B

Solution:

Solution:

$$\lim_{x \rightarrow 0} \frac{2^x - 2^{-x}}{x} = \lim_{x \rightarrow 0} 2^x \log 2 + 2^{-x} \log 2$$

[using L'Hospital's rule]

$$= \log 2 + \log 2 = \log 4$$

Since, function is continuous at $x = 0$.

$$\therefore f(0) = \lim_{x \rightarrow 0} \frac{2^x - 2^{-x}}{x} = \log 4$$

Question 69

If $f(x) = \begin{cases} ax + 3 & x \leq 2 \\ a^2x - 1 & x > 2. \end{cases}$, then the values of a for which f is continuous for all x are

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Options:

- A. 1 and -2
- B. 1 and 2
- C. -1 and 2
- D. -1 and -2

Answer: C

Solution:

Solution:

Given, $f(x) = \begin{cases} ax + 3, & x \leq 2 \\ a^2x - 1, & x > 2 \end{cases}$ end cases
Continuity at $x = 2$,

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} (ax + 3) = 2a + 3$$

$$\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} (a^2x - 1) = 2a^2 - 1$$

Since, $f(x)$ is continuous for all values of x .

$$\therefore \text{LHL} = \text{RHL}$$

$$\Rightarrow 2a + 3 = 2a^2 - 1$$

$$\Rightarrow 2a^2 - 2a - 4 = 0$$

$$\Rightarrow a^2 - a - 2 = 0$$

$$\Rightarrow a^2 - 2a + a - 2 = 0$$

$$\Rightarrow a(a - 2) + 1(a - 2) = 0$$

$$\Rightarrow (a + 1)(a - 2) = 0$$

$$\therefore a = -1, 2$$

Question 70

$\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$ is equal to

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Options:

- A. $\frac{2}{5}$
- B. $\frac{3}{5}$
- C. $\frac{3}{2}$
- D. $\frac{3}{4}$

Answer: C

Solution:

Solution:

$$\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos^2 x + \cos x)}{x^2 \cos x \cdot \frac{\sin x}{x}}$$

$$= 3 \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 3 \times \frac{1}{2} = \frac{3}{2}$$

Question 71

The value of $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{2/x}$, (a, b, c > 0) is

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Options:

- A. $(abc)^3$
- B. abc
- C. $(abc)^{1/3}$
- D. None of these

Answer: D

Solution:

Solution:

$$\begin{aligned} \text{Let } y &= \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{2/x} \\ \Rightarrow \log y &= \lim_{x \rightarrow 0} \frac{2}{x} \log \left(\frac{a^x + b^x + c^x}{3} \right) \\ \Rightarrow \log y &= \lim_{x \rightarrow 0} \frac{2 \log(a^x + b^x + c^x) - \log 3}{x} \\ \Rightarrow \log y &= \log(abc)^{2/3} \\ \therefore \text{ [using L'Hospital's rule]} \\ \therefore y &= (abc)^{2/3} \end{aligned}$$

Question 72

If the vectors $2\hat{i} - \hat{j} - \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is

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Options:

- A. -8
- B. -4
- C. -2
- D. -1

Answer: B

Solution:

Solution:

Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$,

$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$

Now given vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ will be coplanar if

$\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$, i.e. $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

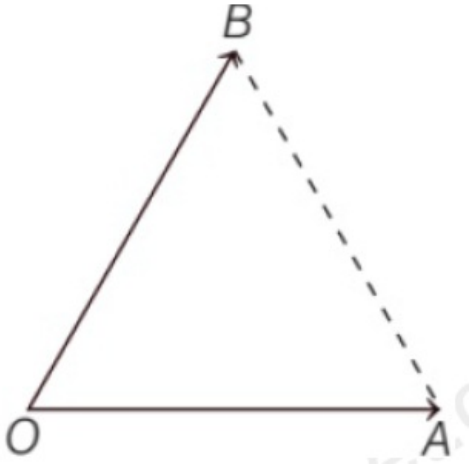
$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

$$\Rightarrow 2(10 + 3\lambda) + 1(5 + 9) + 1(\lambda - 6) = 0$$

$$\Rightarrow 7\lambda = -28 = 0 \Rightarrow \lambda = -4$$

Question 73

A $\triangle OAB$ is determined by the vector $\vec{a}$ and $\vec{b}$ as show in the figure what will be area of triangle?



