

Q1. JEE Main 2026 (28 January Shift 2)

Given below are two statements: **Statement I:** The function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = \frac{x}{1+|x|}$ is one-one.

Statement II: The function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = \frac{x^2+4x-30}{x^2-8x+18}$ is many-one.

In the light of the above statements, choose the correct answer from the options given below :

- (1) Both Statement I and Statement II are true
- (2) Statement I is true but Statement II is false
- (3) Statement I is false but Statement II is true
- (4) Both Statement I and Statement II are false

Q2. JEE Main 2026 (28 January Shift 2)

The sum of all the elements in the range of $f(x) = \text{Sgn}(\sin x) + \text{Sgn}(\cos x) + \text{Sgn}(\tan x) + \text{Sgn}(\cot x)$,

$x \neq \frac{n\pi}{2}, n \in \mathbf{Z}$, where $\text{Sgn}(t) = \begin{cases} 1, & \text{if } t > 0 \\ -1, & \text{if } t < 0 \end{cases}$ is :

- (1) 2
- (2) 0
- (3) 4
- (4) -2

Q3. JEE Main 2026 (24 January Shift 2)

Let f be a function such that $3f(x) + 2f\left(\frac{m}{19x}\right) = 5x, x \neq 0$, where $m = \sum_{i=1}^9 (i)^2$. Then $f(5) - f(2)$ is equal to

- (1) 18
- (2) 36
- (3) 9
- (4) -9

Q4. JEE Main 2026 (22 January Shift 2)

Let the domain of the function $f(x) = \log_3 \log_5 (7 - \log_2 (x^2 - 10x + 85)) + \sin^{-1} \left(\left| \frac{3x-7}{17-x} \right| \right)$ be $(\alpha, \beta]$. Then

$\alpha + \beta$ is equal to :

- (1) 9
- (2) 10
- (3) 12
- (4) 8

Q5. JEE Main 2026 (22 January Shift 2)

Let f and g be functions satisfying $f(x+y) = f(x)f(y)$, $f(1) = 7$ and $g(x+y) = g(xy)$, $g(1) = 1$, for all

$x, y \in \mathbf{N}$. If $\sum_{x=1}^n \left(\frac{f(x)}{g(x)} \right) = 19607$, then n is equal to:

- (1) 7
- (2) 6
- (3) 4
- (4) 5

Q6. JEE Main 2026 (22 January Shift 1)

If the domain of the function $f(x) = \sin^{-1} \left(\frac{5-x}{3+2x} \right) + \frac{1}{\log_e(10-x)}$ is $(-\infty, \alpha] \cup [\beta, \gamma) - \{\delta\}$, then $6(\alpha + \beta + \gamma + \delta)$ is equal to

- (1) 67
- (2) 66
- (3) 70
- (4) 68

Q7. JEE Main 2026 (22 January Shift 1)

The number of distinct real solutions of the equation $x|x+4| + 3|x+2| + 10 = 0$ is

- (1) 2
- (2) 1
- (3) 3
- (4) 0

Q8. JEE Main 2026 (23 January Shift 2)

Consider two sets $A = \{x \in \mathbb{Z} : ||x - 3| - 3| \leq 1\}$ and $B = \left\{x \in \mathbb{R} - \{1, 2\} : \frac{(x-2)(x-4)}{x-1} \log_e(|x-2|) = 0\right\}$.

Then the number of onto functions $f : A \rightarrow B$ is equal to

- (1) 32 (2) 79 (3) 62 (4) 81

Q9. JEE Main 2026 (28 January Shift 1)

If $g(x) = 3x^2 + 2x - 3$, $f(0) = -3$ and $4g(f(x)) = 3x^2 - 32x + 72$, then $f(g(2))$ is equal to:

- (1) $-\frac{7}{2}$ (2) $-\frac{25}{6}$ (3) $\frac{7}{2}$ (4) $\frac{25}{6}$

Q10. JEE Main 2026 (24 January Shift 1)

If the domain of the function $f(x) = \log_{(10x^2 - 17x + 7)}(18x^2 - 11x + 1)$ is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$, then $90(a + b + c + d + e)$ equals:

- (1) 170 (2) 307 (3) 316 (4) 177

ANSWERS AND SOLUTIONS

1. (1) 2. (1) 3. (1) 4. (1) 5. (4) 6. (3) 7. (2) 8. (3)
9. (3) 10. (3)

1. (1) Statement I: $f(x) = \frac{x}{1+|x|}$. For $x \geq 0$, $f(x) = \frac{x}{1+x}$ is strictly increasing. For $x < 0$, $f(x) = \frac{x}{1-x}$ with $f'(x) = \frac{1}{(1-x)^2} > 0$, also strictly increasing. The function is continuous at $x = 0$, so f is strictly increasing on $\mathbb{R}$ and hence one-one. Statement I is true.

