

Q1. JEE Main 2026 (28 January Shift 2)

Let $f(x) = \lim_{\theta \rightarrow 0} \left(\frac{\cos \pi x - x^{\left(\frac{2}{\theta}\right)} \sin(x-1)}{1+x^{\left(\frac{2}{\theta}\right)}(x-1)} \right), x \in \mathbf{R}$. Consider the following two statements :

(I) $f(x)$ is discontinuous at $x = 1$.

(II) $f(x)$ is continuous at $x = -1$.

Then,

(1) Only (II) is True

(2) Neither (I) nor (II) is True

(3) Both (I) and (II) are True

(4) Only (I) is True

ANSWERS AND SOLUTIONS

1. (2)

1. (2) Taking the limit as $\theta \rightarrow 0$, we note $x^{(2/\theta)} \rightarrow 0$ for $|x| < 1$ and $\rightarrow \infty$ for $|x| > 1$. This gives:

$$f(x) = \begin{cases} \cos \pi x & x \rightarrow 1^- \\ \frac{-\sin(x-1)}{(x-1)} & x \rightarrow 1^+ \end{cases}$$

Continuity at $x = 1$:

$$\text{RHL} = \lim_{x \rightarrow 1^+} \frac{-\sin(x-1)}{(x-1)} = -1$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} \cos \pi x = -1, f(1) = -1$$

$\Rightarrow f(x)$ is continuous at $x = 1$. So Statement (I) is false.

Continuity at $x = -1$:

$$f(x) = \begin{cases} \frac{-\sin(x-1)}{-(x-1)} & x \rightarrow -1^- \\ \cos \pi x & x \rightarrow -1^+ \end{cases}$$

$$\text{RHL} = \lim_{x \rightarrow -1^+} \cos \pi x = -1$$

$$\text{LHL} = \lim_{x \rightarrow -1^-} \frac{-\sin(x-1)}{-(x-1)} = \frac{\sin 2}{-2}$$

Since $\text{LHL} \neq \text{RHL}$, $f(x)$ is discontinuous at $x = -1$. So Statement (II) is also false.

Hence, neither (I) nor (II) is true.