Options:

A. $\frac{1}{2} \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - (\vec{a} \cdot \vec{b})}$

B. $\frac{1}{2} \sqrt{|\vec{a}| |\vec{b}| - (\vec{a} \cdot \vec{b})^2}$

C. $\frac{1}{4} \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - (\vec{a} \cdot \vec{b})}$

D. $\frac{1}{4} \sqrt{|\vec{a}| |\vec{b}| - (\vec{a} \cdot \vec{b})^2}$

Answer: B

Solution:

Solution:

We know that area of triangle = $\frac{1}{2} |\vec{OA} \times \vec{OB}|$

$$\Delta^2 = \frac{1}{4} |\vec{a} \times \vec{b}|^2 \dots (i)$$

$$\begin{aligned} \text{Now } |\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 \\ = |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta \end{aligned}$$

$$\begin{aligned}
&= |a|^2 |b|^2 (\sin^2 \theta + \cos^2 \theta) \\
&= |a|^2 |b|^2 \times 1 = |a|^2 |b|^2 \\
&\Rightarrow |a \times b|^2 = |a|^2 |b|^2 - (a \cdot b)^2 \\
&\text{From Eq. (i), we get} \\
\Delta^2 &= \frac{1}{4} [|a|^2 |b|^2 - (a \cdot b)^2] \\
\Delta &= \frac{1}{2} \sqrt{|a|^2 |b|^2 - (a \cdot b)^2}
\end{aligned}$$

Question 74

The line $\frac{x-2}{3} - \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then value of $\alpha\beta$ is

Options:

- A. -42
- B. 1
- C. -2
- D. 42

Answer: A

Solution:

Solution:

Given equation of line

$$\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2} \dots (i)$$

The direction ratios of the normal are $(1, 3, -\alpha)$.

The direction ratios of the line are $(3, -5, 2)$ and equation of given plane

$$x + 3y - \alpha z + \beta = 0 \dots (ii)$$

Four lines are perpendicular

$$\Rightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow 3 - 15 + 2\alpha = 0$$

$$\Rightarrow 2\alpha = -12 \Rightarrow \alpha = -6$$

$(2, 1, -2)$ lies on the plane, so

$$2 + 3 + 6(-2) + \beta = 0 \Rightarrow \beta = 7$$

$$\alpha \cdot \beta = -6 \times 7 = -42$$

Question 75

Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$, $6mn - 2nl + 5lm = 0$

Options:

- A. $\cos^{-1} \frac{1}{6}$

B. $\cos^{-1} \frac{2}{6}$

C. $\sin^{-1} \frac{1}{6}$

D. $\sin^{-1} \frac{2}{6}$

Answer: A

Solution:

Solution:

The given equations are $3l + m + 5n = 0 \dots (i)$

and $6mn - 2nl + 5l/m = -0 \dots (ii)$

Now, from Eq. (i), we get

$$m = -3l - 5n$$

On substituting $m = -3l - 5n$ in Eq. (ii), we get

$$6(-3l - 5n)n - 2nl + 5l(-3l - 5n) = 0$$

$$\Rightarrow 30n^2 + 45ln + 15l^2 = 0$$

$$\Rightarrow 2n^2 + 3ln + l^2 = 0$$

$$\Rightarrow 2n^2 + 2nl + nl + l^2 = 0$$

$$\Rightarrow 2n(n + l) + l(n + l) = 0$$

$$\Rightarrow (n + l)(2n + l) = 0$$

$$\Rightarrow \text{Either } l = -n \text{ or } l = -2n$$

If $l = -n$, then $m = -2n$ and if $l = -2n$, then $m = n$

Thus, the direction ratios of two lines are proportional $(-n, -2n, n)$ and $(-2n, n, n)$ i.e. $(-1, -2, 1)$ and $(-2, 1, 1)$, respectively.

Now, let θ be the acute angle between the lines,

$$\text{Then, } \cos \theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{|2 - 2 + 1|}{\sqrt{1 + 4 + 1} \sqrt{4 + 1 + 1}} = \frac{1}{6}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{1}{6} \right)$$

Question 76

Find the shortest distance between the lines $\mathbf{r} = \hat{i} + \hat{j} + \lambda (2\hat{i} - \hat{j} + \hat{k})$ and $\mathbf{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu (3\hat{i} - 5\hat{j} + 2\hat{k})$.

