

Q1. JEE Main 2026 (21 January Shift 2)

Let $y = y(x)$ be the solution of the differential equation $\sec x \frac{dy}{dx} - 2y = 2 + 3 \sin x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $y(0) = -\frac{7}{4}$. Then $y\left(\frac{\pi}{6}\right)$ is equal to :

- (1) $-\frac{5}{4}$ (2) $-\frac{5}{2}$
(3) $-3\sqrt{3} - 7$ (4) $-3\sqrt{2} - 7$

Q2. JEE Main 2026 (23 January Shift 1)

Let $y = y(x)$ be the solution of the differential equation $x^4 dy + (4x^3 y + 2 \sin x) dx = 0$, $x > 0$, $y\left(\frac{\pi}{2}\right) = 0$. Then $\pi^4 y\left(\frac{\pi}{3}\right)$ is equal to :

- (1) 72 (2) 92 (3) 64 (4) 81

Q3. JEE Main 2026 (23 January Shift 1)

Let f be a twice differentiable non-negative function such that $(f(x))^2 = 25 + \int_0^x \left((f(t))^2 + (f'(t))^2\right) dt$. Then the mean of $f(\log_e(1)), f(\log_e(2)), \dots, f(\log_e(625))$ is equal to _____.

Q4. JEE Main 2026 (28 January Shift 2)

Let $y = y(x)$ be the solution of the differential equation $x \frac{dy}{dx} - y = x^2 \cot x$, $x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then $6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$ is equal to :

- (1) 3π (2) π (3) $-\pi$ (4) -3π

Q5. JEE Main 2026 (24 January Shift 2)

Let $y = y(x)$ be a differentiable function in the interval $(0, \infty)$ such that $y(1) = 2$, and $\lim_{t \rightarrow x} \left(\frac{t^2 y(x) - x^2 y(t)}{x - t}\right) = 3$ for each $x > 0$. Then $2y(2)$ is equal to

- (1) 23 (2) 27 (3) 12 (4) 18

Q6. JEE Main 2026 (22 January Shift 2)

If $y = y(x)$ satisfies the differential equation $16(\sqrt{x+9\sqrt{x}})(4 + \sqrt{9+\sqrt{x}}) \cos y dy = (1 + 2 \sin y) dx$, $x > 0$ and $y(256) = \frac{\pi}{2}$, $y(49) = \alpha$, then $2 \sin \alpha$ is equal to :

- (1) $2(\sqrt{2} - 1)$ (2) $3(\sqrt{2} - 1)$
(3) $2\sqrt{2} - 1$ (4) $\sqrt{2} - 1$

Q7. JEE Main 2026 (22 January Shift 1)

Let the solution curve of the differential equation $xdy - ydx = \sqrt{x^2 + y^2} dx$, $x > 0$, $y(1) = 0$, be $y = y(x)$. Then $y(3)$ is equal to

- (1) 1 (2) 4 (3) 2 (4) 6

Q8. JEE Main 2026 (22 January Shift 1)

Let $f : [1, \infty) \rightarrow \mathbb{R}$ be a differentiable function. If $6 \int_1^x f(t) dt = 3xf(x) + x^3 - 4$ for all $x \geq 1$, then the value of $f(2) - f(3)$ is

- (1) -4 (2) 3 (3) 4 (4) -3

Q9. JEE Main 2026 (23 January Shift 2)

If the solution curve $y = f(x)$ of the differential equation

$$(x^2 - 4)y' - 2xy + 2x(4 - x^2)^2 = 0, x > 2,$$

passes through the point $(3, 15)$, then the local maximum value of f is _____.

