

Q1. JEE Main 2026 (23 January Shift 1)

Let the domain of the function $f(x) = \log_3 \log_5 \log_7 (9x - x^2 - 13)$ be the interval (m, n) . Let the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ have eccentricity $\frac{n}{3}$ and the length of the latus rectum $\frac{8m}{3}$. Then $b^2 - a^2$ is equal to :

- (1) 7 (2) 11 (3) 5 (4) 9

Q2. JEE Main 2026 (22 January Shift 2)

Let $P(10, 2\sqrt{15})$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and S' . If the length of its latus rectum is 8, then the square of the area of $\Delta PSS'$ is equal to :

- (1) 900 (2) 4200 (3) 1462 (4) 2700

Q3. JEE Main 2026 (22 January Shift 1)

If the line $\alpha x + 2y = 1$, where $\alpha \in \mathbb{R}$, does not meet the hyperbola $x^2 - 9y^2 = 9$, then a possible value of α is:

- (1) 0.5 (2) 0.6 (3) 0.7 (4) 0.8

Q4. JEE Main 2026 (23 January Shift 2)

Let PQ be a chord of the hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$, perpendicular to the x -axis such that OPQ is an equilateral triangle, O being the centre of the hyperbola. If the eccentricity of the hyperbola is $\sqrt{3}$, then the area of the triangle OPQ is

- (1) $2\sqrt{3}$ (2) $\frac{11}{5}$ (3) $\frac{9}{5}$ (4) $\frac{8\sqrt{3}}{5}$

Q5. JEE Main 2026 (21 January Shift 1)

Let the foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If the eccentricity of the hyperbola is 5, then the length of its latus rectum is :

- (1) 16 (2) $24\sqrt{5}$ (3) 12 (4) $\frac{96}{\sqrt{5}}$

Q6. JEE Main 2026 (28 January Shift 1)

For some $\theta \in (0, \frac{\pi}{2})$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2 \sec^2 \theta = 8$ be e_1 and l_1 , respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2 \sec^2 \theta + y^2 = 6$ be e_2 and l_2 , respectively. If $e_1^2 = e_2^2 (\sec^2 \theta + 1)$, then $\left(\frac{l_1 l_2}{e_1 e_2}\right) \tan^2 \theta$ is equal to _____

ANSWERS AND SOLUTIONS

1. (1) 2. (4) 3. (4) 4. (4) 5. (4) 6. 8

1. (1) For $f(x) = \log_3 \log_5 \log_7(9x - x^2 - 13)$:

$$\log_7(\cdot) > 0 \Rightarrow 9x - x^2 - 13 > 1 \Rightarrow (x-2)(x-7) < 0 \Rightarrow x \in (2, 7).$$

$$\log_5(\cdot) > 0 \Rightarrow \log_7(9x - x^2 - 13) > 1 \Rightarrow 9x - x^2 - 13 > 7 \Rightarrow (x-4)(x-5) < 0 \Rightarrow x \in (4, 5).$$

$$\text{Domain} = (4, 5), \text{ so } m = 4, n = 5.$$

$$\text{Eccentricity } e = n/3 = 5/3, \text{ latus rectum} = 8m/3 = 32/3.$$

$$e^2 = 1 + b^2/a^2 \Rightarrow b^2/a^2 = 16/9.$$

$$2b^2/a = 32/3 \Rightarrow 32a/9 = 32/3 \Rightarrow a = 3, \text{ so } a^2 = 9, b^2 = 16.$$

$$b^2 - a^2 = 16 - 9 = 7.$$

2. (4) For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, point $P(10, 2\sqrt{15})$ gives $\frac{100}{a^2} - \frac{60}{b^2} = 1$.

$$\text{The latus rectum length } \frac{2b^2}{a} = 8 \text{ gives } b^2 = 4a.$$

$$\text{Substituting: } \frac{100}{a^2} - \frac{15}{a} = 1, \text{ which yields } a^2 + 15a - 100 = 0, \text{ so } a = 5 \text{ and } b^2 = 20.$$

$$\text{Thus } c^2 = 45 \text{ and } c = 3\sqrt{5}.$$

$$\text{The foci are } S(\pm 3\sqrt{5}, 0).$$

$$\text{Triangle } PSS' \text{ has base } SS' = 6\sqrt{5} \text{ and height } 2\sqrt{15}.$$

$$\text{Area} = \frac{1}{2} \cdot 6\sqrt{5} \cdot 2\sqrt{15} = 30\sqrt{3}.$$

$$\text{Therefore } (\text{Area})^2 = 2700.$$

3. (4) Hyperbola: $\frac{x^2}{9} - y^2 = 1$. Line: $y = \frac{1-\alpha x}{2}$.