Options:

A. $\frac{10}{\sqrt{59}}$

B. $\frac{8}{\sqrt{57}}$

C. $\frac{9}{\sqrt{89}}$

D. $\frac{10}{\sqrt{39}}$

Answer: A

Solution:

Solution:

$$r = (\hat{i} + \hat{j}) + \lambda (2\hat{i} - \hat{j} + \hat{k}) \dots (i)$$

$$r = (2\hat{i} + \hat{j} - \hat{k}) + \mu (3\hat{i} - 5\hat{j} + 2\hat{k}) \dots (ii)$$

Compare with vector equation $r = a + \lambda b$

$$a_1 = \hat{i} + \hat{j} = 2\hat{i} - \hat{j} + \hat{k}$$

$$a_2 = 2\hat{i} + \hat{j} - \hat{k}, b_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$$d = \left| \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|} \right| \dots (iii)$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$b_1 \times b_2 = \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3)$$

$$= 3\hat{i} - \hat{j} - 7\hat{k}$$

$$|b_1 \times b_2| = \sqrt{(3)^2 + (-1)^2 + (-7)^2}$$

$$\text{Also } a_2 - a_1 = (2\hat{i} + \hat{j} - \hat{k}) + (\hat{i} + \hat{j}) = \hat{i} - \hat{k} \dots (iv)$$

$$d = \left| \left(3\hat{i} - \hat{j} - 7\hat{k} \right) \left(\hat{i} - \frac{\hat{k}}{\sqrt{59}} \right) \right| = \left| \frac{3 - 0 + 7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}}$$

Question 77

$\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

Options:

A. $\tan x + \cot x + C$

B. $\tan x - \cot x + C$

C. $\tan x - \cot^2 x + C$

D. None of these

Answer: B

Solution:

Solution:

$$\begin{aligned} I &= \int \frac{dx}{\sin^2 x \cos^2 x} \\ &= \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cdot \cos^2 x} [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= \int (\sec^2 x + \operatorname{cosec}^2 x) dx \\ &= \int (\sec^2 x dx + \operatorname{cosec}^2 x dx) \\ &= \tan x - \cot x + C \end{aligned}$$

Question 78

$\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$ is equal to

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Options:

A. $\frac{\tan^{-1}x}{x} + C$

B. $\frac{\tan^{-1}x}{x} + C$

C. $e^{\tan^{-1}x} + C$

D. $\frac{e^{\tan^{-1}x}}{x^2} + C$

Answer: C

Solution:

Solution:

$$\text{Let } I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx$$

$$\text{Put } \tan^{-1}x = t$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

$$\therefore I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx = \int e^t dt$$

$$= e^t + C \quad [\because \int e^x dx = e^x]$$

$$= e^{\tan^{-1}x} + C \quad [\text{put } t = \tan^{-1}x]$$

Question 79

A spherical raindrop evaporates at a rate proportional to its surface originally is 3mm and 1h later has been reduced to 2mm, then radius r of the raindrop at any time t is (where $0 \leq t < 3$)

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Options:

A. $r = t + 3$

B. $r = t + 5$

C. $r = t - 5$

D. $r = 3 - t$

Answer: D

Solution:

Solution:

Let r be the radius, V be the volume and S be the surface area of the spherical raindrop at time t .

Then, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

The rate at which the raindrop evaporates is $\frac{dV}{dt}$

which is proportional to the surface area.

$$\therefore \frac{dV}{dt} \propto S \Rightarrow \frac{dV}{dt} = -kS, \text{ where } k > 0. \dots (i)$$

$$\text{Now, } V = \frac{4}{3}\pi r^3 \text{ and } S = 4\pi r^2$$

$$\therefore \frac{dV}{dt} = \frac{4\pi}{3} \times 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4\pi r^2 \frac{dr}{dt} = -k(4\pi r^2) \quad [\text{from Eq. (i)}]$$

$$\frac{dr}{dt} = -k \Rightarrow dr = -k dt$$

On integrating, we get

$$\int dr = -k \int dt + C$$

$$\therefore r = -kt + C$$

Initially, i.e. when $t = 0$, $r = 3$

$$\therefore 3 = -k \times 0 + C$$

$$\therefore C = 3$$

$$\therefore r = -kt + 3$$

When $t = 1$, $r = 2$

$$\therefore 2 = -k \times 1 + 3$$

$$\therefore k = 1$$

$$\therefore r = -t + 3$$

$$\therefore r = 3 - t, \text{ where } 0 \leq t \leq 3$$

This is the required expression for the radius of the raindrop at any time t .

Question 80

If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, $-\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$ then $\frac{dy}{dx}$ is

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Options:

A. $\frac{6}{\sqrt{1-9x^2}}$

B. $\frac{5}{\sqrt{1-3x^2}}$

C. $\frac{6}{\sqrt{1-3x^2}}$

D. $\frac{5}{\sqrt{1-4x^2}}$

Answer: A

Solution:

Solution:

$$\text{Given, } y = \sin^{-1}(6x\sqrt{1-9x^2})$$

$$\Rightarrow y = \sin^{-1}(2 \cdot 3x\sqrt{1-(3x)^2})$$

$$\text{Put } 3x = \sin \theta \Rightarrow y = \sin^{-1}(2 \sin \theta \cdot \cos \theta)$$

$$\Rightarrow y = \sin^{-1}(\sin 2\theta)$$

$$\Rightarrow y = 2\theta \Rightarrow y = 2\sin^{-1}(3x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-9x^2}}(3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$$

