

Q1. JEE Main 2026 (21 January Shift 2)

If $\int_0^1 4 \cot^{-1}(1 - 2x + 4x^2) dx = a \tan^{-1}(2) - b \log_e(5)$, where $a, b \in \mathbf{N}$, then $(2a + b)$ is equal to _____.

Q2. JEE Main 2026 (23 January Shift 1)

The value of the integral $\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\tan 2x}}$ is :

- (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{12}$ (3) $\frac{\pi}{18}$ (4) $\frac{\pi}{6}$

Q3. JEE Main 2026 (28 January Shift 2)

Let $[\cdot]$ denote the greatest integer function. Then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{12(3 + [x])}{3 + [\sin x] + [\cos x]} \right) dx$ is equal to :

- (1) $13\pi + 1$ (2) $12\pi + 5$ (3) $11\pi + 2$ (4) $15\pi + 4$

Q4. JEE Main 2026 (24 January Shift 2)

If $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 (y + xe^x) f(y) dy$, then $e + f(0)$ is equal to _____.

Q5. JEE Main 2026 (22 January Shift 2)

Let $f(x) = [x]^2 - [x + 3] - 3$, $x \in \mathbf{R}$, where $[\cdot]$ is the greatest integer function. Then

- (1) $\int_0^2 f(x) dx = -6$
(2) $f(x) < 0$ only for $x \in [-1, 3]$
(3) $f(x) > 0$ only for $x \in [4, \infty)$
(4) $f(x) = 0$ for finitely many values of x

Q6. JEE Main 2026 (22 January Shift 2)

Let $[\cdot]$ be the greatest integer function. If $\alpha = \int_0^{64} (x^{1/3} - [x^{1/3}]) dx$, then $\frac{1}{\pi} \int_0^{\alpha\pi} \left(\frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} \right) d\theta$ is equal to _____.

Q7. JEE Main 2026 (22 January Shift 1)

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{[x]+4} \right) dx$, where $[\cdot]$ denotes the greatest integer function, is

- (1) $\frac{1}{60}(21\pi - 1)$ (2) $\frac{7}{60}(\pi - 3)$
(3) $\frac{7}{60}(3\pi - 1)$ (4) $\frac{1}{60}(\pi - 7)$

Q8. JEE Main 2026 (23 January Shift 2)

The number of elements in the

set $S = \{x : x \in [0, 100] \text{ and } \int_0^x t^2 \sin(x - t) dt = x^2\}$ is _____.

Q9. JEE Main 2026 (21 January Shift 1)

The value of $\int_{-\pi/6}^{\pi/6} \left(\frac{\pi+4x^{11}}{1-\sin(|x|+\pi/6)} \right) dx$ is equal to:

(1) 8π

(2) 6π

(3) 2π

(4) 4π

Q10. JEE Main 2026 (21 January Shift 1)

$6 \int_0^\pi |(\sin 3x + \sin 2x + \sin x)| dx$ is equal to _____.

Q11. JEE Main 2026 (28 January Shift 1)

Let f be a polynomial function such that $f(x^2 + 1) = x^4 + 5x^2 + 2$, for all $x \in \mathbb{R}$. Then $\int_0^3 f(x) dx$ is equal to

(1) $\frac{5}{3}$

(2) $\frac{41}{3}$

(3) $\frac{27}{2}$

(4) $\frac{33}{2}$

Q12. JEE Main 2026 (28 January Shift 1)

If $\int \left(\frac{1-5\cos^2 x}{\sin^5 x \cos^2 x} \right) dx = f(x) + C$, where C is the constant of integration, then $f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{1}{\sqrt{3}}(26 + \sqrt{3})$

(2) $\frac{1}{\sqrt{3}}(26 - \sqrt{3})$

(3) $\frac{4}{\sqrt{3}}(8 - \sqrt{6})$

(4) $\frac{2}{\sqrt{3}}(4 + \sqrt{6})$

Q13. JEE Main 2026 (28 January Shift 1)

The value of $\sum_{r=1}^{20} \left(\left| \sqrt{\pi \left(\int_0^r x |\sin \pi x| dx \right)} \right| \right)$ is _____

Q14. JEE Main 2026 (24 January Shift 1)

Let a differentiable function f satisfy the equation $\int_0^{36} f\left(\frac{tx}{36}\right) dt = 4\alpha f(x)$. If $y = f(x)$ is a standard parabola passing through the points $(2, 1)$ and $(-4, \beta)$, then β^α is equal to _____.

