

Q1. JEE Main 2026 (21 January Shift 2)

If $\left(\frac{1}{{}^{15}C_0} + \frac{1}{{}^{15}C_1}\right) \left(\frac{1}{{}^{15}C_1} + \frac{1}{{}^{15}C_2}\right) \cdots \left(\frac{1}{{}^{15}C_{12}} + \frac{1}{{}^{15}C_{13}}\right) = \frac{\alpha^{13}}{{}^{14}C_0 {}^{14}C_1 \cdots {}^{14}C_{12}}$, then 30α is equal to ____.

Q2. JEE Main 2026 (23 January Shift 1)

The value of $\frac{{}^{100}C_{50}}{51} + \frac{{}^{100}C_{51}}{52} + \cdots + \frac{{}^{100}C_{100}}{101}$ is :

(1) $\frac{2^{101}}{100}$

(2) $\frac{2^{100}}{100}$

(3) $\frac{2^{101}}{101}$

(4) $\frac{2^{100}}{101}$

Q3. JEE Main 2026 (23 January Shift 1)

The sum of all possible values of $n \in \mathbb{N}$, so that the coefficients of x , x^2 and x^3 in the expansion of

$(1 + x^2)^2(1 + x)^n$, are in arithmetic progression is :

(1) 12

(2) 7

(3) 3

(4) 9

Q4. JEE Main 2026 (28 January Shift 2)

Given below are two statements :

Statement I: $25^{13} + 20^{13} + 8^{13} + 3^{13}$ is divisible by 7.

Statement II: The integral part of $(7 + 4\sqrt{3})^{25}$ is an odd number.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

Q5. JEE Main 2026 (28 January Shift 2)

The sum of the coefficients of x^{499} and x^{500} in $(1 + x)^{1000} + x(1 + x)^{999} + x^2(1 + x)^{998} + \cdots + x^{1000}$ is :

(1) ${}^{1002}C_{500}$

(2) ${}^{1002}C_{501}$

(3) ${}^{1001}C_{501}$

(4) ${}^{1000}C_{501}$

Q6. JEE Main 2026 (22 January Shift 2)

Let C_r denote the coefficient of x^r in the binomial expansion of $(1 + x)^n$, $n \in \mathbb{N}$, $0 \leq r \leq n$. If

$P_n = C_0 - C_1 + \frac{2^2}{3}C_2 - \frac{2^3}{4}C_3 + \cdots + \frac{(-2)^n}{n+1}C_n$, then the value of $\sum_{n=1}^{25} \frac{1}{P_{2n}}$ equals.

(1) 650

(2) 675

(3) 525

(4) 580

Q7. JEE Main 2026 (22 January Shift 1)

The coefficient of x^{48} in $(1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + \cdots + 100(1 + x)^{100}$ is equal to

(1) $100 \cdot {}^{100}C_{49} - {}^{100}C_{48}$

(2) ${}^{100}C_{50} + {}^{101}C_{49}$

(3) ${}^{100 \cdot 101}C_{49} - {}^{101}C_{50}$

(4) $100 \cdot {}^{100}C_{49} - {}^{100}C_{50}$

Q8. JEE Main 2026 (21 January Shift 1)

If the coefficient of x in the expansion of $(ax^2 + bx + c)(1 - 2x)^{26}$ is -56 and the coefficients of x^2 and x^3 are both zero, then $a + b + c$ is equal to :

(1) 1500

(2) 1300

(3) 1403

(4) 1483

Q9. JEE Main 2026 (24 January Shift 1)

Let $S = \frac{1}{25!} + \frac{1}{3!23!} + \frac{1}{5!21!} + \dots$ up to 13 terms. If $13S = \frac{2^k}{n!}$, $k \in \mathbb{N}$, then $n + k$ is equal to

(1) 50

(2) 49

(3) 52

(4) 51

ANSWERS AND SOLUTIONS

1. 32 2. (4) 3. (4) 4. (2) 5. (1) 6. (2) 7. (3) 8. (3)

9. (2)

1. (32) The general term of the product is $T_r = \frac{1}{{}^{15}C_{r-1}} + \frac{1}{{}^{15}C_r}$ for $r = 1, 2, \dots, 13$.