Question 81

If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then $\frac{dy}{dx}$

Options:

A. $(\sin x)^x[\cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

B. $(\sin x)[x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

C. $(\sin x)^x[x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

D. None of the above

Answer: C

Solution:

Solution:

Given, $y = (\sin x)^x + \sin^{-1}x \dots (i)$

Let $u = (\sin x)^x \dots (ii)$

Then, Eq. (i) becomes,

$y = u + \sin^{-1}\sqrt{x} \dots (iii)$

On taking log both sides of Eq. (ii), we get

$\log u = x \log \sin x$

On differentiating both sides w.r.t., x , we get

$$\frac{1}{u} \frac{du}{dx} = x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(x)$$

$$\Rightarrow \frac{du}{dx} = u \left[x \times \frac{1}{\sin x} \frac{d}{dx}(\sin x) + \log \sin x(1) \right]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x \left[\frac{x}{\sin x} \times \cos x + \log \sin x \right]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x [x \cot x + \log \sin x] \dots (iv)$$

On differentiating both sides of Eq. (iii) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx}(\sqrt{x})$$

$$\therefore \frac{dy}{dx} = (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$$

[from Eq. (iv)]

Question 82

The side of an equilateral triangle is increasing at the rate of 2cm / s. If the side of the triangle is 20cm² the rate of area increasing is

Options:

- A. $20\sqrt{3}\text{cm}^2$
- B. 20cm^2
- C. 60cm^2
- D. $\frac{20\sqrt{3}}{3}\text{cm}^2$

Answer: A

Solution:

Solution:

Let a be the side of an equilateral triangle and A be the area of an equilateral triangle. Then, $\frac{da}{dt} = 2\text{cm/s}$

We know that, area of an equilateral triangle

$$A = \frac{\sqrt{3}}{4}a^2$$

On differentiating both sides w.r.t. t , we get

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times \frac{da}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2 \times 20 \times 2 \quad [\text{given } a = 20]$$

$$\therefore \frac{dA}{dt} = 20\sqrt{3}\text{cm}^2/\text{s}$$

Thus, the rate of area increasing is $20\sqrt{3}\text{cm}^2/\text{s}$.

Question 83

The maximum value $Z = 11x + 7y$, subject to $x \leq 3$, $y \leq 2$, $x \geq 0$ and $y \geq 0$ is

Options:

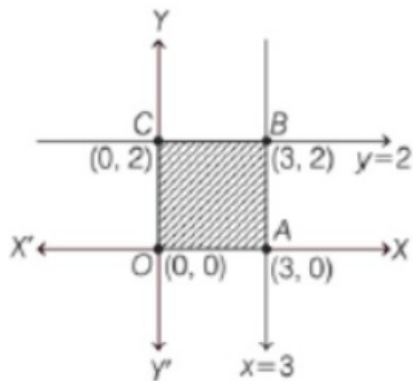
- A. 44
- B. 46
- C. 54
- D. 47

Answer: D

Solution:

Solution:

Maximise $Z = 11x + 7y$, subject to the constraints $x \leq 3$, $y \leq 2$, $x \geq 0$, $y \geq 0$



The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are $(0, 0)$, $(3, 0)$, $(3, 2)$ and $(0, 2)$ respectively.

Corner points	Corresponding value of Z
$(0, 0)$	0
$(3, 0)$	33
$(3, 2)$	47 ← Maximum
$(0, 2)$	14

Hence, Z is maximum at $(3, 2)$ and its maximum value is 47 .

Question 84

The minimum value $Z = 13x - 15y$ subject to $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$ and $y \geq 0$ is

Options:

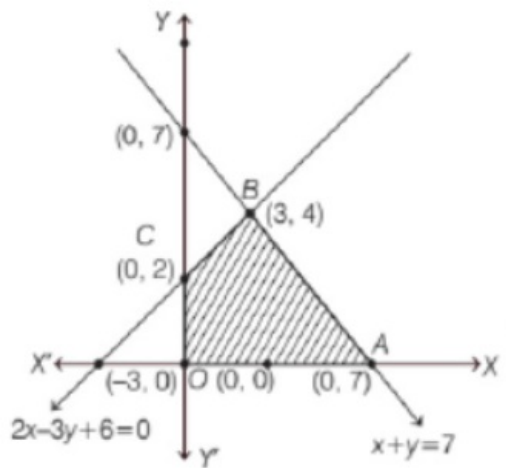
- A. 80
- B. -21
- C. -30
- D. 91

Answer: C

Solution:

Solution:

Minimize $Z = 13x - 15y$ subject to the constraints $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$



Shaded region shown as OABC is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively,

Corner points	Corresponding value of Z
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	-30 ← Minimum

Hence, the minimum value of Z is (−30) at (0, 2).