ANSWERS AND SOLUTIONS

1. 9 2. (2) 3. (3) 4. 2 5. (2) 6. 36 7. (3) 8. 16

9. (4) 10. 17 11. (4) 12. (3) 13. 210 14. 64

1. (9) Let $I = \int_0^1 \cot^{-1}(1 - 2x + 4x^2) dx$
 $I = \int_0^1 (\cot^{-1}(2x - 1) - \cot^{-1}(2x)) dx$

Applying king

$$I = \int_0^1 (-\cot^{-1}(2x - 1) + \cot^{-1}(2x - 2)) dx$$

From (1) and (2)

$$2I = \int_0^1 (\cot^{-1}(2x - 2) - \cot^{-1}(2x)) dx$$

$$= \int_0^1 \cot^{-1}(2x - 2) dx - \int_0^1 \cot^{-1}(2x) dx$$

Applying King

$$= \int_0^1 \cot^{-1}(-2x) dx - \int_0^1 \cot^{-1}(2x) dx$$

$$= \int_0^1 (\pi - \cot^{-1}(2x)) dx - \int_0^1 \cot^{-1}(2x) dx$$

$$= \int_0^1 (\pi - 2 \cot^{-1}(2x)) dx$$

$$= \pi - 2 \int_0^1 (\cot^{-1} 2x) \cdot 1 dx$$

By parts

$$= \pi - 2 \left[(x \cot^{-1} 2x)_0^1 + \int_0^1 \frac{2x}{1+4x^2} dx \right]$$

$$\text{Let } 1 + 4x^2 = t$$

$$8x dx = dt$$

$$= \pi - 2 \left[\cot^{-1} 2 + \frac{1}{4} \int_1^5 \frac{dt}{t} \right]$$

$$= \pi - 2 \cot^{-1} 2 - \frac{1}{2} \ln 5$$

$$2I = 2 \tan^{-1} 2 - \frac{1}{2} \ln 5$$

$$\Rightarrow 4I = 4 \tan^{-1} 2 - \ln 5$$

$$\therefore 2a + b = 8 + 1 = 9$$

2. (2) Let $I = \int_{\pi/24}^{5\pi/24} \frac{dx}{1 + \sqrt[3]{\tan 2x}}$.

Apply King's rule ($x \rightarrow \pi/4 - x$):

$$I = \int_{\pi/24}^{5\pi/24} \frac{dx}{1 + \sqrt[3]{\cot 2x}} = \int_{\pi/24}^{5\pi/24} \frac{\sqrt[3]{\tan 2x}}{1 + \sqrt[3]{\tan 2x}} dx.$$

$$\text{Adding: } 2I = \int_{\pi/24}^{5\pi/24} dx = \frac{5\pi}{24} - \frac{\pi}{24} = \frac{\pi}{6}.$$

$$I = \frac{\pi}{12}.$$

3. (3) $I = \int_{-\pi/2}^{\pi/2} \frac{12(3 + [x]) dx}{3 + [\sin x] + [\cos x]}$

$$I = \int_{-\pi/2}^{-1} \frac{12(1) dx}{2} + \int_{-1}^0 \frac{12(2) dx}{2} + \int_0^1 \frac{12(3) dx}{3} + \int_1^{\pi/2} \frac{12(4) dx}{3}$$

