

Q1. JEE Main 2026 (21 January Shift 2)

For the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix}$, if $(A^{15} + B) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then among the following which one is true?

- (1) $x = 5, y = 7$ (2) $x = 18, y = 11$
 (3) $x = 11, y = 2$ (4) $x = 16, y = 3$

Q2. JEE Main 2026 (23 January Shift 1)

Let $|A| = 6$, where A is a 3×3 matrix. If $|\text{adj}(3 \text{adj}(A^2 \cdot \text{adj}(2A)))| = 2^m \cdot 3^n, m, n \in \mathbb{N}$, then $m + n$ is equal to

_____.

Q3. JEE Main 2026 (28 January Shift 2)

Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and B be two matrices such that $A^{100} = 100B + I$. Then the sum of all the elements of B^{100} is

_____.

Q4. JEE Main 2026 (24 January Shift 2)

Let $f(x) = \int \frac{7x^{10} + 9x^8}{(1+x^2+2x^9)^2} dx, x > 0, \lim_{x \rightarrow 0} f(x) = 0$ and $f(1) = \frac{1}{4}$. If $A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$ and $B = \text{adj}(\text{adj } A)$ be such that $|B| = 81$, then α^2 is equal to

- (1) 2 (2) 1 (3) 3 (4) 4

Q5. JEE Main 2026 (24 January Shift 2)

Let $P = [p_{ij}]$ and $Q = [q_{ij}]$ be two square matrices of order 3 such that $q_{ij} = 2^{(i+j-1)} p_{ij}$ and $\det(Q) = 2^{10}$. Then the value of $\det(\text{adj}(\text{adj } P))$ is:

- (1) 81 (2) 16 (3) 124 (4) 32

Q6. JEE Main 2026 (22 January Shift 2)

If $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is a solution of the system of equations $AX = B$, where $\text{adj } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, then $|x + y + z|$ is equal to :

- (1) 2 (2) $\frac{3}{2}$ (3) 1 (4) 3

Q7. JEE Main 2026 (22 January Shift 1)

If $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$, then the determinant of the matrix $(A^{2025} - 3A^{2024} + A^{2023})$ is

- (1) 12 (2) 28 (3) 24 (4) 16

Q8. JEE Main 2026 (22 January Shift 1)

Let A be a 3×3 matrix such that $A + A^T = O$. If $A \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$, $A^2 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 19 \\ -24 \end{bmatrix}$ and

$\det(\text{adj}(2 \text{adj}(A + I))) = (2)^\alpha \cdot (3)^\beta \cdot (11)^\gamma$, α, β, γ are non-negative integers, then $\alpha + \beta + \gamma$ is equal to _____

Q9. JEE Main 2026 (23 January Shift 2)

Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and B be a matrix such that $B(I - A) = I + A$. Then the sum of the diagonal elements of $B^T B$ is equal to _____.

Q10. JEE Main 2026 (21 January Shift 1)

For some $\alpha, \beta \in \mathbf{R}$, let $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$ be such that $A^2 - 4A + 2I = B^2 - 3B + I = O$. Then $(\det(\text{adj}(A^3 - B^3)))^2$ is equal to _____.

Q11. JEE Main 2026 (28 January Shift 1)

Let A, B and C be three 2×2 matrices with real entries such that $B = (I + A)^{-1}$ and $A + C = I$. If

$BC = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}$ and $CB \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$, then $x_1 + x_2$ is

- (1) 2 (2) -2 (3) 0 (4) 4

Q12. JEE Main 2026 (24 January Shift 1)

The number of 3×2 matrices A , which can be formed using the elements of the set $\{-2, -1, 0, 1, 2\}$ such that the sum of all the diagonal elements of $A^T A$ is 5, is

ANSWERS AND SOLUTIONS

1. (3) 2. 62 3. 0 4. (4) 5. (2) 6. (1) 7. (4) 8. 18
 9. 3 10. 225 11. (3) 12. 312

1. (3) Here $A^n = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix}$
 $\Rightarrow A^{15} = \begin{bmatrix} 31 & -60 \\ 15 & -29 \end{bmatrix}$
 $\Rightarrow A^{15} + B = \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}$
 Now $(A^{15} + B) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
 $\Rightarrow \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
 $\Rightarrow 2x - 11y = 0 \Rightarrow x = \frac{11y}{2}$.
 For $y = 2, x = 11$.

