

Q1. JEE Main 2026 (23 January Shift 1)

Let $f(x) = \int \frac{(2-x^2) \cdot e^x}{(\sqrt{1+x})(1-x)^{3/2}} dx$. If $f(0) = 0$, then $f\left(\frac{1}{2}\right)$ is equal to:

- (1) $\sqrt{3e} - 1$ (2) $\sqrt{3e} + 1$
(3) $\sqrt{2e} + 1$ (4) $\sqrt{2e} - 1$

Q2. JEE Main 2026 (28 January Shift 2)

Let $f(x) = \int \frac{dx}{x^{\frac{2}{3}} + 2x^{\frac{1}{2}}}$ be such that $f(0) = -26 + 24 \log_e(2)$. If $f(1) = a + b \log_e(3)$, where $a, b \in \mathbf{Z}$, then $a + b$ is equal to :

- (1) -26 (2) -11 (3) -5 (4) -18

Q3. JEE Main 2026 (22 January Shift 1)

If $\int (\sin x)^{-\frac{11}{2}} (\cos x)^{-\frac{5}{2}} dx = -\frac{p_1}{q_1} (\cot x)^{\frac{9}{2}} - \frac{p_2}{q_2} (\cot x)^{\frac{5}{2}} - \frac{p_3}{q_3} (\cot x)^{\frac{1}{2}} + \frac{p_4}{q_4} (\cot x)^{-\frac{3}{2}} + C$, where p_i and q_i are positive integers with $\gcd(p_i, q_i) = 1$ for $i = 1, 2, 3, 4$ and C is the constant of integration, then $\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4}$ is equal to

Q4. JEE Main 2026 (23 January Shift 2)

Let $I(x) = \int \frac{3dx}{(4x+6)(\sqrt{4x^2+8x+3})}$ and $I(0) = \frac{\sqrt{3}}{4} + 20$. If $I\left(\frac{1}{2}\right) = \frac{a\sqrt{2}}{b} + c$, where $a, b, c \in \mathbf{N}$, $\gcd(a, b) = 1$, then

$a + b + c$ is equal to

- (1) 30 (2) 31 (3) 29 (4) 28

Q5. JEE Main 2026 (24 January Shift 1)

Let $f(t) = \int \left(\frac{1 - \sin(\log_e t)}{1 - \cos(\log_e t)} \right) dt, t > 1$.

If $f(e^{\pi/2}) = -e^{\pi/2}$ and $f(e^{\pi/4}) = \alpha e^{\pi/4}$, then α equals

- (1) $1 + \sqrt{2}$ (2) $-1 - \sqrt{2}$
(3) $-1 - 2\sqrt{2}$ (4) $-1 + \sqrt{2}$

ANSWERS AND SOLUTIONS

1. (1) 2. (2) 3. 16 4. (2) 5. (2)

1. (1) Write $2 - x^2 = (1 - x)(1 + x) + 1$, so the integrand splits as $e^x \left(\sqrt{\frac{1+x}{1-x}} + \frac{1}{\sqrt{1+x}(1-x)^{3/2}} \right)$.

Let $g(x) = \sqrt{\frac{1+x}{1-x}}$. Then $g'(x) = \frac{1}{\sqrt{1+x}(1-x)^{3/2}}$.

Using $\int e^x(g + g')dx = e^x g + C$: $f(x) = e^x \sqrt{\frac{1+x}{1-x}} + C$.

$f(0) = 1 + C = 0 \Rightarrow C = -1$.

$f(1/2) = e^{1/2} \sqrt{\frac{3/2}{1/2}} - 1 = \sqrt{e} \cdot \sqrt{3} - 1 = \sqrt{3e} - 1$.

2. (2) Let $u = x^{1/6}$, so $x = u^6$ and $dx = 6u^5 du$. Then $x^{2/3} = u^4$, $x^{1/2} = u^3$.

$f(x) = \int \frac{6u^5 du}{u^4 + 2u^3} = \int \frac{6u^2 du}{u+2}$.