$$\text{Substituting: } x^2 - 9 \cdot \frac{(1-\alpha x)^2}{4} = 9$$

$$\Rightarrow (4 - 9\alpha^2)x^2 + 18\alpha x - 45 = 0.$$

$$\text{For no intersection (when } 4 - 9\alpha^2 \neq 0): \text{discriminant} < 0.$$

$$D = 324\alpha^2 + 180(4 - 9\alpha^2) < 0$$

$$\Rightarrow -1296\alpha^2 + 720 < 0 \Rightarrow \alpha^2 > \frac{5}{9}.$$

$$|\alpha| > \frac{\sqrt{5}}{3} \approx 0.745.$$

Among the options, only $\alpha = 0.8$ satisfies this condition.

4. (4) Hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$ with $e = \sqrt{3}$. $e^2 = 1 + \frac{b^2}{4} = 3 \Rightarrow b^2 = 8$.

$$\text{Let } P = (x_0, y_0), Q = (x_0, -y_0).$$

For equilateral $\triangle OPQ$:

$$OP = PQ \Rightarrow x_0^2 + y_0^2 = 4y_0^2 \Rightarrow x_0^2 = 3y_0^2.$$

From the hyperbola:

$$\frac{3y_0^2}{4} - \frac{y_0^2}{8} = 1 \Rightarrow \frac{5y_0^2}{8} = 1 \Rightarrow y_0^2 = \frac{8}{5}.$$

$$\text{Area} = \frac{\sqrt{3}}{4} (2y_0)^2 = \sqrt{3} \cdot y_0^2 = \frac{8\sqrt{3}}{5}.$$

5. (4) For ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$: $a = 6$, $b = 4$, $c = \sqrt{36 - 16} = 2\sqrt{5}$.

Foci of ellipse: $(\pm 2\sqrt{5}, 0)$.

For hyperbola with eccentricity $e = 5$ and foci at $(\pm 2\sqrt{5}, 0)$:

$$e = \frac{c}{A} \Rightarrow A = \frac{2\sqrt{5}}{5}.$$

$$C^2 = A^2 + B^2 \Rightarrow 20 = \frac{4}{5} + B^2 \Rightarrow B^2 = \frac{96}{5}.$$

$$\text{Latus rectum} = \frac{2B^2}{A} = \frac{2 \cdot \frac{96}{5}}{\frac{2\sqrt{5}}{5}} = \frac{96}{\sqrt{5}}.$$

6. (8) $\frac{x^2}{8} - \frac{y^2}{8 \cos^2 \theta} = 1$, $e_1 = \sqrt{1 + \frac{8 \cos^2 \theta}{8}}$

$$\ell_1 = \frac{2b^2}{a} = \frac{2 \cdot (8 \cos^2 \theta)}{2\sqrt{2}}$$

$$\frac{x^2}{6} + \frac{y^2}{6 \cos^2 \theta} = 1; e_2 = \sqrt{1 - \frac{6 \cos^2 \theta}{6}} = \sin \theta$$

$$\ell_2 = \frac{2b^2}{a} = \frac{2 \cdot 6 \cos^2 \theta}{\sqrt{6}}$$

$$e_1^2 = e_2^2(1 + \sec^2 \theta)$$

$$1 + \cos^2 \theta = \sin^2 \theta \left(1 + \frac{1}{\cos^2 \theta} \right)$$

$$1 + \cos^2 \theta = \sin^2 \theta + \tan^2 \theta$$

$$\text{Solving we get } \theta = \frac{\pi}{4}$$

$$\ell_1 = 2\sqrt{2}$$

$$e_1 = \sqrt{\frac{3}{2}}$$

$$\ell_2 = \sqrt{6}$$

$$e_2 = \frac{1}{\sqrt{2}}$$

$$\left(\frac{\ell_1 \ell_2}{e_1 e_2} \right) \tan^2 \theta = 8 \text{ (By putting values)}$$