2. (62) For 3×3 matrix: $|\text{adj}(M)| = |M|^2, |kM| = k^3|M|$.
 $|2A| = 8 \cdot 6 = 48, |\text{adj}(2A)| = 48^2 = 2^8 \cdot 3^2$.
 $|A^2 \cdot \text{adj}(2A)| = |A|^2 \cdot |\text{adj}(2A)| = 2^2 \cdot 3^2 \cdot 2^8 \cdot 3^2 = 2^{10} \cdot 3^4$.
 Let $B = A^2 \cdot \text{adj}(2A)$. $|\text{adj}(B)| = |B|^2 = 2^{20} \cdot 3^8$.
 $|3 \text{adj}(B)| = 3^3 \cdot 2^{20} \cdot 3^8 = 2^{20} \cdot 3^{11}$.
 $|\text{adj}(3 \text{adj}(B))| = (2^{20} \cdot 3^{11})^2 = 2^{40} \cdot 3^{22}$.
 $m = 40, n = 22 \Rightarrow m + n = 62$.

3. (0) $A = I + \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$, let $M = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$
 $M^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = M^3 = M^4 = \dots = M^{100}$
 $A^{100} = (I + M)^{100} = \sum_{r=0}^{100} \binom{100}{r} M^r \cdot I$
 $A^{100} = I + 100M = I + 100B$
 $\therefore M = B \Rightarrow M^{100} = B^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4. (4) Divide numerator and denominator by x^{18} , let $t = x^{-9} + x^{-7} + 2$, then $dt = -(9x^{-10} + 7x^{-8})dx$.

$f(x) = \frac{1}{t} + C = \frac{x^9}{1+x^2+2x^9} + C$.
 $\lim_{x \rightarrow 0} f(x) = 0 \Rightarrow C = 0$. $f(1) = \frac{1}{4}$ (verified).
 $f'(x) = \frac{9x^8(1+x^2+2x^9) - x^9(2x+18x^8)}{(1+x^2+2x^9)^2}$.
 $f'(1) = \frac{36-20}{16} = 1$.

$$|A| = \begin{vmatrix} 0 & 0 & 1 \\ 1/4 & 1 & 1 \\ \alpha^2 & 4 & 1 \end{vmatrix} = 1 - \alpha^2.$$

$$|B| = |\text{adj}(\text{adj } A)| = |A|^{(3-1)^2} = (1 - \alpha^2)^4 = 81 = 3^4.$$

$$|1 - \alpha^2| = 3 \Rightarrow \alpha^2 = 4.$$

$$5. \quad (2) \quad \begin{vmatrix} 2^2 p_{11} & 2^2 p_{12} & 2^3 p_{13} \\ 2^2 p_{21} & 2^3 p_{22} & 2^4 p_{23} \\ 2^3 p_{31} & 2^4 p_{32} & 2^5 p_{33} \end{vmatrix} = 2^{10}$$

$$2^2 \cdot 2 \cdot 2^3 \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ 2p_{21} & 2p_{22} & 2p_{23} \\ 2^2 p_{31} & 2^2 p_{32} & 2^2 p_{33} \end{vmatrix} = 2^{10}$$

$$2^9 \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix} = 2^{10} \Rightarrow |P| = 2$$

$$|\text{adj}(\text{adj}(P))| = |P|^{(n-1)^2} = |P|^4 = 2^4 = 16$$

$$6. \quad (1) \quad X = \frac{1}{|A|} \text{adj}(A) \cdot B.$$

$$|\text{adj}(A)| = 4(10) - 2(-20) + 2(10) = 100, \text{ so } |A|^2 = 100 \Rightarrow |A| = \pm 10.$$

$$\text{adj}(A) \cdot B = \begin{pmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{pmatrix} = \begin{pmatrix} 20 \\ -10 \\ 10 \end{pmatrix}$$

$$x + y + z = \frac{20 - 10 + 10}{|A|} = \frac{20}{\pm 10} = \pm 2$$

$$|x + y + z| = 2$$

$$7. \quad (4) \quad A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 13 & 21 \\ 21 & 34 \end{bmatrix}$$

$$|A^{2025} - 3A^{2024} + A^{2023}|$$

$$= |A^{2023}(A^2 - 3A + I)|$$

$$= |A|^{2023} |A^2 - 3A + I|$$

$$= 1 \cdot \begin{vmatrix} 8 & 12 \\ 12 & 20 \end{vmatrix} = 160 - 144 = 16$$

$$8. \quad (18) \quad A \text{ is skew-symmetric: } A = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix}.$$

$$\text{From } A \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} : c = 3, a + b = -2.$$

$$\text{From } A^2 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 19 \\ -24 \end{bmatrix} : b = 3, a = -5.$$

$$A + I = \begin{bmatrix} 1 & -3 & 3 \\ 3 & 1 & 5 \\ -3 & -5 & 1 \end{bmatrix}, \det(A + I) = 44.$$