Q10. JEE Main 2026 (21 January Shift 1)

Let $y = y(x)$ be the solution curve of the differential equation $(1 + x^2)dy + (y - \tan^{-1} x)dx = 0, y(0) = 1$. Then the value of $y(1)$ is :

- (1) $\frac{4}{e^{\pi/4}} + \frac{\pi}{2} - 1$ (2) $\frac{2}{e^{\pi/4}} + \frac{\pi}{4} - 1$
(3) $\frac{2}{e^{\pi/4}} - \frac{\pi}{4} - 1$ (4) $\frac{4}{e^{\pi/4}} - \frac{\pi}{2} - 1$

Q11. JEE Main 2026 (28 January Shift 1)

Let $y = y(x)$ be the solution of the differential equation $x \frac{dy}{dx} - \sin 2y = x^3 (2 - x^3) \cos^2 y, x \neq 0$. If $y(2) = 0$, then $\tan(y(1))$ is equal to

- (1) $-\frac{3}{4}$ (2) $\frac{3}{4}$ (3) $-\frac{7}{4}$ (4) $\frac{7}{4}$

ANSWERS AND SOLUTIONS

1. (2) 2. (4) 3. 1565 4. (3) 5. (1) 6. (3) 7. (2) 8. (2)
9. 16 10. (2) 11. (4)

1. (2) The differential equation is $\sec x \frac{dy}{dx} - 2y = 2 + 3 \sin x$, or $\frac{dy}{dx} - 2 \cos x \cdot y = (2 + 3 \sin x) \cos x$.

This is a linear ODE with integrating factor $\mu(x) = e^{-2 \sin x}$.

Multiplying both sides and integrating: $\frac{d}{dx}[ye^{-2 \sin x}] = (2 + 3 \sin x) \cos x \cdot e^{-2 \sin x}$

Computing the integral: $\int 2 \cos x \cdot e^{-2 \sin x} dx = -e^{-2 \sin x}$

For the second term, using integration by parts: $\int 3 \sin x \cos x \cdot e^{-2 \sin x} dx = \frac{3}{4} e^{-2 \sin x} (-2 \sin x - 1)$

Thus: $ye^{-2 \sin x} = -e^{-2 \sin x} + \frac{3}{4} e^{-2 \sin x} (-2 \sin x - 1) + C$

Simplifying: $y = -\frac{7}{4} - \frac{3}{2} \sin x + Ce^{2 \sin x}$

Using $y(0) = -\frac{7}{4}$ gives $C = 0$.

Therefore: $y(\frac{\pi}{6}) = -\frac{7}{4} - \frac{3}{2} \cdot \frac{1}{2} = -\frac{5}{2}$

2. (4) $x^4 dy + 4x^3 y dx + 2 \sin x dx = 0 \Rightarrow d(x^4 y) + 2 \sin x dx = 0$.

Integrating: $x^4 y - 2 \cos x = C$.

$y(\pi/2) = 0$: $(\pi/2)^4 \cdot 0 - 2 \cos(\pi/2) = 0 \Rightarrow C = 0$.

$x^4 y = 2 \cos x$.

At $x = \pi/3$: $(\pi/3)^4 y(\pi/3) = 2 \cos(\pi/3) = 1$.

$\pi^4 y(\pi/3) = 81 \cdot (\pi/3)^4 y(\pi/3) \cdot \frac{1}{(\pi/3)^4} \cdot \frac{\pi^4}{81} \dots$ Directly: $y(\pi/3) = \frac{81}{\pi^4}$, so $\pi^4 y(\pi/3) = 81$.

3. (1565) Differentiating $(f(x))^2 = 25 + \int_0^x ((f(t))^2 + (f'(t))^2) dt$:

$2ff' = f^2 + (f')^2 \Rightarrow (f - f')^2 = 0 \Rightarrow f' = f$.

$f(x) = Ce^x$. At $x = 0$: $f(0)^2 = 25 \Rightarrow f(0) = 5$ (non-negative), so $f(x) = 5e^x$.

$f(\log_e k) = 5k$ for $k = 1, 2, \dots, 625$.

Mean = $\frac{5(1+2+\dots+625)}{625} = \frac{5 \times 625 \times 626}{2 \times 625} = \frac{5 \times 626}{2} = 1565$.

4. (3) The differential equation $x \frac{dy}{dx} - y = x^2 \cot x$ can be rewritten as $\frac{dy}{dx} - \frac{y}{x} = x \cot x$.

The integrating factor is $\mu(x) = \frac{1}{x}$. Multiplying by $\mu(x)$ gives $\frac{d}{dx}\left(\frac{y}{x}\right) = \cot x$.

