

Q1. JEE Main 2026 (21 January Shift 2)

If P is a point on the circle $x^2 + y^2 = 4$, Q is a point on the straight line $5x + y + 2 = 0$ and $x - y + 1 = 0$ is the perpendicular bisector of PQ, then 13 times the sum of abscissa of all such points P is _____.

Q2. JEE Main 2026 (28 January Shift 2)

Let the circle $x^2 + y^2 = 4$ intersect x -axis at the points A(a, 0), $a > 0$ and B(b, 0). Let $P(2 \cos \alpha, 2 \sin \alpha)$, $0 < \alpha < \frac{\pi}{2}$ and $Q(2 \cos \beta, 2 \sin \beta)$ be two points such that $(\alpha - \beta) = \frac{\pi}{2}$. Then the point of intersection of AQ and BP lies on :

- (1) $x^2 + y^2 - 4y - 4 = 0$
- (2) $x^2 + y^2 - 4x - 4y = 0$
- (3) $x^2 + y^2 - 4x - 4 = 0$
- (4) $x^2 + y^2 - 4x - 4y - 4 = 0$

Q3. JEE Main 2026 (22 January Shift 1)

Let the set of all values of r , for which the circles $(x + 1)^2 + (y + 4)^2 = r^2$ and $x^2 + y^2 - 4x - 2y - 4 = 0$ intersect at two distinct points be the interval (α, β) . Then $\alpha\beta$ is equal to

- (1) 21
- (2) 24
- (3) 20
- (4) 25

Q4. JEE Main 2026 (23 January Shift 2)

If the points of intersection of the ellipses $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and $4x^2 + 2y^2 - 20x - 12y + 35 = 0$ lie on a circle of radius r and centre (a, b) , then the value of $ab + 18r^2$ is

- (1) 52
- (2) 53
- (3) 51
- (4) 55

Q5. JEE Main 2026 (21 January Shift 1)

Let $\vec{c}$ and $\vec{d}$ be vectors such that $|\vec{c} + \vec{d}| = \sqrt{29}$ and $\vec{c} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{d}$. If λ_1, λ_2 ($\lambda_1 > \lambda_2$) are the possible values of $(\vec{c} + \vec{d}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$, then the equation $K^2x^2 + (K^2 - 5K + \lambda_1)xy + \left(3K + \frac{\lambda_2}{2}\right)y^2 - 8x + 12y + \lambda_2 = 0$ represents a circle, for K equal to :

- (1) 1
- (2) 4
- (3) -1
- (4) 2

Q6. JEE Main 2026 (21 January Shift 1)

Let PQ and MN be two straight lines touching the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ at the points A and B respectively. Let O be the centre of the circle and $\angle AOB = \pi/3$. Then the locus of the point of intersection of the lines PQ and MN is :

(1) $x^2 + y^2 - 12x - 18y + 25 = 0$

(2) $3(x^2 + y^2) - 18x - 12y + 25 = 0$

(3) $3(x^2 + y^2) - 12x - 18y - 25 = 0$

(4) $x^2 + y^2 - 18x - 12y - 25 = 0$

Q7. JEE Main 2026 (28 January Shift 1)

Let $y = x$ be the equation of a chord of the circle C_1 (in the closed half-plane $x \geq 0$) of diameter 10 passing through the origin. Let C_2 be another circle described on the given chord as its diameter. If the equation of the chord of the circle C_2 , which passes through the point $(2, 3)$ and is farthest from the center of C_2 , is $x + ay + b = 0$, then $a - b$ is equal to

(1) -2

(2) 10

(3) -6

(4) 6

Q8. JEE Main 2026 (24 January Shift 1)

Let $S = \{z \in \mathbb{C} : \left| \frac{z-6i}{z-2i} \right| = 1 \text{ and } \left| \frac{z-8+2i}{z+2i} \right| = \frac{3}{5} \}$.

Then $\sum_{z \in S} |z|^2$ is equal to

(1) 385

(2) 398

(3) 413

(4) 423

Q9. JEE Main 2026 (24 January Shift 1)

Let a circle of radius 4 pass through the origin O, the points $A(-\sqrt{3}a, 0)$ and $B(0, -\sqrt{2}b)$, where a and b are real parameters and $ab \neq 0$. Then the locus of the centroid of $\triangle OAB$ is a circle of radius

(1) $\frac{7}{3}$

(2) $\frac{8}{3}$

(3) $\frac{11}{3}$

(4) $\frac{5}{3}$

ANSWERS AND SOLUTIONS

1. 2 2. (1) 3. (4) 4. (4) 5. (1) 6. (3) 7. (1) 8. (1)