Question 85

If $z_1, z_2, \dots, z_n$ are complex numbers such that $|z_1| = |z_2| = \dots = |z_n| = 1$, then $|z_1 + z_2 + \dots + z_n|$ is equal to

Options:

- A. $|z_1 z_2 z_3 \dots z_n|$
- B. $|z_1| + |z_2| + \dots + |z_n|$
- C. $\left| \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right|$
- D. n

Answer: C

Solution:

Solution:
 Given, $|z_1| = |z_2| = \dots = |z_n| = 1$
 $\Rightarrow |z_1|^2 = |z_2|^2 = \dots = |z_n|^2 = 1$
 $\Rightarrow z_1 \bar{z}_1 = z_2 \bar{z}_2 = \dots = z_n \bar{z}_n = 1$

$$\Rightarrow \bar{z}_1 = \frac{1}{z_1}, \bar{z}_2 = \frac{1}{z_2}, \dots, \bar{z}_n = \frac{1}{z_n} \dots (i)$$

$$\text{Now, } |z_1 + z_2 + \dots + z_n|$$

$$= |z_1 + z_2 + \dots + z_n| = |\bar{z}_1 + \bar{z}_2 + \dots + \bar{z}_n|$$

$$= \left| \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right| \quad [\text{using Eq. (i)}]$$

Question 86

If $2\alpha = -1 - i\sqrt{3}$ and $2\beta = -1 + i\sqrt{3}$, then $5\alpha^4 + 5\beta^4 + 7\alpha^{-1}\beta^{-1}$ is equal to

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Options:

A. -1

B. 2

C. 0

D. 1

Answer: B

Solution:

Solution:

$$\text{Given, } 2\alpha = -1 - i\sqrt{3} \text{ and } 2\beta = -1 + i\sqrt{3}$$

$$\therefore \alpha + \beta = -1 \text{ and } \alpha\beta = 1$$

$$\text{Now, } 5\alpha^4 + 5\beta^4 + \frac{7}{\alpha\beta} = 5[\{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2]$$

$$= 5[\{(-1)^2 - 2 \times 1\}^2 - 2(1)^2] + \frac{7}{1}$$

$$= 5[(1 - 2)^2 - 2] + 7 = 2$$

Question 87

If a fair coin is tossed 20 times and we get head n times, then probability that n is odd, is

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Options:

A. $\frac{1}{2}$

B. $\frac{1}{6}$

C. $\frac{5}{8}$

D. $\frac{7}{8}$

Answer: A

Solution:

Solution:

Probability of getting head in one trial, $p = \frac{1}{2}$ and probability of not getting head,

$$q = \frac{1}{2}$$

Probability of getting head odd times

$$\begin{aligned} &= {}^{20}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{19} + {}^{20}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{17} \\ &+ \dots + {}^{20}C_{19} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{19} \\ &= \frac{1}{2^{20}} [{}^{20}C_1 + {}^{20}C_3 + \dots + {}^{20}C_{19}] \\ &= \frac{1}{2^{20}} \times 2^{20-1} = \frac{2^{19}}{2^{20}} = \frac{1}{2} \end{aligned}$$

Question 88

If the records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only three will die is

Options:

A. 8748×10^{-5}

B. 1458×10^{-5}

C. 1468×10^{-6}

D. 41×10^{-6}

Answer: B

Solution:

Solution:

Since, the probability of person die, due to suffering from a disease is 10%.

$$\therefore p = \frac{10}{100} = \frac{1}{10} \text{ and } q = \frac{9}{10}$$

Total number of patients, $n = 6$

$$\begin{aligned} \therefore \text{Required probability} &= {}^6C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^3 \\ &= \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \times \frac{1}{1000} \times \frac{9 \times 9 \times 9}{1000} \\ &= \frac{2}{10^5} \times 729 = 1458 \times 10^{-5} \end{aligned}$$