Using the formula ${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$, we have ${}^{15}C_r = \frac{15}{r} \cdot {}^{14}C_{r-1}$ and

$${}^{15}C_{r-1} = \frac{15}{15-(r-1)} \cdot {}^{14}C_{r-1} = \frac{15}{16-r} \cdot {}^{14}C_{r-1}.$$

Substituting these into T_r :

$$T_r = \frac{16-r}{15 \cdot {}^{14}C_{r-1}} + \frac{r}{15 \cdot {}^{14}C_{r-1}} = \frac{16-r+r}{15 \cdot {}^{14}C_{r-1}} = \frac{16}{15 \cdot {}^{14}C_{r-1}}.$$

$$\text{The product is } \prod_{r=1}^{13} T_r = \prod_{r=1}^{13} \frac{16}{15 \cdot {}^{14}C_{r-1}} = \frac{16^{13}}{15^{13} \cdot \prod_{r=1}^{13} {}^{14}C_{r-1}}.$$

The denominator of the product is $15^{13} \cdot ({}^{14}C_0 \cdot {}^{14}C_1 \dots {}^{14}C_{12})$.

Comparing this with the given expression $\frac{\alpha^{13}}{{}^{14}C_0 \cdot {}^{14}C_1 \dots {}^{14}C_{12}}$, we get:

$$\alpha^{13} = \frac{16^{13}}{15^{13}} \implies \alpha = \frac{16}{15}.$$

We need to find the value of 30α :

$$30\alpha = 30 \times \frac{16}{15} = 2 \times 16 = 32.$$

2. (4) Using $\frac{{}^n(r)}{r+1} = \frac{1}{n+1} {}^{(n+1)}(r+1)$:

$$\sum_{r=50}^{100} \frac{{}^{(100)}(r)}{r+1} = \frac{1}{101} \sum_{r=50}^{100} {}^{(101)}(r+1) = \frac{1}{101} \sum_{k=51}^{101} {}^{(101)}(k).$$

Since 101 is odd, by symmetry $\sum_{k=51}^{101} {}^{(101)}(k) = \sum_{k=0}^{50} {}^{(101)}(k) = \frac{2^{101}}{2} = 2^{100}$.

$$\text{Required value} = \frac{2^{100}}{101}.$$

3. (4) $(1+x^2)^2 = 1 + 2x^2 + x^4$. Multiply with $(1+x)^n$:

Coefficient of x : n .

$$\text{Coefficient of } x^2: \binom{n}{2} + 2 = \frac{n(n-1)}{2} + 2.$$

$$\text{Coefficient of } x^3: \binom{n}{3} + 2n = \frac{n(n-1)(n-2)}{6} + 2n.$$

$$\text{For AP: } 2 \left(\frac{n(n-1)}{2} + 2 \right) = n + \frac{n(n-1)(n-2)}{6} + 2n.$$

$$\text{Simplifying: } n^3 - 9n^2 + 26n - 24 = 0.$$

$$(n-2)(n-3)(n-4) = 0 \implies n = 2, 3, 4.$$

$$\text{Sum} = 2 + 3 + 4 = 9.$$

4. (2) Statement I:

$$25^{13} + 3^{13} \rightarrow \text{divisible by } (25 + 3)$$

$$20^{13} + 8^{13} \rightarrow \text{divisible by } (20 + 8)$$

$\therefore$ divisible by 7. TRUE

Statement II:

For $(7 + 4\sqrt{3})^{25}$, let $\alpha = 7 + 4\sqrt{3}$ and $\beta = 7 - 4\sqrt{3}$ (conjugate).

Then $\alpha\beta = 1$ and $\alpha + \beta = 14$.

Since $0 < \beta < 1$, we have $\alpha^{25} + \beta^{25}$ is an integer.

The fractional part of α^{25} is $1 - \beta^{25}$, making the integral part odd. TRUE.

Both Statement I and Statement II are true.