$$I = 6 \left(\frac{\pi}{2} - 1 \right) + 12(0 + 1) + 12(1 - 0) + 16 \left(\frac{\pi}{2} - 1 \right)$$

$$I = 3\pi - 6 + 12 + 12 + 8\pi - 16$$

$$I = 11\pi + 2$$

4. (2) Given: $f(x) = e^x + \int_0^1 (y + xe^y) f(y) dy$

$$\text{Let } A = \int_0^1 f(y) dy \text{ and } B = \int_0^1 ye^y f(y) dy.$$

$$\text{Then } f(x) = e^x + Ax + Be^x = (1 + B)e^x + Ax$$

Integrating both sides from 0 to 1:

$$A = \int_0^1 [(1 + B)e^y + Ay] dy = (1 + B)(e - 1) + \frac{A}{2}$$

$$\text{This gives } \frac{A}{2} = (1 + B)(e - 1), \text{ so } A = 2(1 + B)(e - 1)$$

From $f(0) = 1 + B$ and solving the coupled equations for A and B :

$$f(0) = 1 - e$$

$$\text{Therefore } e + f(0) = e + (1 - e) = 2$$

5. (2) For $x \in [n, n + 1)$ where n is an integer, $[x] = n$ and $[x + 3] = n + 3$.

$$\text{Thus } f(x) = n^2 - (n + 3) - 3 = n^2 - n - 6 = (n - 3)(n + 2).$$

$$\text{For } f(x) < 0, \text{ we need } (n - 3)(n + 2) < 0, \text{ which gives } -2 < n < 3, \text{ i.e., } n \in \{-1, 0, 1, 2\}.$$

$$\text{This corresponds to } x \in [-1, 3).$$

We can verify:

$$\text{For } n = -1: f = 1 - 2 - 3 = -4 < 0$$

$$\text{For } n = 0, 1: f = -6 < 0$$

$$\text{For } n = 2: f = 4 - 5 - 3 = -4 < 0$$

$$\text{For } n = 3: f = 9 - 6 - 3 = 0$$

$$\text{For } n \geq 4, (n - 3)(n + 2) > 0.$$

$$\text{Therefore } f(x) < 0 \text{ only for } x \in [-1, 3).$$

6. (36) $\therefore \int_0^{64} x^{\frac{1}{3}} dx = \frac{3}{4} \left[x^{\frac{4}{3}} \right]_0^{64} = 192$ &

$$\int_0^{64} [x^{1/3}] dx = \int_0^1 [x^{1/3}] dx + \int_1^8 [x^{1/3}] dx + \int_8^{27} [x^{1/3}] dx + \int_{27}^{64} [x^{1/3}] dx = 156$$

$$\text{So } \alpha = 192 - 156 = 36$$

$$\text{Now } E = \frac{1}{\pi} \int_0^{36\pi} \frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta$$

$$= \frac{36}{\pi} \int_0^{\pi} \frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta$$

$$\Rightarrow E = \frac{36 \cdot 2}{\pi} \int_0^{\pi/2} \frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta$$

$$\text{Let } J = \int_0^{\pi/2} \frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta \dots (1)$$

Applying King

$$J = \int_0^{\pi/2} \frac{\cos^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta \dots (2)$$

$$\text{Now } 2J = \int_0^{\pi/2} \frac{1}{\sin^6 \theta + \cos^6 \theta} d\theta \text{ (add (1) \& (2))}$$

$$= \int_0^{\pi/2} \frac{\sec^6 \theta}{\tan^6 \theta + 1} d\theta$$

$$= \int_0^{\infty} \frac{(1 + \lambda^2)}{\lambda^4 - \lambda^2 + 1} d\lambda$$

$$= \int_0^{\infty} \frac{1 + \frac{1}{\lambda^2}}{\lambda^2 - 1 + \frac{1}{\lambda^2}} d\lambda$$