Question 89

Area bounded by the curve $x = 0$ and $x + 2|y| = 1$ is

Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

Answer: B

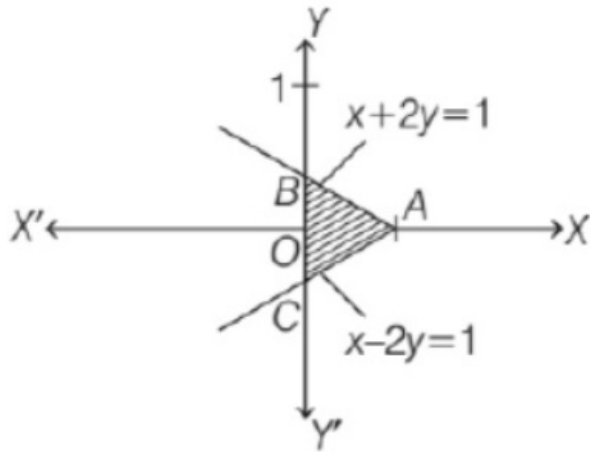
Solution:

Solution:

Given curves are $x = 0$ and $x + 2|y| = 1$

Now, $x + 2|y| = 0$

When $y > 0$, $x + 2y = 1$; when $y < 0$, $x - 2y = 1$



$\therefore$ Area of bounded region ABC

$$= 2AOB = 2 \int_0^1 \left(\frac{1-x}{2} \right) dx$$

$$= \left[x - \frac{x^2}{2} \right]_0^1 = \left[1 - \frac{1}{2} - (0 - 0) \right] = \frac{1}{2}$$

Question 90

The area bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ is

Options:

A. $\left(\frac{5\pi}{4} - 2 \right)$ sq units

B. $\frac{(5\pi - 2)}{4}$ sq units

C. $\frac{(5\pi - 2)}{2}$ sq units

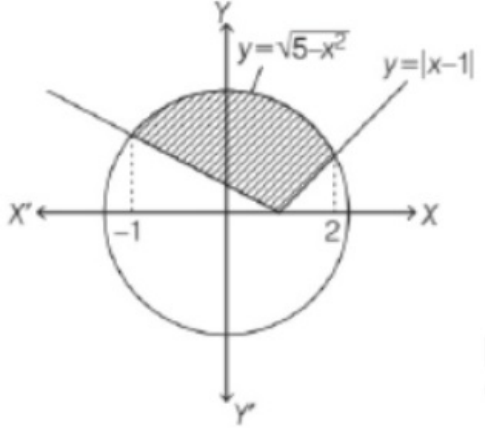
D. $\left(\frac{\pi}{2} - 5\right)$ sq units

Answer: B

Solution:

Solution:

Given, $y = \sqrt{5 - x^2} \Rightarrow y^2 + x^2 = 5$
and $y = |x - 1|$



$\therefore$ Required area

$$\begin{aligned} &= \int_{-1}^2 \sqrt{5 - x^2} dx - \int_{-1}^1 (1 - x) dx - \int_1^2 (x - 1) dx \\ &= \left[\frac{x}{2} \sqrt{5 - x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_{-1}^2 - \left[x - \frac{x^2}{2} \right]_{-1}^1 - \left[\frac{x^2}{2} - x \right]_1^2 \\ &= \left[1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} + 1 + \frac{5}{2} \sin^{-1} \frac{1}{\sqrt{5}} \right] \\ &\quad - \left[1 - \frac{1}{2} - \left(-1 - \frac{1}{2} \right) \right] - \left[2 - 2 - \left(\frac{1}{2} - 1 \right) \right] \\ &= 2 + \frac{5}{2} \sin^{-1} \left(\frac{2}{\sqrt{5}} \sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}} \sqrt{1 - \frac{4}{5}} \right) - \frac{5}{2} \\ &= \frac{5}{2} \sin^{-1}(1) - \frac{1}{2} = \frac{5\pi}{4} - \frac{1}{2} = \left(\frac{5\pi - 2}{4} \right) \text{ sq units} \end{aligned}$$

Question 91

The value of $\int_3^5 \frac{x^2}{x^2 - 4} dx$ is

Options:

A. $2 - \log_e \left(\frac{15}{7} \right)$

B. $2 + \log_e \left(\frac{15}{7} \right)$

C. $2 + 4\log_e 3 - 4\log_e 7 + 4\log_e 5$

D. $2 - \tan^{-1} \left(\frac{15}{7} \right)$

Answer: B

Solution:

Solution:

$$\begin{aligned}\int_3^5 \frac{x^2}{x^2-4} dx &= \int_3^5 \left(\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4} \right) dx \\&= \int_3^5 \left(1 + \frac{4}{x^2-4} \right) dx = \left[x + \frac{4}{2 \times 2} \log_e \left(\frac{x-2}{x+2} \right) \right]_3^5 \\&= \left[5 + \log_e \left(\frac{5-2}{5+2} \right) - 3 - \log_e \left(\frac{3-2}{3+2} \right) \right] \\&= 2 + \log_e \left(\frac{3}{7} \right) - \log_e \left(\frac{1}{5} \right) \\&= 2 + \log_e \left(\frac{3}{7} \times \frac{5}{1} \right) = 2 + \log_e \left(\frac{15}{7} \right)\end{aligned}$$

