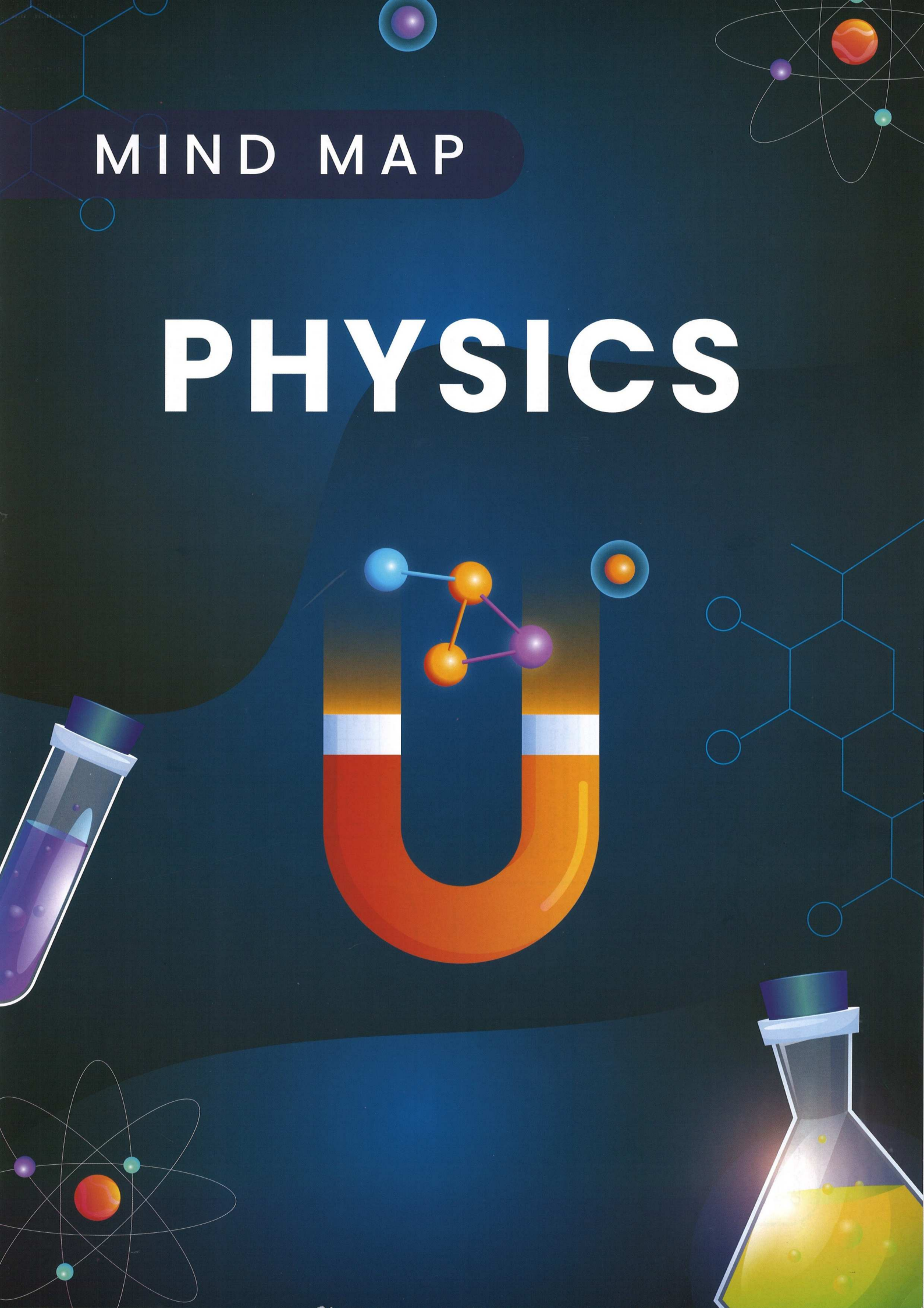


MIND MAP

PHYSICS





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UNITS & MEASUREMENTS

MEASUREMENT OF MASS & TIME

MASS

• Unified atomic mass unit (amu) is used to measure mass of atoms & molecules

- $1\text{amu} = (1/12)^{\text{th}}$ mass of one C^{12} atom
- $1\text{amu} = 1.66 \times 10^{-27} \text{ kg}$
- Electron mass- 10^{-30} kg
- Earth mass : 10^{25} kg
- Observable Universe 10^{55} kg