5. (1) $S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \dots + x^{1000}$

This is a geometric series with first term $(1+x)^{1000}$, common ratio $\frac{x}{1+x}$, and 1001 terms.

$$S = (1+x)^{1000} \cdot \frac{1 - \left(\frac{x}{1+x}\right)^{1001}}{1 - \frac{x}{1+x}}$$

$$= (1+x)^{1001} - x^{1001}$$

Required sum = coefficient of x^{499} + coefficient of x^{500} in $(1+x)^{1001} - x^{1001}$

$$= {}^{1001}C_{499} + {}^{1001}C_{500} = {}^{1002}C_{500}$$

6. (2) Using the identity $\frac{1}{r+1} \binom{n}{r} = \frac{1}{n+1} \binom{n+1}{r+1}$,

$$\text{we get } P_n = \sum_{r=0}^n \frac{(-2)^r}{r+1} C_r = \frac{1}{n+1} \sum_{s=1}^{n+1} (-2)^s \binom{n+1}{s}.$$

$$\text{Since } (1-2)^{n+1} = (-1)^{n+1} = \sum_{s=0}^{n+1} (-2)^s \binom{n+1}{s},$$

$$\text{we have } \sum_{s=1}^{n+1} (-2)^s \binom{n+1}{s} = (-1)^{n+1} - 1.$$

$$\text{For even } n = 2k: P_{2k} = \frac{1 - (-1)}{2(2k+1)} = \frac{1}{2k+1}.$$

$$\text{Thus } \frac{1}{P_{2n}} = 2n+1, \text{ and } \sum_{n=1}^{25} (2n+1) = 2 \cdot \frac{25 \cdot 26}{2} + 25 = 675.$$

7. (3) Let $1+x=r$

$$\therefore S = 1 \cdot r + 2 \cdot r^2 + 3 \cdot r^3 + \dots + 100r^{100} \dots (1) \text{ (AP)}$$

$$rS = 1 \cdot r^2 + 2 \cdot r^3 + \dots + 99r^{100} + 100r^{101} \dots (2)$$

(1) - (2) gives

$$S = -\frac{(1+x)^{101}}{x^2} + \frac{1}{x^2} + \frac{100(1+x)^{101}}{x}$$

$\therefore$ coefficient x^{48} in S

$$= -\text{coefficient of } x^{48} \text{ in } \frac{(1+x)^{101}}{x^2} + 100 \cdot \text{Coefficient of } x^{48} \text{ in } \frac{(1+x)^{101}}{x}$$

$$= 100 \cdot {}^{101}C_{49} - {}^{101}C_{50}$$

8. (3) In $(1-2x)^{26}$: $T_0 = 1, T_1 = -52, T_2 = 1300, T_3 = -20800$.

$$\text{Coefficient of } x: b - 52c = -56 \dots (1)$$

$$\text{Coefficient of } x^2: a - 52b + 1300c = 0 \dots (2)$$

$$\text{Coefficient of } x^3: -52a + 1300b - 20800c = 0 \dots (3)$$

$$\text{From (1): } b = 52c - 56. \text{ Substituting in (2): } a = 1404c - 2912.$$

$$\text{Substituting in (3): } -26208c + 78624 = 0 \Rightarrow c = 3.$$

$$\text{Thus } b = 100, a = 1300.$$

$$a + b + c = 1300 + 100 + 3 = 1403.$$

9. (2) The general term is $\frac{1}{(2r-1)!(27-2r)!}$ for $r = 1$ to 13.

Multiply and divide by $26!$: $\frac{1}{(2r-1)!(27-2r)!} = \frac{1}{26!} \binom{26}{2r-1}$

$$S = \frac{1}{26!} \sum_{r=1}^{13} \binom{26}{2r-1} = \frac{1}{26!} \left[\binom{26}{1} + \binom{26}{3} + \cdots + \binom{26}{25} \right]$$

Sum of odd-indexed binomial coefficients: $\sum \binom{26}{\text{odd}} = 2^{25}$

$$S = \frac{2^{25}}{26!}$$

$$13S = \frac{13 \cdot 2^{25}}{26!} = \frac{13 \cdot 2^{25}}{26 \cdot 25!} = \frac{2^{25}}{2 \cdot 25!} = \frac{2^{24}}{25!}$$

So $k = 24$, $n = 25$, giving $n + k = 49$.