Statement II: $f(x) = \frac{x^2+4x-30}{x^2-8x+18}$. The denominator $(x-4)^2 + 2 > 0$ for all x .
 $f'(x) = \frac{-12(x^2-8x+14)}{(x^2-8x+18)^2}$
 $f'(x) = 0 \Rightarrow x = 4 \pm \sqrt{2}$. Since f' changes sign, f is not monotonic and hence many-one. Statement II is true.
Both statements are true.

2. (1) $x \in (0, \pi/2) \Rightarrow y = 1 + 1 + 1 + 1 = 4$
 $x \in (\pi/2, \pi) \Rightarrow y = 1 - 1 - 1 - 1 = -2$
 $x \in (\pi, 3\pi/2) \Rightarrow y = -1 - 1 + 1 + 1 = 0$
 $x \in (3\pi/2, 2\pi) \Rightarrow y = -1 + 1 - 1 - 1 = -2$
 $\therefore$ Range of y is $\{-2, 0, 4\}$
Required sum $= -2 + 0 + 4 = 2$

3. (1) $m = \sum_{i=1}^9 i^2 = \frac{9 \cdot 10 \cdot 19}{6} = 285$, so $\frac{m}{19} = 15$.
The equation becomes $3f(x) + 2f\left(\frac{15}{x}\right) = 5x \dots(i)$
Replace x by $\frac{15}{x}$: $3f\left(\frac{15}{x}\right) + 2f(x) = \frac{75}{x} \dots(ii)$
From $3 \times (i) - 2 \times (ii)$: $5f(x) = 15x - \frac{150}{x}$, so $f(x) = 3x - \frac{30}{x}$.
 $f(5) = 15 - 6 = 9$, $f(2) = 6 - 15 = -9$.
 $f(5) - f(2) = 9 - (-9) = 18$.

4. (1) For $\log_3 \log_5(\cdot)$ we need $\log_5(\cdot) > 0$
So $7 - \log_2(x^2 - 10x + 85) > 1$, giving $x^2 - 10x + 85 < 64$
 $(x-3)(x-7) < 0$
 $\Rightarrow 3 < x < 7$.
For $\sin^{-1}\left(\left|\frac{3x-7}{17-x}\right|\right)$ we need $|3x-7| \leq |17-x|$
i.e., $(x-6)(x+5) \leq 0 \Rightarrow -5 \leq x \leq 6$.
Intersection: $(3, 6]$, so $\alpha = 3, \beta = 6$.
 $\alpha + \beta = 9$

5. (4) $f(x+y) = f(x)f(y)$ with $f(1) = 7$ gives $f(x) = 7^x$.
For g : put $y = 1$ in $g(x+y) = g(xy)$ to get $g(x+1) = g(x)$.
So $g(x) = g(1) = 1$ for all $x \in \mathbb{N}$.

$$\sum_{x=1}^n \frac{f(x)}{g(x)} = \sum_{x=1}^n 7^x = \frac{7(7^n-1)}{6} = 19607$$

$$7^{n+1} - 7 = 117642$$

$$\Rightarrow 7^{n+1} = 117649 = 7^6$$

$$n+1 = 6 \Rightarrow n = 5$$

6. (3) $-1 \leq \frac{5-x}{2x+3} \leq 1$ & $10-x > 0$, $10-x \neq 1$

$$\left| \frac{5-x}{2x+3} \right| \geq 1 \text{ \& } x < 10 \text{ \& } x \neq 9$$

$$(5-x)^2 - (2x+3)^2 \leq 0 \text{ \& } x < 10 \text{ \& } 4x \neq 9$$

$$(x+8)(3x-2) \geq 0 \text{ \& } x < 10 \text{ \& } x \neq 9$$

$$\Rightarrow (-\infty, -8] \cup \left[\frac{2}{3}, 10 \right) - \{9\}$$

$$\Rightarrow (\alpha + \beta + \gamma + \delta) = 6 \left(-8 + \frac{2}{3} + 10 + 9 \right)$$

$$= 70$$

7. (2) Equation: $x|x+4| + 3|x+2| + 10 = 0$.

Critical points: $x = -4, -2$.