TIME

- SI unit is second (based on caesium clock with an uncertainty less than 1 part in 10^{-13} ie, 3µs loss every year)
- Timespan of unstable particle: 10^{-24} s
- Age of universe: 10^{17} s

MEASUREMENT OF LENGTH

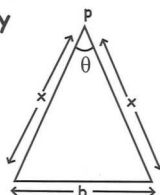
• Large distance is measured by parallax method

$$\text{Parallax angle} = \frac{\text{BASIS}}{\text{DISTANCE}} = \frac{b}{x}$$

- $1^\circ = 1.745 \times 10^{-2} \text{ rad}$
- $1' = 2.91 \times 10^{-4} \text{ rad}$
- $1'' = 4.85 \times 10^{-6} \text{ rad}$

• For very small sizes, optical microscope, tunneling microscope, electron microscope are used.

- $1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$
- $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$
- $1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$
- Size of proton: 10^{-15} m
- Radius Of Earth: 10^7 m
- Distance to Boundary Of Observable Universe: 10^{26} m

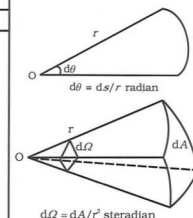


SI SYSTEM

7 Base units and 2 supplementary units

NO.	Base Units		
	Quantity	Unit	Symbol
1	Length	meter	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	K
5	Electric current	ampere	A
6	Luminous intensity	candela	cd
7	Amount of substance	mole	mol

NO.	Supplementary Units		
	Quantity	Unit	Symbol
1	Plane angle	radian	rad
2	Solid angle	steradian	sr



What is the unit of permittivity of free space ϵ_0 ?

- colomb/newton-metre
- newton-metre² /colomb²
- colomb²/newton-metre²
- colomb²/(newton-metre)²

SIGNIFICANT FIGURES

The digits in a measured quantity which are reliable and confidence in our measurement + the digit which is uncertain.

RULES FOR SIGNIFICANT FIGURES

- All non-zero digits are significant. For example, 42.3 has three significant figures; 243.4 has four significant figures; and 24.123 has five significant figures.
- A zero becomes significant figure if it appears between two non-zero digits. For example, 5.03 has three significant figures; 5.604 has four significant figures; and 4.004 has four significant figures.
- Leading zeros or the zeros placed to the left of the number are never significant. For example, 0.543 has three significant figures; 0.045 has two significant figures; and 0.006 has one significant figure.
- Trailing zeros or the zeros placed to the right of the number are significant. For example, 4.330 has four significant figures; 433.00 has five significant figures; and 343.000 has six significant figures.
- In exponential notation, the numerical portion gives the number of significant figures. For example, 1.32×10^{-2} has three significant figures and 1.32×10^4 has three significant figures.

RULES FOR ROUNDING OF A MEASUREMENT

- If the digit to be dropped is less than 5, then the preceding digit is left unchanged. For example, $x = 7.82$ is rounded off to 7.8 and $x = 3.94$ is rounded off to 3.9.
- If the digit to be dropped is more than 5, then the preceding digit is raised by one. For example, $x = 6.87$ is rounded off to 6.9 and $x = 12.78$ is rounded off to 12.8.
- If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one. For example, $x = 16.351$ is rounded off to 16.4 and $x = 6.758$ is rounded off to 6.8.
- If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is even, is left unchanged. For example, $x = 3.250$ becomes 3.2 on rounding off and $x = 12.650$ becomes 12.6 on rounding off.
- If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is odd, is raised by one. For example, $x = 3.750$ is rounded off to 3.8, again $x = 16.150$ is rounded off to 16.2.