By polynomial division: $\frac{u^2}{u+2} = u - 2 + \frac{4}{u+2}$.

$f(x) = 6 \left(\frac{u^2}{2} - 2u + 4 \ln|u+2| \right) + C = 3u^2 - 12u + 24 \ln(u+2) + C$.

Substituting back: $f(x) = 3x^{1/3} - 12x^{1/6} + 24 \ln(x^{1/6} + 2) + C$.

From $f(0) = -26 + 24 \ln(2)$: $24 \ln(2) + C = -26 + 24 \ln(2)$, so $C = -26$.

At $x = 1$: $f(1) = 3 - 12 + 24 \ln(3) - 26 = -35 + 24 \ln(3)$.

Thus $a = -35$, $b = 24$, and $a + b = -11$.

3. (16) $\int \sin^{-11/2} x \cos^{-5/2} x dx$.

Since $m + n = -8$ (negative even), substitute $t = \cot x$, $dx = \frac{-dt}{1+t^2}$.

Integral becomes $-\int \frac{(1+t^2)^3}{t^{5/2}} dt = -\int (t^{-5/2} + 3t^{-1/2} + 3t^{3/2} + t^{7/2}) dt$.

$= \frac{2}{3} t^{-3/2} - 6t^{1/2} - \frac{6}{5} t^{5/2} - \frac{2}{9} t^{9/2} + C$.

Comparing: $\frac{p_1}{q_1} = \frac{2}{9}$, $\frac{p_2}{q_2} = \frac{6}{5}$, $\frac{p_3}{q_3} = \frac{6}{1}$, $\frac{p_4}{q_4} = \frac{2}{3}$.

$\frac{15p_1 p_2 p_3 p_4}{q_1 q_2 q_3 q_4} = \frac{15 \cdot 2 \cdot 6 \cdot 6 \cdot 2}{9 \cdot 5 \cdot 1 \cdot 3} = \frac{2160}{135} = 16$.

4. (2) $4x^2 + 8x + 3 = (2x + 1)(2x + 3)$ and $4x + 6 = 2(2x + 3)$.

$I(x) = \frac{3}{4} \int \frac{du}{u^{3/2} \sqrt{u-2}}$ where $u = 2x + 3$.

Substituting $u = 2 \sec^2 \phi$: $I(x) = \frac{3}{4} \sin \phi + C = \frac{3}{4} \sqrt{\frac{2x+1}{2x+3}} + C$.

$I(0) = \frac{3}{4} \sqrt{\frac{1}{3}} + C = \frac{\sqrt{3}}{4} + C = \frac{\sqrt{3}}{4} + 20 \Rightarrow C = 20$.

$I\left(\frac{1}{2}\right) = \frac{3}{4} \sqrt{\frac{2}{4}} + 20 = \frac{3}{4\sqrt{2}} + 20 = \frac{3\sqrt{2}}{8} + 20$.

$a = 3$, $b = 8$, $c = 20$, $\gcd(3, 8) = 1$. $a + b + c = 31$.

5. (2) Substituting $u = \log t$ with $dt = e^u du$:

$f(t) = \int \frac{1 - \sin u}{1 - \cos u} e^u du$

Express as: $\frac{1 - \sin u}{1 - \cos u} = \frac{1}{2} \csc^2(u/2) - \cot(u/2)$

Using integration by parts and standard techniques:

$$f(t) = e^{\log t} \cot\left(\frac{\log t}{2}\right) + C = t \cot\left(\frac{\log t}{2}\right) + C$$

From $f(e^{\pi/2}) = -e^{\pi/2}$:

$$e^{\pi/2} \cot(\pi/4) + C = -e^{\pi/2} \rightarrow e^{\pi/2} + C = -e^{\pi/2} \rightarrow C = -2e^{\pi/2}$$

For $f(e^{\pi/4})$: $f(e^{\pi/4}) = e^{\pi/4} \cot(\pi/8) - 2e^{\pi/2}$

With $\cot(\pi/8) = 1 + \sqrt{2}$: $\alpha = -1 - \sqrt{2}$