

Q1. JEE Main 2026 (21 January Shift 2)

Let $[\cdot]$ denote the greatest integer function and $f(x) = \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n \left[\frac{k^2}{3^x} \right]$. Then $12 \sum_{j=1}^{\infty} f(j)$ is equal to ____.

Q2. JEE Main 2026 (22 January Shift 2)

If $\lim_{x \rightarrow 0} \frac{e^{(a-1)x} + 2 \cos bx + (c-2)e^{-x}}{x \cos x - \log_e(1+x)} = 2$, then $a^2 + b^2 + c^2$ is equal to :

- (1) 5 (2) 9 (3) 3 (4) 7

Q3. JEE Main 2026 (21 January Shift 1)

Let $f : \mathbf{R} \rightarrow (0, \infty)$ be a twice differentiable function such that $f(3) = 18$, $f'(3) = 0$ and $f''(3) = 4$. Then

$\lim_{x \rightarrow 1} \left(\log_e \left(\frac{f(2+x)}{f(3)} \right)^{\frac{18}{(x-1)^2}} \right)$ is equal to :

- (1) 1 (2) 18 (3) 2 (4) 9

Q4. JEE Main 2026 (28 January Shift 1)

The value of $\lim_{x \rightarrow 0} \frac{\log_e(\sec(ex) \cdot \sec(e^2x) \cdot \dots \cdot \sec(e^{10}x))}{e^2 - e^{2 \cos x}}$ is equal to

- (1) $\frac{(e^{20}-1)}{2(e^2-1)}$ (2) $\frac{(e^{10}-1)}{2(e^2-1)}$ (3) $\frac{(e^{10}-1)}{2e^2(e^2-1)}$ (4) $\frac{(e^{20}-1)}{2e^2(e^2-1)}$

ANSWERS AND SOLUTIONS

1. 2

2. (4)

3. (3)

4. (1)

1. (2) Let $[\cdot]$ denote the greatest integer function. We have $f(x) = \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n \left[\frac{k^2}{3^x} \right]$.

For large n : $\sum_{k=1}^n \left[\frac{k^2}{3^x} \right] \approx \sum_{k=1}^n \frac{k^2}{3^x} = \frac{1}{3^x} \cdot \frac{n(n+1)(2n+1)}{6} \approx \frac{n^3}{3 \cdot 3^x}$

Therefore: $f(x) = \lim_{n \rightarrow \infty} \frac{1}{n^3} \cdot \frac{n^3}{3 \cdot 3^x} = \frac{1}{3^{x+1}}$

Computing the infinite series:

$$\sum_{j=1}^{\infty} f(j) = \sum_{j=1}^{\infty} \frac{1}{3^{j+1}} = \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \dots$$

$$= \frac{1}{9} \cdot \left(1 + \frac{1}{3} + \frac{1}{9} + \dots \right) = \frac{1}{9} \cdot \frac{1}{1 - 1/3} = \frac{1}{9} \cdot \frac{3}{2} = \frac{1}{6}$$

Therefore: $12 \sum_{j=1}^{\infty} f(j) = 12 \cdot \frac{1}{6} = 2$

$$2. \quad (4) \lim_{x \rightarrow 0} \frac{\left(1 + (a-1)x + \frac{(a-1)^2 x^2}{2!} \right) + 2 \left(1 - \frac{b^2 x^2}{2!} \right) + (c-2) \left(1 - x + \frac{x^2}{2!} \right)}{x \left(1 - \frac{x^2}{2!} \right) - \left(x - \frac{x^2}{2} \dots \right)} = 2$$

$$\lim_{x \rightarrow 0} \frac{(1+2+c-2) + x(a-1-c+2) + x^2 \left(\frac{(a-1)^2}{2} - b^2 + \left(\frac{c-2}{2} \right) \right)}{\frac{x^2}{2} - \frac{x^3}{2!} + \dots} = 2$$

For which

$$\therefore c + 1 = 0 \Rightarrow c = -1$$

$$\therefore a - c = -1 \Rightarrow a = -2$$

$$\therefore \frac{(a-1)^2}{2} - b^2 + \left(\frac{c-2}{2} \right) = 1$$

$$\frac{9}{2} - b^2 - \frac{3}{2} = 1 \Rightarrow b^2 = 2$$

$$a^2 + b^2 + c^2 = 4 + 2 + 1 = 7$$

$$3. \quad (3) \text{ Let } T = \lim_{x \rightarrow 1} \left(\frac{f(x+2)}{f(3)} \right)^{\frac{18}{(x-1)^2}}; 1^\infty \text{ form}$$

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \frac{18}{(x-1)^2} \left(\frac{f(x+2) - f(3)}{f(3)} \right)}$$

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \frac{18}{(x-1)^2} \left(\frac{f(x+2) - f(3)}{18} \right)}$$

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \left(\frac{f(x+2) - f(3)}{(x-1)^2} \right) \cdot 0} \text{ form, apply L'Hital}$$

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \frac{f'(x+2)}{2(x-1)} \cdot \frac{0}{0} \text{ form, apply L'Hital}}$$

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \frac{f''(x+2)}{2} \cdot \frac{4}{2}} = e^{\frac{4}{2}} = e^2$$

$$\Rightarrow \log_e(T) = 2$$

4. (1) Numerator: $\log_e(\sec(ex) \sec(e^2x) \cdots \sec(e^{10}x)) = \sum_{k=1}^{10} \log_e(\sec(e^kx))$.

Using $\sec(u) \approx 1 + \frac{u^2}{2}$ for small u , we get $\log_e(\sec(e^kx)) \approx \frac{e^{2k}x^2}{2}$.

Thus the numerator is $\approx \frac{x^2}{2} \sum_{k=1}^{10} e^{2k} = \frac{x^2 e^2 (e^{20} - 1)}{2(e^2 - 1)}$.

Denominator: Using $\cos x \approx 1 - \frac{x^2}{2}$, we have $e^{2 \cos x} \approx e^2(1 - x^2)$, so $e^2 - e^{2 \cos x} \approx e^2 x^2$.

Therefore: $\lim_{x \rightarrow 0} \frac{\frac{x^2 e^2 (e^{20} - 1)}{2(e^2 - 1)}}{e^2 x^2} = \frac{e^{20} - 1}{2(e^2 - 1)}$.