RULES FOR ROUNDING OF A MEASUREMENT

ADDITION & SUBTRACTION

In addition or subtraction, the final result should be reported to the same number of decimal places as that of the original number with minimum number of decimal places

$$\begin{array}{r} 3.1421 \\ 0.241 \\ +0.09 \end{array} \leftarrow \begin{array}{l} \text{(has two decimal places)} \\ \text{(Answer should be reported to two decimal} \\ \text{places after rounding off)} \end{array}$$

Answer = 3.47

MULTIPLICATION & DIVISION

When numbers are multiplied or divided, the number of significant figures in the answer equals the smallest number of significant figures in any of the original numbers

$$\begin{array}{r} 51.028 \\ \times 1.31 \end{array} \leftarrow \begin{array}{l} \text{(Three significant figures)} \\ \text{(Answer should have three significant figures} \\ \text{after rounding off)} \end{array}$$

Answer = 66.8

If $L = 2.331 \text{ cm}$, $B = 2.1 \text{ cm}$, then $L+B = ?$

- 4.431 cm
- 4.43 cm
- 4.4 cm
- 4 cm

Dimensional Analysis

Dimensions of a physical quantity are the powers to which units of base quantity are raised. Eg: $[M]^a [L]^b [T]^c [A]^d [K]^e$

APPLICATIONS

checking the correctness of various formulae
Eg: If $Z=A+B$, $[Z]=[A]=[B]$

conversion of one system of unit into another

$$n_1 u_1 = n_2 u_2$$

$$\text{Eg: } n_1 [M_1^A L_1^B T_1^C] = n_2 [M_2^A L_2^B T_2^C]$$

$$n_1 = n_2 \left[\frac{M_2}{M_1} \right]^A \left[\frac{L_2}{L_1} \right]^B \left[\frac{T_2}{T_1} \right]^C$$

Deducing relation among physical quantity

DIMENSIONAL FORMULA

- 1) Pressure=stress=Young's modulus= $ML^{-1} T^{-2}$
- 2) Work=Energy=Torque= $M L^2 T^{-2}$
- 3) Power $P=M L^2 T^{-3}$
- 4) Gravitational constant $G=M^{-1} L^3 T^{-2}$
- 5) Force constant=Spring constant= $M T^{-2}$
- 6) Coefficient of viscosity= $M L^{-1} T^{-1}$
- 7) Latent heat = $L^2 T^{-2}$
- 8) Electric potential= $\frac{P}{I} = M L^2 T^{-3} A^{-1}$
- 9) Resistance= $\sqrt{\frac{\mu_0}{\epsilon_0}} = M L^2 T^{-3} A^{-2}$
- 10) Capacitance= $M^{-1} L^{-2} T^4 A^2$
- 11) Permittivity $\epsilon_0=M^{-1} L^{-3} T^4 A^2$
- 12) Angular momentum = planck's constant = $M^1 L^2 T^{-1}$
- 13) $M = k \sqrt{\frac{hc}{G}}$ $L = k \sqrt{\frac{hG}{c^2}}$ $T = k \sqrt{\frac{hG}{c^3}}$

$$\text{Time period } \frac{T}{R} = RC = \sqrt{LC}$$

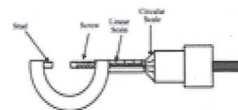
DIMENSIONLESS QUANTITIES

- 1) Strain
- 2) Refractive index
- 3) Relative density
- 4) Plane angle
- 5) Solid angle

In SI Units, the dimensions of $\sqrt{\frac{G}{\mu_0}}$ is:

- a) $A^{-1} T M L^3$
- b) $A T^2 M^{-1} L^{-1}$
- c) $A T^{-3} M L^{3/2}$
- d) $A^2 T^3 M^{-1} L^{-2}$

SCREW GAUGE



$$\text{Pitch} = \frac{\text{Main Scale Reading}}{\text{No. of rotations}}$$

$$\text{Least Count} = \frac{\text{pitch}}{\text{Total no. of divisions on circular scale}}$$

$$\text{Total Reading} = \text{Linear Scale Reading} + \text{circular scale reading} \times \text{least count}$$

