

Q1. JEE Main 2026 (21 January Shift 2)

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$ and $f'(a-1) = 0$, where a is a real number. Let $g(x) = f(\tan^2 x - 2 \tan x + a)$, $0 < x < \frac{\pi}{2}$.

Consider the following two statements:

(I) g is increasing in $(0, \frac{\pi}{4})$

(II) g is decreasing in $(\frac{\pi}{4}, \frac{\pi}{2})$. Then,

(1) Neither (I) nor (II) is True

(2) Only (I) is True

(3) Both (I) and (II) are True

(4) Only (II) is True

Q2. JEE Main 2026 (28 January Shift 2)

Let f be a differentiable function satisfying $f(x) = 1 - 2x + \int_0^x e^{(x-t)} f(t) dt$, $x \in \mathbb{R}$ and let $g(x) = \int_0^x (f(t) + 2)^{15} (t-4)^6 (t+12)^{17} dt$, $x \in \mathbb{R}$. If p and q are respectively the points of local minima and local maxima of g , then the value of $|p+q|$ is equal to ____.

Q3. JEE Main 2026 (24 January Shift 2)

Consider the following three statements for the function $f : (0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = |\log_e x| - |x-1|$:

(I) f is differentiable at all $x > 0$.

(II) f is increasing in $(0, 1)$.

(III) f is decreasing in $(1, \infty)$.

Then,

(1) All (I), (II) and (III) are TRUE.

(2) Only (II) and (III) are TRUE.

(3) Only (I) is TRUE.

(4) Only (I) and (III) are TRUE.

Q4. JEE Main 2026 (22 January Shift 1)

Let $f(x) = x^{2025} - x^{2000}$, $x \in [0, 1]$ and the minimum value of the function $f(x)$ in the interval $[0, 1]$ be

$(80)^{80}(n)^{-81}$. Then n is equal to

(1) -40

(2) -41

(3) -80

(4) -81

Q5. JEE Main 2026 (21 January Shift 1)

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that the quadratic equation $f(x)m^2 - 2f'(x)m + f''(x) = 0$ in m , has two equal roots for every $x \in \mathbb{R}$. If $f(0) = 1$, $f'(0) = 2$, and (α, β) is the largest interval in which the function $f(\log_e x - x)$ is increasing, then $\alpha + \beta$ is equal to ____.

Q6. JEE Main 2026 (24 January Shift 1)

Let $(2\alpha, \alpha)$ be the largest interval in which the function $f(t) = \frac{|t+1|}{t^2}$, $t < 0$, is strictly decreasing. Then the local maximum value of the function $g(x) = 2 \log_e(x-2) + \alpha x^2 + 4x - \alpha$, $x > 2$, is _____

ANSWERS AND SOLUTIONS

1. (1) 2. 9 3. (4) 4. (4) 5. 1 6. 4

1. (1) Given $f''(x) > 0$ for all $x \in \mathbb{R}$, which means $f'(x)$ is a strictly increasing function.

We are given $f'(a-1) = 0$. Since $f'(x)$ is strictly increasing, $f'(x) < 0$ for $x < a-1$ and $f'(x) > 0$ for $x > a-1$.

The function $g(x)$ is defined as $g(x) = f(\tan^2 x - 2 \tan x + a)$ for $x \in (0, \pi/2)$.

Let $u(x) = \tan^2 x - 2 \tan x + a = (\tan x - 1)^2 + a - 1$.

Differentiating $g(x)$ with respect to x :

$$g'(x) = f'(u(x)) \cdot u'(x) = f'((\tan x - 1)^2 + a - 1) \cdot (2 \tan x \sec^2 x - 2 \sec^2 x)$$

$$g'(x) = f'((\tan x - 1)^2 + a - 1) \cdot 2 \sec^2 x (\tan x - 1).$$

Case 1: $x \in (0, \pi/4)$.

In this interval, $0 < \tan x < 1$, so $(\tan x - 1) < 0$.

Also, $(\tan x - 1)^2 > 0$, so $u(x) = (\tan x - 1)^2 + a - 1 > a - 1$.

Since $f'(x)$ is increasing and $f'(a-1) = 0$, for $u(x) > a-1$, we have $f'(u(x)) > 0$.

Thus, $g'(x) = (+)(+)(-) < 0$. So $g(x)$ is decreasing in $(0, \pi/4)$.

Statement (I) is False.

Case 2: $x \in (\pi/4, \pi/2)$.

In this interval, $\tan x > 1$, so $(\tan x - 1) > 0$.

Also, $(\tan x - 1)^2 > 0$, so $u(x) = (\tan x - 1)^2 + a - 1 > a - 1$.

Thus, $f'(u(x)) > 0$.

Then $g'(x) = (+)(+)(+) > 0$. So $g(x)$ is increasing in $(\pi/4, \pi/2)$.