9. (2)

Mid point of PQ lies on $x - y + 1 = 0$

$$\frac{2 \cos \theta + \alpha}{2} - \frac{2 \sin \theta - 5\alpha - 2}{2} + 1 = 0$$

$$2 \cos \theta + \alpha - 2 \sin \theta + 5\alpha + 2 + 2 = 0$$

$$\cos \theta - \sin \theta + 3\alpha + 2 = 0 \dots (1)$$

 $\therefore$ Slope of PQ is -1

$$\frac{2 \sin \theta + 5\alpha + 2}{2 \cos \theta - \alpha} = -1$$

$$2 \sin \theta + 5\alpha + 2 = -2 \cos \theta + \alpha$$

$$\sin \theta + \cos \theta + 2\alpha + 1 = 0 \dots (2)$$

eliminate α from (1) and (2)

$$\Rightarrow \cos \theta + 5 \sin \theta = 1, \theta \in [0, 2\pi]$$

$$\Rightarrow 5 \times 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 2 \sin^2 \frac{\theta}{2}$$

$$\therefore \sin \frac{\theta}{2} = 0 \Rightarrow \cos \theta = 1$$

or

$$\sin \frac{\theta}{2} = 5 \Rightarrow \cos \theta = -\frac{12}{13}$$

Sum of all possible values of abscissa of point P is

$$= 2 \times 1 + 2 \left(-\frac{12}{13} \right) = \frac{2}{13}$$

 $\therefore$ 13 times sum of all possible values of abscissa of point P is 2.2. (1) Let point of intersection $R(h, k)$

$$m_{BR} = m_{BP} \Rightarrow \frac{k}{h+2} = \frac{2 \sin \alpha}{2 \cos \alpha + 2} \Rightarrow \frac{k}{h+2} = \tan \frac{\alpha}{2}$$

$$m_{AR} = m_{AQ} \Rightarrow \frac{k}{h-2} = \frac{2 \sin \beta}{2 \cos \beta - 2} = \frac{\sin \beta}{\cos \beta - 1} = -\cot \frac{\beta}{2}$$

$$\frac{\alpha}{2} - \frac{\beta}{2} = \frac{\pi}{4}$$

$$\tan\left(\frac{\alpha}{2} - \frac{\beta}{2}\right) = \tan \frac{\pi}{4} = 1$$

$$\frac{\tan \frac{\alpha}{2} - \tan \frac{\beta}{2}}{1 + \tan \frac{\alpha}{2} \tan \frac{\beta}{2}} = 1$$

$$\frac{\frac{k}{h+2} + \frac{h-2}{k}}{1 + \left(\frac{k}{h+2}\right)\left(\frac{h-2}{k}\right)} = 1 \Rightarrow \frac{k^2 + h^2 - 4}{\frac{k(h+2)}{4}} = 1$$

$$\frac{h^2 + k^2 - 4}{4k} = 1$$

$$x^2 + y^2 - 4y - 4 = 0$$

3. (4) Circle 1: center $C_1 = (-1, -4)$, radius r .

Circle 2: $(x-2)^2 + (y-1)^2 = 9$, center $C_2 = (2, 1)$, radius 3.

$$\text{Distance } d = \sqrt{9 + 25} = \sqrt{34}.$$

For two intersection points: $|r-3| < \sqrt{34} < r+3$.

$$r+3 > \sqrt{34} \Rightarrow r > \sqrt{34} - 3 \text{ and } |r-3| < \sqrt{34} \Rightarrow r < \sqrt{34} + 3.$$

$$r \in (\sqrt{34} - 3, \sqrt{34} + 3), \text{ so } \alpha = \sqrt{34} - 3, \beta = \sqrt{34} + 3.$$

$$\alpha\beta = (\sqrt{34})^2 - 9 = 34 - 9 = 25.$$

4. (4) The circle through intersection of the ellipses is $S_1 + \lambda S_2 = 0$.

$$(1+4\lambda)x^2 + (2+2\lambda)y^2 - (6+20\lambda)x - (12+12\lambda)y + (23+35\lambda) = 0.$$

$$\text{For a circle: } 1+4\lambda = 2+2\lambda \Rightarrow \lambda = \frac{1}{2}.$$

$$\text{Circle: } 3x^2 + 3y^2 - 16x - 18y + \frac{81}{2} = 0 \Rightarrow x^2 + y^2 - \frac{16}{3}x - 6y + \frac{27}{2} = 0.$$

$$\text{Centre } (a, b) = \left(\frac{8}{3}, 3\right), r^2 = \frac{64}{9} + 9 - \frac{27}{2} = \frac{47}{18}.$$

$$ab + 18r^2 = \frac{8}{3} \times 3 + 18 \times \frac{47}{18} = 8 + 47 = 55.$$

5. (1) From $\vec{c} \times \vec{a} = \vec{a} \times \vec{d}$ where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, we get $(\vec{c} + \vec{d}) \times \vec{a} = 0$.