The least count of the main scale of a screw gauge is 1mm. The minimum no. of divisions on its circular scale required to measure 5μm diameter of wire is;

- a) 200
- b) 50
- c) 400
- d) 100

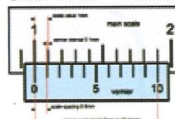
INSTRUMENTS

Least Count:
Smallest quantity an instrument can measure

mm scale	vernier scale	screw gauge
1mm	0.1mm	0.01mm

VERNIER CALIPERS

Least Count = 1 MSD - 1VSD



If n VSD Coincides with (n-1) MSD,
then (n-1) MSD = n VSD
∴ 1VSD = $\frac{n-1}{n}$ MSD

$$\text{Least Count} = 1\text{MSD} - \frac{n-1}{n}\text{MSD} = \frac{1\text{MSD}}{n}$$

$$\text{Total Reading} = \text{Main Scale Reading} + (\text{Coinciding Vernier Scale division} \times \text{least count})$$

In a vernier calipers, one main scale division is x cm & n division of vernier scale coincide with n-1 divisions of the main scale. The least count (in cm) of the calipers is;

- a) $\left(\frac{n-1}{n}\right)x$
- b) $\frac{nx}{(n-1)}$
- c) $\frac{x}{(n-1)}$
- d) $\frac{x}{n}$

COMBINATION OF ERRORS

Operations	Formula Z	Absolute error ΔZ	Relative error ΔZ/Z	Percentage error 100 × ΔZ/Z
Sum	A+B	ΔA+ΔB	$\frac{\Delta A + \Delta B}{A+B}$	$\frac{\Delta A + \Delta B}{A+B} \times 100$
Difference	A-B	ΔA+ΔB	$\frac{\Delta A + \Delta B}{A-B}$	$\frac{\Delta A + \Delta B}{A-B} \times 100$
Multiplication	A×B	AΔB+ BΔA	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A + A\Delta B}{B^2}$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Power	A ⁿ	n A ⁿ⁻¹ ΔA	$n \frac{\Delta A}{A}$	$n \frac{\Delta A}{A} \times 100$
Root	A ^{1/n}	$\frac{1}{n} A^{\frac{1}{n}-1} \Delta A$	$\frac{1}{n} \frac{\Delta A}{A}$	$\frac{1}{n} \frac{\Delta A}{A} \times 100$

General rule:

If $Z = \frac{A^p B^q}{C^r}$, Then the maximum fractional relative error in Z will be:

$$\frac{\Delta Z}{Z} = p \frac{\Delta A}{A} + q \frac{\Delta B}{B} + r \frac{\Delta C}{C}$$

ERRORS IN MEASUREMENT

Difference between true value & measured value of a quantity

Systematic Errors

Errors which tend to occur only in one direction, either positive or negative

Random Errors

Irregular and random in magnitude & direction

Instrumental

Due to inbuilt defect of measuring instrument

Experimental

Limitations in experimental technique

Personal

Due to individual bias, Lack of proper setting of apparatus

• Least count error is the smallest value that can be measured by instrument (occurs with random & systematic errors)

• Absolute Error :- $\Delta a = a_1 - a_{\text{mean}}$, $a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$

• Relative Error :- $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots + |\Delta a_n|}{n}$

• Percentage Error :- $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100$

In an experiment four quantities a,b,c and d are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity P is calculated as shown below. What is the percentage error in P?

$$P = \frac{a^2 b^2}{cd}$$

(a) 14%

(b) 10%

(c) 7%

(d) 4%

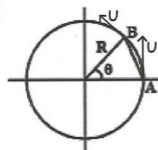
UNITS & MEASUREMENTS

- Distance = Length of actual path
- Displacement = Length of shortest path
- Distance $\geq$ |displacement|



A particle moves from A to B in a circular path of radius R covering an angle θ with uniform speed U