For 3×3 matrix: $\det(\text{adj}(M)) = (\det M)^2$.

$$\det(2 \text{adj}(A + I)) = 8 \cdot 44^2 = 8 \cdot 1936 = 15488 = 2^7 \cdot 11^2.$$

$$\det(\text{adj}(2 \text{adj}(A + I))) = 15488^2 = 2^{14} \cdot 11^4.$$

$$\alpha = 14, \beta = 0, \gamma = 4. \alpha + \beta + \gamma = 18.$$

9. (3) A is skew-symmetric ($A^T = -A$).

$$\text{From } B(I - A) = I + A:$$

$$B = (I + A)(I - A)^{-1}.$$

$$B^T = ((I - A)^T)^{-1}(I + A)^T = (I + A)^{-1}(I - A).$$

Since A is skew-symmetric, $(I - A)$ and $(I + A)$ commute:

$$(I - A)(I + A) = I - A^2 = (I + A)(I - A).$$

$$B^T B = (I + A)^{-1}(I - A)(I + A)(I - A)^{-1} = (I + A)^{-1}(I + A)(I - A)(I - A)^{-1} = I.$$

$$\text{Trace of } B^T B = \text{tr}(I) = 3.$$

10. (225) From $A^2 - 4A + 2I = O$: characteristic equation is $\lambda^2 - 4\lambda + 2 = 0$.

$$\text{For } A: \alpha + 2 = 4 \Rightarrow \alpha = 2.$$

$$\text{From } B^2 - 3B + I = O: \text{characteristic equation is } \lambda^2 - 3\lambda + 1 = 0.$$

$$\text{For } B: \beta + 1 = 3 \Rightarrow \beta = 2.$$

$$A^3 = 14A - 8I, B^3 = 8B - 3I.$$

$$A^3 - B^3 = 14A - 8B - 5I = \begin{bmatrix} 15 & 20 \\ 6 & 7 \end{bmatrix}.$$

$$\det(A^3 - B^3) = 105 - 120 = -15.$$

For 2×2 matrix: $\det(\text{adj}(M)) = \det(M)$.

$$(\det(\text{adj}(A^3 - B^3)))^2 = 225.$$

11. (3) From $B = (I + A)^{-1}$ we get $B(I + A) = I$, so $BA = I - B$.

$$\text{From } A + C = I: C = I - A.$$

$$\text{Calculating } CB = C(I + A)^{-1} = (I - A)(I + A)^{-1}.$$

$$\text{Since } B(I + A) = I, \text{ we have } (I + A) = B^{-1}, \text{ so } C = (I - A)B^{-1}.$$

$$\text{Multiplying by } B: CB = (I - A).$$

$$\text{However, using the given } BC = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix} \text{ and } B(I + A) = I \text{ implies}$$

$$B - BA = B - (I - B) = 2B - I = BC, \text{ i.e. } BC = 2B - I.$$

$$\text{Therefore } B = \frac{1}{2}(BC + I) = \frac{1}{2} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -5/2 \\ -1/2 & 3/2 \end{bmatrix}.$$

$$\text{Also } CB = 2B - I = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}.$$

$$\text{From } CB \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}: (CB)^{-1} = -\frac{1}{3} \begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix}.$$

$$\text{So } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} -6 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}.$$

Therefore $x_1 + x_2 = 0$

12. (312) For a 3×2 matrix A, the diagonal sum of $A^T A$ equals the sum of squares of all 6 elements of A. We need:

$$a_{11}^2 + a_{21}^2 + a_{31}^2 + a_{12}^2 + a_{22}^2 + a_{32}^2 = 5 \text{ where each element is from } \{-2, -1, 0, 1, 2\}.$$

Possible squares: $0^2 = 0$, $(\pm 1)^2 = 1$, $(\pm 2)^2 = 4$.

Case 1:

Distribution $4 + 1 + 0 + 0 + 0 + 0$ (one ± 2 , one ± 1 , four 0's).

Arrangements: $\binom{6}{1} \times \binom{5}{1} = 30$.

Sign choices: $2 \times 2 = 4$. Total: $30 \times 4 = 120$.

Case 2:

Distribution $1 + 1 + 1 + 1 + 1 + 0$ (five ± 1 , one 0).

Arrangements: $\binom{6}{1} = 6$ ways to place the 0.

Sign choices for five ± 1 : $2^5 = 32$. Total: $6 \times 32 = 192$.

Total matrices: $120 + 192 = 312$