Integrating: $\frac{y}{x} = \ln |\sin x| + C$, so $y = x \ln(\sin x) + Cx$.

Using $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$: we get $\frac{\pi}{2} = 0 + C \frac{\pi}{2}$, thus $C = 1$.

Therefore $y = x(1 + \ln(\sin x))$.

Now: $y\left(\frac{\pi}{6}\right) = \frac{\pi}{6}(1 - \ln 2)$ and $y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\left(1 - \frac{\ln 2}{2}\right)$.

Thus $6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right) = \pi(1 - \ln 2) - 2\pi\left(1 - \frac{\ln 2}{2}\right) = -\pi$.

5. (1) Given the limit $\lim_{t \rightarrow x} \frac{t^2 y(x) - x^2 y(t)}{x - t} = 3$.

The limit is in $\frac{0}{0}$ form. Applying L'Hopital's Rule with respect to t :

$$\lim_{t \rightarrow x} \frac{\frac{d}{dt}(t^2 y(x) - x^2 y(t))}{\frac{d}{dt}(x - t)} = 3$$

$$\lim_{t \rightarrow x} \frac{2ty(x) - x^2 y'(t)}{-1} = 3$$

Substituting $t = x$:

$$\frac{2xy(x) - x^2 y'(x)}{-1} = 3$$

$$x^2 y'(x) - 2xy(x) = 3$$

Dividing by x^4 to make it a linear differential equation or recognizing the quotient rule form:

$$\frac{x^2 y'(x) - 2xy(x)}{x^4} = \frac{3}{x^4}$$

$$\frac{d}{dx} \left(\frac{y(x)}{x^2} \right) = \frac{3}{x^4}$$

Integrating both sides with respect to x :

$$\frac{y(x)}{x^2} = \int 3x^{-4} dx = \frac{3x^{-3}}{-3} + C = -\frac{1}{x^3} + C$$

Given $y(1) = 2$:

$$\frac{2}{1^2} = -\frac{1}{1^3} + C \implies 2 = -1 + C \implies C = 3$$

$$\text{So, } \frac{y(x)}{x^2} = -\frac{1}{x^3} + 3$$

$$y(x) = -\frac{1}{x} + 3x^2$$

We need to find $2y(2)$:

$$y(2) = -\frac{1}{2} + 3(2^2) = -\frac{1}{2} + 12 = \frac{23}{2}$$

$$2y(2) = 2 \times \frac{23}{2} = 23$$

$$6. (3) \int \frac{\cos y}{1+2 \sin y} dy = \int \frac{dx}{16(\sqrt{9 \sqrt{x}+x})(4+\sqrt{9+\sqrt{x}})}$$

$$4 + \sqrt{9 + \sqrt{x}} = t$$

$$\frac{1}{2\sqrt{9+\sqrt{x}}} \times \frac{dx}{2\sqrt{x}} = 1dx$$

$$\frac{1}{2} \ln |1 + 2 \sin y| = \int \frac{4dt}{16t} + C$$

$$\frac{1}{2} \ln |1 + 2 \sin y| = \frac{1}{4} \ln |4 + \sqrt{9 + \sqrt{x}}| + C$$

$$\frac{1}{2} \ln (2 \sin y + 1) = \frac{1}{4} \ln |4 + \sqrt{9 + \sqrt{x}}| + C$$

Substituting $\left(256, \frac{\pi}{2}\right)$

$$\frac{1}{2} \ln 3 = \frac{1}{4} \ln 3 + C \quad C = 0$$

Substituting $(49, \alpha)$

$$\frac{1}{2} \ln (2 \sin \alpha + 1) = \frac{1}{4} \ln 8$$

$$\ln (2 \sin \alpha + 1) = \frac{1}{2} \ln 8$$

$$\ln (2 \sin \alpha + 1) = \ln 2\sqrt{2}$$

$$2 \sin \alpha + 1 = 2\sqrt{2}$$

$$2 \sin \alpha = 2\sqrt{2} - 1$$

7. (2) $xdy - ydx = \sqrt{x^2 + y^2} dx$.