- Distance = $\widehat{AB} = R\theta$ • Displacement = $AB = 2R\sin(\frac{\theta}{2})$
- Ratio of Displacement to Distance = $\frac{\sin(\frac{\theta}{2})}{\frac{\theta}{2}}$
- Time $t = \frac{R\theta}{U}$
- Average Velocity = $\frac{2U\sin(\frac{\theta}{2})}{\frac{\theta}{2}}$
- Average Acceleration = $\frac{U^2\sin(\frac{\theta}{2})}{R\frac{\theta}{2}}$



- For uniform motion

Displacement = velocity $\times$ time

Average speed = |average velocity| = |instantaneous velocity|

- Time average speed

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{S_1 + S_2 + S_3 + \dots + S_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

If $t_1 = t_2 = t_3 = \dots = t_n$

$$\text{then } v_{av} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n}$$

$$\text{for } v_1 \text{ \& } v_2, \quad v_{av} = \frac{v_1 + v_2}{2} \quad (\text{Arithmetic mean of speeds})$$

- Distance average speed

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{S_1 + S_2 + S_3 + \dots + S_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{\frac{S_1}{v_1} + \frac{S_2}{v_2} + \frac{S_3}{v_3} + \dots + \frac{S_n}{v_n}}{\frac{S_1}{v_1} + \frac{S_2}{v_2} + \frac{S_3}{v_3} + \dots + \frac{S_n}{v_n}}$$

If $S_1 = S_2 = S_3 = \dots = S_n$

$$\text{then } v_{av} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \dots + \frac{1}{v_n}} \quad \text{for } v_1 \text{ \& } v_2, \quad v_{av} = \frac{2v_1 v_2}{v_1 + v_2} \quad (\text{Harmonic mean of speeds})$$

$$\text{Instantaneous Velocity } v = \frac{dx}{dt} \quad \Delta x = v \Delta t$$

$$\text{Instantaneous Acceleration } a = \frac{dv}{dt} \quad \Delta v = a \Delta t$$

Case 1

$v = f(t)$ or $x = f(t)$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Case 2

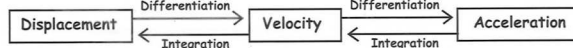
$V = f(x)$

$$a = v \frac{dv}{dx}$$

Case 3

$t = f(x)$

$$\text{then } a = -(\text{double diff. of } t \text{ w.r.t. } x) \times v^3$$



Motion with constant acceleration: Equations of motion

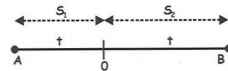
$$(i) \quad v = u + at$$

$$(ii) \quad s = ut + \frac{1}{2} at^2$$

- A Person travels from A to B covers unequal distances in equal interval of time with constant acceleration a then

$$\text{initial velocity } U = \frac{3S_1 - S_2}{2t}$$

$$\text{Acceleration } a = \frac{S_2 - S_1}{t^2}$$



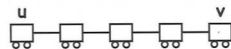
$$(iii) \quad v^2 = u^2 + 2as$$

- The number of planks required to stop the bullet

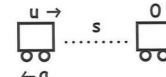
$$N = \frac{u^2}{u^2 - v^2}$$

- The two ends of a train moving with constant acceleration pass a certain point with velocities u and v. The velocity with which the middle point of the train passes the same point is

$$v_{mid} = \sqrt{\frac{u^2 + v^2}{2}}$$



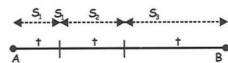
- Calculation of stopping distance $s = \frac{u^2}{2a}$



$$(iv) \quad s_n = u + \frac{a}{2} (2n-1)$$

- Ratio of distance travelled in equal interval of time in a uniformly accelerated motion from rest

$$S_1 : S_2 : S_3 = 1 : 3 : 5$$



- for uniform accelerated motion

$$v_{avg} = \frac{u+v}{2}$$

Different Cases	v-t graph	s-t graph
1. Uniform motion		
2. Uniformly accelerated motion with u = 0 at t = 0		
3. Uniformly accelerated with u ≠ 0 at t = 0 & s = 0 at t = 0		
4. Uniformly accelerated motion with u = 0 and s = s_0 at t = 0		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