$$= \pi$$

$$\Rightarrow J = \frac{\pi}{2}$$

$$\Rightarrow E = \frac{36 \cdot 2}{\pi} \times J = 36$$

7. (3) $I = \int_{-\pi/2}^{\pi/2} \frac{1}{[x] + 4} dx$

$$I = \int_{-\pi/2}^{-1} \frac{dx}{2} + \int_{-1}^0 \frac{dx}{3} + \int_0^1 \frac{dx}{4} + \int_1^{\pi/2} \frac{dx}{5}$$

$$= \frac{1}{2} \left(-1 + \frac{\pi}{2} \right) + \frac{1}{3} (1) + \frac{1}{4} (1) + \frac{1}{5} \left(\frac{\pi}{2} - 1 \right)$$

$$= \frac{7\pi}{20} - \frac{7}{60} = \frac{7}{60} (3\pi - 1)$$

8. (16) Using $u = x - t$: $\int_0^x t^2 \sin(x - t) dt = \int_0^x (x - u)^2 \sin u du$.

Expanding and integrating term by term:

$$= x^2(1 - \cos x) - 2x(-x \cos x + \sin x) + (-x^2 \cos x + 2x \sin x + 2 \cos x - 2)$$

$$= x^2 + 2 \cos x - 2.$$

$$\text{Setting equal to } x^2: 2 \cos x - 2 = 0 \Rightarrow \cos x = 1 \Rightarrow x = 2n\pi.$$

$$\text{In } [0, 100]: 2n\pi \leq 100 \Rightarrow n \leq \frac{50}{\pi} \approx 15.9.$$

So $n = 0, 1, 2, \dots, 15$, giving 16 elements.

9. (4) Split: $\int_{-\pi/6}^{\pi/6} \frac{\pi}{1 - \sin(|x| + \pi/6)} dx + \int_{-\pi/6}^{\pi/6} \frac{4x^{11}}{1 - \sin(|x| + \pi/6)} dx$.

Second integral = 0 (odd function over symmetric interval).

$$\text{First integral: } 2\pi \int_0^{\pi/6} \frac{1}{1 - \sin(x + \pi/6)} dx.$$

$$\text{Let } u = x + \pi/6: = 2\pi \int_{\pi/6}^{\pi/3} \frac{1}{1-\sin u} du.$$

$$\frac{1}{1-\sin u} = \sec^2 u + \sec u \tan u.$$

$$= 2\pi [\tan u + \sec u]_{\pi/6}^{\pi/3} = 2\pi [(\sqrt{3} + 2) - \sqrt{3}] = 4\pi.$$

10. (17) Using $\sin 3x + \sin x = 2 \sin 2x \cos x$:

$$\sin 3x + \sin 2x + \sin x = \sin 2x(2 \cos x + 1) = 2 \sin x \cos x(2 \cos x + 1)$$

$$\text{Zeros in } [0, \pi]: x = 0, \pi/2, 2\pi/3, \pi$$

Expression is positive on $(0, \pi/2)$, negative on $(\pi/2, 2\pi/3)$, positive on $(2\pi/3, \pi)$

$$\text{Let } F(x) = -\frac{\cos 3x}{3} - \frac{\cos 2x}{2} - \cos x$$

$$F(0) = -\frac{11}{6}, F(\pi/2) = \frac{1}{2}, F(2\pi/3) = \frac{5}{12}, F(\pi) = \frac{5}{6}$$

$$\int_0^{\pi/2} = \frac{7}{3}, \left| \int_{\pi/2}^{2\pi/3} \right| = \frac{1}{12}, \int_{2\pi/3}^{\pi} = \frac{5}{12}$$

$$\text{Total} = \frac{7}{3} + \frac{1}{12} + \frac{5}{12} = \frac{17}{6}$$

$$6 \times \frac{17}{6} = 17$$

11. (4) Let $u = x^2 + 1$, then $x^2 = u - 1$.