Question 92

The particular solution of the differential equation $\frac{dy}{dx} = \frac{y+1}{x^2-x}$, when $x = 2$ and $y = 1$ is

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Options:

- A. $xy = 3x - 4$
- B. $xy = 2x - 2$
- C. $xy = 4x - 6$
- D. $xy = -x + 4$

Answer: A

Solution:

Solution:

Given, differential equation

$$\frac{dy}{dx} = \frac{y+1}{x^2-x} \Rightarrow \frac{dy}{y+1} = \frac{dx}{x^2-x}$$

On integrating both sides, we get

$$\int \frac{dy}{y+1} = \int \frac{dx}{x^2-x}$$

$$\text{Now, } \frac{1}{x^2-x} = \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$\therefore \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \dots (i)$$

$$\Rightarrow 1 = A(x-1) + B(x)$$

Putting $x = 0$, then

$$1 = A(0-1) \Rightarrow A = -1$$

Putting $x-1 = 0$, then $x = 1$

$$\therefore 1 = A(0) + B(1)$$

$$\Rightarrow B = 1$$

From Eq. (i), we get

$$\int \frac{dy}{y+1} = \int -\frac{1dx}{x} + \int \frac{1}{x-1} dx$$

$$\Rightarrow \log(y+1) = -\log x + \log(x-1) + \log C$$

$$\Rightarrow \log(y+1) + \log x - \log(x-1) = \log C$$

$$\Rightarrow \log \left\{ \frac{x(y+1)}{x-1} \right\} = \log C$$

$$\Rightarrow \frac{x(y+1)}{x-1} = C \dots (ii)$$

On putting $x = 2$ and $y = 1$ in Eq. (ii), we get

$$\frac{2(1+1)}{2-1} = C \Rightarrow C = (2)(2) = 4$$

Putting value of $C = 4$ in Eq. (ii), we get

$$\frac{x(y+1)}{x-1} = 4$$

$$\Rightarrow xy + x = 4x - 4 \Rightarrow xy = 3x - 4$$

Question 93

The differential equation of all circles which passes through the origin and whose centre lies on Y-axis, is

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Options:

A. $(x^2 - y^2) \frac{dy}{dx} - 2xy = 0$

B. $(x^2 - y^2) \frac{dy}{dx} + 2xy = 0$

C. $(x^2 - y^2) \frac{dy}{dx} - xy = 0$

D. $(x^2 - y^2) \frac{dy}{dx} + xy = 0$

Answer: A

Solution:

Solution:

Let $x^2 + y^2 - 2ky = 0$

On differentiating w.r.t x , we get

$$2x + 2y \frac{dy}{dx} - 2k \frac{dy}{dx} = 0$$

$$\Rightarrow k = \frac{x}{\left(\frac{dy}{dx}\right)} + y$$

From Eq. (i),

$$x^2 + y^2 - 2 \left(\frac{x}{(dy/dx)} + y \right) y = 0$$

$$\Rightarrow (x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

Question 94

The solution of the equation $(x^2 + xy)dy = (x^2 + y^2)dx$ is

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Options:

A. $\log x = \log(x - y) + \frac{y}{x} + C$

B. $\log x = 2 \log(x - y) + \frac{y}{x} + C$

C. $\log x = \log(x - y) + \frac{y}{x} + C$

D. None of the above

Answer: B

Solution:

Solution:

Given, $(x^2 + xy)dy = (x^2 + y^2)dx$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{x^2 + x^2 v}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{1 + v^2}{1 + v}$$

$$\Rightarrow \frac{xdv}{dx} = \frac{1 + v^2}{1 + v} - v$$

$$\Rightarrow = \frac{1 + v^2 - v - v^2}{1 + v} = \frac{1 - v}{1 + v}$$

$$\Rightarrow dv \left(\frac{1 + v}{1 - v} \right) = \frac{dx}{x} \Rightarrow dv \left(-1 + \frac{2}{1 - v} \right) = \frac{dx}{x}$$

On integrating both sides, we get

$$-v - 2 \log(1 - v) = \log x + C$$

$$\Rightarrow \frac{y}{x} - 2 \log \left(1 - \frac{y}{x} \right) = \log x + C$$

$$\Rightarrow \frac{y}{x} - 2 \log(x - y) + 2 \log(x - y) + C = \log x + C$$

Question 95

The value of $\int_{\pi/6}^{\pi/2} \left(\frac{1 + \sin 2x + \cos 2x}{\sin x \cos x} \right) dx$ is equal to

Options:

A. 16

B. 8

C. 4

D. 1

Answer: D

Solution:

Solution:

$$\int_{\pi/6}^{\pi/2} \left(\frac{1 + \sin 2x + \cos 2x}{\sin x \cos x} \right) dx$$

$$\begin{aligned}
&= \int_{\pi/6}^{\pi/2} \left(\frac{1 + \sin x \cos x + 2\cos^2 x - 1}{(\sin x + \cos x)} \right) dx \\
&= \int_{\pi/6}^{\pi/2} \left(\frac{2\cos x(\sin x + \cos x)}{(\sin x + \cos x)} \right) dx \\
&= \int_{\pi/6}^{\pi/2} 2\cos x dx = 2[\sin x]_{\pi/6}^{\pi/2} \\
&= 2 \left(\sin \frac{\pi}{2} - \sin \frac{\pi}{6} \right) = 2 \left(1 - \frac{1}{2} \right) = 2 \times \frac{1}{2} = 1
\end{aligned}$$

Question 96

How many 5-digit telephone number can be constructed using the digits 0 to 9 , if each number starts with 67 and no digits appears more than once?

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Options:

- A. 335
- B. 336
- C. 338
- D. 337

Answer: B

Solution:

Solution:
Since, telephone number start with 67 , so two digits is already fixed. Now, we have to do arrangement of three digits from remaining eight digits.
 $\therefore$ Possible number of ways = 8P_3
 $= \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$ ways

Question 97

The number of selecting atleast 4 candidates from 8 candidates is

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Options:

- A. 270
- B. 70
- C. 163
- D. None of these

Answer: C

Solution:

Solution:

$$\begin{aligned} &\text{Required number of selections} \\ &= {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8 \\ &= 70 + 56 + 28 + 8 + 1 = 163 \end{aligned}$$

Question 98

If $f(x) = \frac{2x-1}{x+5}$, $x \neq -5$, then $f^{-1}(x)$ is equal to

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Options:

A. $\frac{x+5}{2x-1}$, $x \neq \frac{1}{2}$

B. $\frac{5x+1}{2-x}$, $x \neq 2$

C. $\frac{x-3}{2x+1}$, $x \neq \frac{1}{2}$

D. $\frac{5x-1}{2-x}$, $x \neq 2$

Answer: D

Solution:

Solution:

$$\begin{aligned} \text{Let } y &= \frac{2x-1}{x+5} \\ \Rightarrow x &= \frac{5y+1}{2-y} \\ \therefore f^{-1}(x) &= \frac{5x-1}{2-x}, x \neq 2 \end{aligned}$$

Question 99

If $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, for what values of α is $f(f(x)) = x$?

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Options:

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. -1

D. 2

Answer: C

Solution:

Solution:

$$f(x) = \frac{\alpha x}{x+1}, x \neq -1$$

$$\therefore f(f(x)) = f\left(\frac{\alpha x}{x+1}\right) = \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right) + 1} = \frac{\alpha^2 x}{\alpha x + x + 1}$$

$$\Rightarrow \frac{\alpha^2 x}{\alpha x + x + 1} = x \quad [\text{given}]$$

$$\Rightarrow x[\alpha^2 - \alpha x - x - 1] = 0$$

$$\Rightarrow x(\alpha + 1)(\alpha - 1 - x) = 0$$

$$\Rightarrow x = 0 \text{ or } \alpha + 1 = 0$$

$$\text{or } \alpha = 1 + x \quad [\because \alpha - 1 - x \neq 0]$$

$$\Rightarrow \alpha = -1 \text{ or } \alpha = 1 + x$$

$$\therefore \alpha = -1$$

$$[\because \alpha = 1 + x \text{ gives value for particular } x, \text{ not for all } x]$$

Question 100

If A and B are two events with $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$. Then, $P[(B) / (A \cap B^c)]$ is equal to

Options:

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

Answer: A

Solution:

Solution:

$$\text{Given, } P(A^c) = 0.3, P(B) = 0.4$$

$$\text{and } P(A \cap B^c) = 0.5$$

$$\therefore P(A^c) = 0.3$$

$$\Rightarrow P(A) = 1 - P(A^c) = 0.7$$

$$\text{and } P(B) = 0.4 \Rightarrow P(B^c) = 1 - P(B) = 0.6$$

$$\text{Consider, } P(A \cap B^c) = P(A) - P(A \cap B)$$

$$0.5 = 0.7 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 0.2$$

$$\text{Now, } P\left[\frac{B}{(A \cup B^c)}\right] = \frac{P[B \cap (A \cup B^c)]}{P(A \cup B^c)}$$

$$= \frac{P[(B \cap A) \cup (A \cap B^c)]}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{P(B \cap A)}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$