Statement (II) is False.

Therefore, neither (I) nor (II) is true.

2. (9) Given $f(x) = 1 - 2x + \int_0^x e^{(x-t)} f(t) dt$.

Differentiating and simplifying:

$$e^{-x} f'(x) - e^{-x} f(x) = -2e^{-x} + (1 - 2x)e^{-x}(-1) + e^{-x} f(x)$$

This reduces to:

$$f'(x) - 2f(x) = 2x - 3$$

Solving the linear ODE $\frac{dy}{dx} - 2y = 2x - 3$:

$$y \cdot e^{-2x} = \int e^{-2x}(2x - 3) dx$$

On solving, we get $f(x) = 1 - x$.

$$\text{Now, } g(x) = \int_0^x (3 - t)^{15} (t - 4)^6 (t + 12)^{17} dt$$

$$g'(x) = (3-x)^{15}(x-4)^6(x+12)^{17}$$

$$= -(x-3)^{15}(x-4)^6(x+12)^{17}$$

Sign analysis of $g'(x)$:

$g'(x)$ changes from $+$ $\rightarrow$ $-$ at $x = 3 \rightarrow$ local maxima ($q = 3$)

$g'(x)$ changes from $-$ $\rightarrow$ $+$ at $x = -12 \rightarrow$ local minima ($p = -12$)

$$|p+q| = |-12+3| = 9$$

3. (4) For $0 < x < 1$: $f(x) = -\ln x + x - 1$, $f'(x) = -\frac{1}{x} + 1 = \frac{x-1}{x} < 0$ (decreasing).

For $x > 1$: $f(x) = \ln x - x + 1$, $f'(x) = \frac{1}{x} - 1 = \frac{1-x}{x} < 0$ (decreasing).

At $x = 1$: LHD = $\frac{1-1}{1} = 0$, RHD = $\frac{1-1}{1} = 0$. So f is differentiable at $x = 1$.

(I) TRUE: f is differentiable for all $x > 0$.

(II) FALSE: f is decreasing in $(0, 1)$.

(III) TRUE: f is decreasing in $(1, \infty)$.

Only (I) and (III) are TRUE.

4. (4) $f(x) = x^{2025} - x^{2000}$, $f'(x) = x^{1999}(2025x^{25} - 2000) = 0$.

Critical point: $x^{25} = \frac{2000}{2025} = \frac{80}{81}$, i.e., $x = \left(\frac{80}{81}\right)^{1/25}$.

$f(0) = f(1) = 0$. At the critical point:

$$f = x^{2000}(x^{25} - 1) = \left(\frac{80}{81}\right)^{80} \left(\frac{80}{81} - 1\right) = -\frac{80^{80}}{81^{81}}.$$

This equals $(80)^{80}(n)^{-81}$, so $n^{-81} = -81^{-81} \Rightarrow n = -81$.

5. (1) For equal roots in $f(x)m^2 - 2f'(x)m + f''(x) = 0$, discriminant = 0:

$$[f'(x)]^2 = f(x)f''(x)$$

$$\text{This gives } \frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$$

Integrating: $f'(x) = kf(x)$, so $f(x) = Ae^{kx}$

Using $f(0) = 1$ and $f'(0) = 2$: $A = 1$, $k = 2$

$$f(x) = e^{2x}$$

For $g(x) = f(\ln x - x)$:

$$g'(x) = 2e^{2(\ln x - x)} \cdot \left(\frac{1}{x} - 1\right)$$

$g'(x) > 0$ when $\frac{1-x}{x} > 0$, i.e., $0 < x < 1$

$$(\alpha, \beta) = (0, 1)$$

$$\alpha + \beta = 1$$

6. (4) For $t < 0$, analyze $f(t) = \frac{|t+1|}{t^2}$ in two regions.

For $-1 < t < 0$:

$$f(t) = \frac{t+1}{t^2}, \text{ so } f'(t) = \frac{-(t+2)}{t^3} > 0 \text{ (increasing).}$$

For $t < -1$:

$$f(t) = \frac{-t-1}{t^2}, \text{ so } f'(t) = \frac{t+2}{t^3}.$$

For $-2 < t < -1$:

$f'(t) < 0$ (decreasing).

For $t < -2$:

$f'(t) > 0$ (increasing).

Largest decreasing interval is $(-2, -1) = (2\alpha, \alpha)$, so $\alpha = -1$.

For $g(x) = 2 \ln(x - 2) - x^2 + 4x + 1$:

$g'(x) = \frac{2}{x-2} - 2x + 4 = 0$ gives $(x - 2)^2 = 1$, so $x = 3$.

Since $g''(x) = -\frac{2}{(x-2)^2} - 2 < 0$, $x = 3$ is maximum.

$g(3) = 2 \ln(1) - 9 + 12 + 1 = 4$