Substituting into the given condition:

$$f(u) = (u - 1)^2 + 5(u - 1) + 2 =$$

$$u^2 - 2u + 1 + 5u - 5 + 2$$

$$= u^2 + 3u - 2.$$

Therefore, $f(x) = x^2 + 3x - 2$.

$$\text{Calculating the integral: } \int_0^3 f(x) dx = \int_0^3 (x^2 + 3x - 2) dx$$

$$= \left[\frac{x^3}{3} + \frac{3x^2}{2} - 2x \right]_0^3 = 9 + \frac{27}{2} - 6 = \frac{33}{2}.$$

$$12. (3) \int \frac{1-5\cos^2 x}{\sin^5 x \cos^2 x} dx = \int \frac{dx}{\sin^5 x \cos^2 x} - 5 \int \frac{dx}{\sin^5 x}$$

$$= \int \frac{\sec^2 x dx}{\sin^5 x} - 5 \int \frac{dx}{\sin^5 x}$$

By IBP:

$$= \frac{\tan x}{\sin^5 x} - \int \left(-\frac{5}{\sin^6 x} \right) \cos x \cdot \tan x dx - 5 \int \frac{dx}{\sin^5 x}$$

$$= \frac{\tan x}{\sin^5 x} + C$$

$$\text{So } f(x) = \frac{\tan x}{\sin^5 x}$$

$$f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right) = \frac{2^5}{\sqrt{3}} - (\sqrt{2})^5 = \frac{32}{\sqrt{3}} - 4\sqrt{2} = \frac{4}{\sqrt{3}}(8 - \sqrt{6})$$

13. (210) Let $I_r = \int_0^r x |\sin \pi x| dx \dots (1)$

Apply King Property

$$= \int_0^r (r-x) |\sin \pi x| dx \dots (2)$$

By (1) + (2)

$$2I_r = \int_0^r r |\sin \pi x| dx \Rightarrow I_r = \frac{r}{2} \int_0^r |\sin \pi x| dx$$

$$I_1 = \frac{1}{2} \int_0^1 |\sin \pi x| dx = \frac{1}{2\pi} \int_0^\pi |\sin t| dt = \frac{1}{2\pi} (2)$$

$$I_2 = \frac{2}{2} \int_0^2 |\sin \pi x| dx = \frac{2}{2\pi} \int_0^{2\pi} |\sin t| dt = \frac{2}{2\pi} (4)$$

$$S = \sqrt{\pi \cdot \frac{1}{2\pi} \cdot 2} + \sqrt{\pi \cdot \frac{2}{2\pi} \cdot 4} + \sqrt{\pi \cdot \frac{3}{2\pi} \cdot 6} + \dots + \sqrt{\pi \cdot \frac{20}{2\pi} (2 \cdot 20)}$$

$$= 1 + 2 + 3 + \dots + 20$$

$$= \frac{20 \times 21}{2} = 210$$

14. (64) From $\int_0^{36} f\left(\frac{tx}{36}\right) dt = 4\alpha f(x)$, substitute $u = \frac{tx}{36}$ to get $\frac{36}{x} \int_0^x f(u) du = 4\alpha f(x)$.

Differentiating: $f(x) = \frac{\alpha}{9} [f(x) + xf'(x)]$, which simplifies to $(9 - \alpha)f(x) = \alpha x f'(x)$.

This gives $\frac{f'(x)}{f(x)} = \frac{9-\alpha}{\alpha x}$, integrating to $f(x) = Kx^{\frac{9-\alpha}{\alpha}}$.

For f to be a standard parabola: $\frac{9-\alpha}{\alpha} = 2 \Rightarrow \alpha = 3$.

Thus $f(x) = Kx^2$. Using point $(2, 1)$: $K = \frac{1}{4}$, so $f(x) = \frac{x^2}{4}$.

At $(-4, \beta)$: $\beta = \frac{16}{4} = 4$.

Therefore $\beta^\alpha = 4^3 = 64$