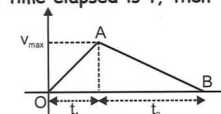
Important points about graphical analysis of motion

- Instantaneous velocity is the slope of position-time curve $\left[v = \frac{dx}{dt} \right]$
- Area of v-t curve gives displacement $\left[\Delta x = \int v dt \right]$
- Slope of velocity-time curve = instantaneous acceleration $\left[a = \frac{dv}{dt} \right]$
- Area of a-t curve gives change in velocity $\left[\Delta v = \int a dt \right]$

A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β , to come to rest. If the total time elapsed is t, then

$$v_{max} = \frac{\alpha\beta}{\alpha+\beta} t$$

$$\text{Total Distance} = \frac{1}{2} \left(\frac{\alpha\beta}{\alpha+\beta} \right) t^2$$



MOTION UNDER GRAVITY

Sign Convention

- (i) initial velocity

+ve = upward motion
-ve = downward motion

- (ii) Acceleration

Always -ve

- (iii) Displacement

+ve = final position is above initial position
-ve = final position is below initial position
Zero = final position & initial position are at same level

- Object is dropped from top of a tower

- (i) Ratio of displacement in equal interval of time $S_1 : S_2 : S_3 : \dots = 1 : 3 : 5 : \dots$

- (ii) Ratio of time of covering equal distance

$$t_1 : (t_2 - t_1) : (t_3 - t_2) : \dots : (t_n - t_{n-1}) = 1 : (\sqrt{2} - \sqrt{1}) : (\sqrt{3} - \sqrt{2}) : \dots : (\sqrt{n} - \sqrt{n-1})$$

- (iii) Ratio of total distance covered at the end of time $t : 2t : 3t : \dots = 1^2 : 2^2 : 3^2 : \dots$

- If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then

- (i) Maximum height attained $H = \frac{u^2}{2g}$

- (ii) Time of ascent = time of descent $\frac{u}{g}$

- (iii) Total time of flight = $\frac{2u}{g}$

- (iv) Velocity of fall at the point of projection = u (downwards)

- At any point on its path the body will have same speed for upward journey and downward journey. If a body thrown upwards crosses a point in time t_1 & t_2 respectively then

$$\text{height of point } h = \frac{1}{2} g t_1 t_2 \quad \text{Maximum height } H = \frac{1}{8} g (t_1 + t_2)^2$$

$$\text{Time of flight} = t_1 + t_2 = \frac{2u}{g}$$

- A body is thrown upward, downward & horizontally with same speed takes time t_1 , t_2 & t_3 respectively to reach the ground then

$$t_3 = \sqrt{t_1 t_2} \quad \text{height from where the particle was throw is } h = \frac{1}{2} g t_1 t_2$$

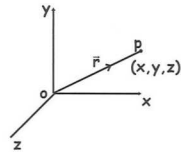
MOTION ALONG A STRAIGHT LINE

VECTORS

Position Vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

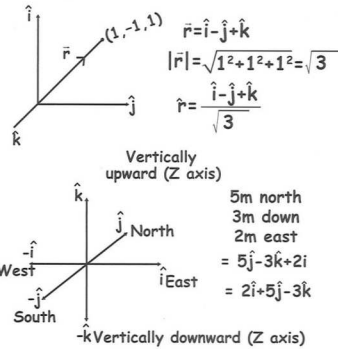
$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{r}| \text{ or } r = \sqrt{x^2 + y^2 + z^2}$$

$$\text{unit vector} = \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$



Displacement Vector

Particle displaces from position P_1 to position P_2

$$\Delta \vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\Delta \vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Parallel Vectors

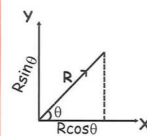
$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{a} = m\vec{b}$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = m$$

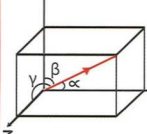
Components of Vector



In two dimensions

$$\vec{R} = R \cos \theta \hat{i} + R \sin \theta \hat{j}$$

In three dimensions



$$\vec{R} = R \cos \alpha \hat{i} + R \cos \beta \hat{j} + R \cos \gamma \hat{k}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