Put $y = vx$: $x^2 dv = x\sqrt{1+v^2} dx$.

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}.$$

Integrating: $\ln(v + \sqrt{1+v^2}) = \ln x + C$.

$y(1) = 0 \Rightarrow v = 0$ at $x = 1$: $C = 0$.

$v + \sqrt{1+v^2} = x \Rightarrow y + \sqrt{x^2 + y^2} = x^2$.

At $x = 3$: $y + \sqrt{9 + y^2} = 9$

$\Rightarrow 9 + y^2 = (9 - y)^2$

$\Rightarrow 18y = 72$

$\Rightarrow y = 4$.

8. (2) Differentiating $6 \int_1^x f(t) dt = 3xf(x) + x^3 - 4$:

$6f(x) = 3f(x) + 3xf'(x) + 3x^2 \Rightarrow f(x) - xf'(x) = x^2$.

$$\frac{d}{dx} \left(\frac{f(x)}{x} \right) = \frac{xf'(x) - f(x)}{x^2} = -1.$$

Integrating: $\frac{f(x)}{x} = -x + C \Rightarrow f(x) = -x^2 + Cx$.

At $x = 1$: $0 = 3f(1) - 3 \Rightarrow f(1) = 1$. So $-1 + C = 1 \Rightarrow C = 2$.

$f(x) = -x^2 + 2x$. $f(2) = 0$, $f(3) = -3$.

$f(2) - f(3) = 3$.

9. (16) Rewriting: $\frac{dy}{dx} - \frac{2x}{x^2-4}y = -2x(x^2-4)$.

IF = $e^{-\int \frac{2x}{x^2-4} dx} = \frac{1}{x^2-4}$.

$$\frac{d}{dx} \left(\frac{y}{x^2-4} \right) = -2x \Rightarrow \frac{y}{x^2-4} = -x^2 + C.$$

$y = (x^2 - 4)(C - x^2)$. At $(3, 15)$: $15 = 5(C - 9) \Rightarrow C = 12$.

$y = (x^2 - 4)(12 - x^2)$.

$y' = 4x(8 - x^2) = 0 \Rightarrow x = 2\sqrt{2}$ (for $x > 2$).

$y'' = 32 - 12x^2 = -64 < 0$ at $x = 2\sqrt{2}$, confirming local maximum.

$f(2\sqrt{2}) = (8 - 4)(12 - 8) = 16$.

10. (2) Rewrite as $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1} x}{1+x^2}$.

IF = $e^{\tan^{-1} x}$.

$$y \cdot e^{\tan^{-1} x} = \int \frac{\tan^{-1} x}{1+x^2} \cdot e^{\tan^{-1} x} dx.$$

Let $t = \tan^{-1} x$: $\int te^t dt = e^t(t - 1) + C$.

$y = \tan^{-1} x - 1 + Ce^{-\tan^{-1} x}$.

Using $y(0) = 1$: $C = 2$.

$y(1) = \frac{\pi}{4} - 1 + \frac{2}{e^{\pi/4}}$.

11. (4) Divide the equation by $\cos^2 y$:

$x \sec^2 y \frac{dy}{dx} - 2 \tan y = x^3(2 - x^3)$.

Let $v = \tan y$, so $\sec^2 y \frac{dy}{dx} = \frac{dv}{dx}$.

The equation becomes $x \frac{dv}{dx} - 2v = x^3(2 - x^3)$.

Dividing by x : $\frac{dv}{dx} - \frac{2v}{x} = x^2(2 - x^3)$.

Using integrating factor $\mu = x^{-2}$: $\frac{d}{dx}(x^{-2}v) = 2 - x^3$.

Integrating: $x^{-2}v = 2x - \frac{x^4}{4} + C$.

From $y(2) = 0$, we have $\tan 0 = 0 = 16 - 16 + 4C$, giving $C = 0$.

Therefore $\tan y = 2x^3 - \frac{x^6}{4}$.

At $x = 1$: $\tan(y(1)) = 2 - \frac{1}{4} = \frac{7}{4}$.