So $\vec{c} + \vec{d} = t\vec{a}$. Given $|\vec{c} + \vec{d}| = \sqrt{29}$: $|t| \cdot \sqrt{29} = \sqrt{29}$, so $t = \pm 1$.

$$(\vec{c} + \vec{d}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k}) = \pm(-14 + 6 + 12) = \pm 4.$$

$$\lambda_1 = 4, \lambda_2 = -4.$$

$$\text{For circle: Coeff of } x^2 = \text{Coeff of } y^2: K^2 = 3K - 2 \Rightarrow K = 1, 2.$$

$$\text{Coeff of } xy = 0: K^2 - 5K + 4 = 0 \Rightarrow K = 1, 4.$$

$$\text{Common value: } K = 1.$$

6. (3) Circle: $(x-2)^2 + (y-3)^2 = 16$, center $O = (2, 3)$, radius $r = 4$.

For tangents from external point P touching at A and B with $\angle AOB = \pi/3$:

In quadrilateral OAPB: $\angle APB = \pi - \pi/3 = 2\pi/3$, so $\angle OPA = \pi/3$.

$$\text{In right triangle OAP: } \sin(\pi/3) = \frac{4}{OP}.$$

$$OP = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}.$$

$$\text{Locus: } (x-2)^2 + (y-3)^2 = \frac{64}{3}.$$

$$3(x^2 + y^2) - 12x - 18y - 25 = 0.$$

7. (1) Circle C_1 has radius 5 and chord on line $y = x$ through origin.

Setting center at $(5, 0)$ (in region $x \geq 0$), the chord endpoints are $(0, 0)$ and $(5, 5)$ by solving $(t-5)^2 + t^2 = 25$.

Circle C_2 has this chord as diameter, so center is at $(\frac{5}{2}, \frac{5}{2})$ with radius $\frac{5\sqrt{2}}{2}$.

The chord of C_2 through $(2, 3)$ that is farthest from center is perpendicular to the radial direction from center to $(2, 3)$.

The radial direction is $(2 - \frac{5}{2}, 3 - \frac{5}{2}) = (-\frac{1}{2}, \frac{1}{2})$ with normal direction $(1, -1)$.

The chord equation is $1(x-2) - 1(y-3) = 0$, giving $x - y + 1 = 0$.

In form $x + ay + b = 0$, we have $a = -1$ and $b = 1$, so $a - b = -1 - 1 = -2$.

8. (1) Solving $\left| \frac{z-6i}{z-2i} \right| = 1 \Rightarrow y = 4 \dots (1)$

(where $z = x + iy$)

$$\text{Now solving } \left| \frac{z-8+2i}{z+2i} \right| = \frac{3}{5}$$

$$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0 \dots (2)$$

$$\text{Solving (1) \& (2) } \Rightarrow z = 17 + 4i \text{ \& } 8 + 4i$$

$$\Rightarrow \sum |z|^2 = (17)^2 + (4)^2 + (8)^2 + (4)^2 = 385$$

9. (2) Circle of radius 4 passes through O and points $A(-\sqrt{3a}, 0)$, $B(0, -\sqrt{2b})$.

Let center be $C(h, k)$ on circle: $h^2 + k^2 = 16$.

Since A, B lie on circle: $3a^2 + 2\sqrt{3a}h + h^2 + k^2 = 16$ gives $h = -\frac{\sqrt{3a}}{2}$

And $h^2 + 2b + 2\sqrt{2b}k + k^2 = 16$ gives $k = -\frac{\sqrt{2b}}{2}$

Centroid $G = \left(\frac{-\sqrt{3a}}{3}, \frac{-\sqrt{2b}}{3} \right)$. Setting $x = \frac{-\sqrt{3a}}{3}$ and $y = \frac{-\sqrt{2b}}{3}$:

$$a = 3x^2 \text{ and } b = \frac{9y^2}{2}$$

$$\text{From } h^2 + k^2 = 16: \frac{3a}{4} + \frac{2b}{4} = 16 \rightarrow 3a + 2b = 64$$

$$\text{Substituting: } 9x^2 + 9y^2 = 64 \rightarrow x^2 + y^2 = \frac{64}{9}$$

$$\text{Radius} = \frac{8}{3}$$