Case 1:

$$x \geq -2: x(x+4) + 3(x+2) + 10 = 0$$

$$\Rightarrow x^2 + 7x + 16 = 0.$$

$$\text{Discriminant} = 49 - 64 = -15 < 0. \text{ No real roots.}$$

Case 2:

$$-4 \leq x < -2: x(x+4) - 3(x+2) + 10 = 0$$

$$\Rightarrow x^2 + x + 4 = 0.$$

$$\text{Discriminant} = 1 - 16 = -15 < 0. \text{ No real roots.}$$

Case 3:

$$x < -4: -x(x+4) - 3(x+2) + 10 = 0$$

$$\Rightarrow x^2 + 7x - 4 = 0.$$

$$x = \frac{-7 \pm \sqrt{65}}{2}.$$

$$x = \frac{-7 + \sqrt{65}}{2} \approx 0.53 \text{ (rejected, not in } x < -4). \quad x = \frac{-7 - \sqrt{65}}{2} \approx -7.53 \text{ (valid).}$$

Number of distinct real solutions = 1.

8. (3) For set A:

$$||x-3|-3| \leq 1 \Rightarrow 2 \leq |x-3| \leq 4.$$

$$\text{If } x \geq 3: 5 \leq x \leq 7 \Rightarrow x \in \{5, 6, 7\}.$$

$$\text{If } x < 3: -1 \leq x \leq 1 \Rightarrow x \in \{-1, 0, 1\}.$$

So $A = \{-1, 0, 1, 5, 6, 7\}$, $|A| = 6$.

For set B :

$$\frac{(x-2)(x-4)}{x-1} \log_e(|x-2|) = 0 \text{ with } x \in \mathbb{R} - \{1, 2\}.$$

Either $\frac{(x-2)(x-4)}{x-1} = 0 \Rightarrow x = 4$ (since $x \neq 1, 2$), or $\log_e|x-2| = 0 \Rightarrow |x-2| = 1 \Rightarrow x = 3$ (since $x \neq 1$).

So $B = \{3, 4\}$, $|B| = 2$.

Number of onto functions from A to $B = 2^6 - 2 = 62$.

9. (3) Let $f(x) = ax + b$. Then $4g(f(x)) = 12(ax + b)^2 + 8(ax + b) - 12$.

Comparing with $3x^2 - 32x + 72$:

$$x^2: 12a^2 = 3 \Rightarrow a = \pm \frac{1}{2}$$

$$\text{Constant: } 12b^2 + 8b - 12 = 72 \Rightarrow 3b^2 + 2b - 21 = 0 \Rightarrow b = \frac{7}{3} \text{ or } b = -3. \text{ Since } f(0) = -3, b = -3.$$

$$x: 8a(3(-3) + 1) = -32 \Rightarrow -8a = -4 \Rightarrow a = \frac{1}{2}.$$

$$\text{So } f(x) = \frac{x}{2} - 3.$$

$$g(2) = 3(4) + 4 - 3 = 13.$$

$$f(g(2)) = f(13) = \frac{13}{2} - 3 = \frac{7}{2}.$$

10. (3) For $f(x) = \log_{(10x^2-17x+7)}(18x^2-11x+1)$, we need:

$$\text{Base} > 0: 10x^2 - 17x + 7 > 0 \Rightarrow 10(x-1)(x-7/10) > 0 \Rightarrow x < 7/10 \text{ or } x > 1$$

$$\text{Base} \neq 1: 10x^2 - 17x + 6 = 0 \Rightarrow x = 1/2 \text{ or } x = 6/5$$

$$\text{Argument} > 0: 18x^2 - 11x + 1 > 0 \Rightarrow 18(x-1/2)(x-1/9) > 0 \Rightarrow x < 1/9 \text{ or } x > 1/2$$

$$\text{Intersecting conditions 1 and 3: } (-\infty, 1/9) \cup (1/2, 7/10) \cup (1, \infty)$$

Removing base = 1 points: $x = 1/2$ is already excluded; $x = 6/5 \in (1, \infty)$ must be removed.

$$\text{Domain: } (-\infty, 1/9) \cup (1/2, 7/10) \cup (1, \infty) - \{6/5\}$$

$$\text{So } a = 1/9, b = 1/2, c = 7/10, d = 1, e = 6/5.$$

$$90(a + b + c + d + e) = 90\left(\frac{1}{9} + \frac{1}{2} + \frac{7}{10} + 1 + \frac{6}{5}\right) = 10 + 45 + 63 + 90 + 108 = 316$$