Addition of Vectors

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$R_{\max} = A + B$$

$$R_{\min} = |A - B|$$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

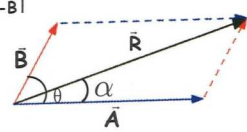
Vector product

$$\vec{C} = \vec{A} \times \vec{B} \quad |\vec{A} \times \vec{B}| = AB \sin \theta$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}[A_y B_z - B_y A_z] - \hat{j}[A_x B_z - A_z B_x] + \hat{k}[A_x B_y - A_y B_x]$$

Dot product

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$



PROJECTILE MOTION

Projectile motion

Horizontal component = $U \cos \theta$

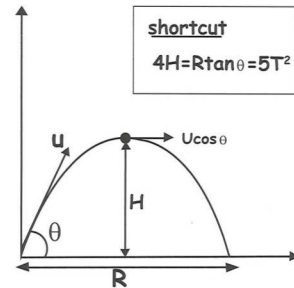
Vertical component = $U \sin \theta$

$$H = \frac{U^2 \sin^2 \theta}{2g} = \frac{(U \sin \theta)^2}{2g} = \frac{U_y^2}{2g}$$

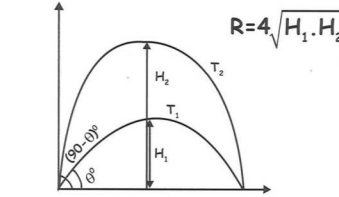
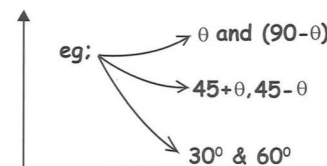
$$R = \frac{U^2 \sin 2\theta}{g} = \frac{2U \sin \theta U \cos \theta}{g} = \frac{2U_x U_y}{g}$$

shortcut

$$4H = R \tan \theta = 5T^2$$



Same range for θ and $(90 - \theta)$



$$\frac{T_1}{T_2} = \tan \theta$$

$$\frac{H_1}{H_2} = \tan^2 \theta$$

$$T_1 \times T_2 = \frac{2R}{g}$$

$$H_1 \times H_2 = \frac{R^2}{16}$$

$$T_1^2 + T_2^2 = \frac{4R_{\max}}{g}$$

$$H_1 + H_2 = \frac{R_{\max}}{2}$$

Maximum range

For $\theta = 45^\circ$

$$R_{\max} = \frac{U^2}{g}$$

From the relation,
 $4H = R \tan \theta = 5T^2$

$$4H = R_{\max} \tan 45^\circ \Rightarrow H = \frac{R_{\max}}{4}$$

$$4H = R_{\max} \Rightarrow H = \frac{U^2}{4g}$$

KE at maximum height = $K \cos^2 \theta$

Momentum

$$P_i = m(u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j})$$

$$v = [u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}]$$

$$P_i = m(u \cos \theta \hat{i} + (u \sin \theta) \hat{j})$$

$$\Delta p = -mgt \hat{j}$$

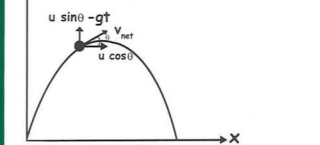
$$= -mg \times \frac{u \sin \theta}{g} \hat{j}$$

$$= -mu \sin \theta \hat{j}$$

Equation of Velocity

$$V_{\text{net}} = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{(u \cos \theta)^2 + (u \sin \theta - gt)^2}$$



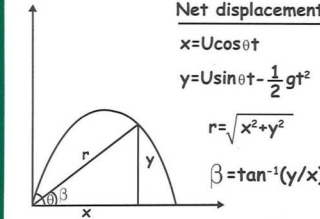
Net displacement

$$x = U \cos \theta t$$

$$y = U \sin \theta t - \frac{1}{2}gt^2$$

$$r = \sqrt{x^2 + y^2}$$

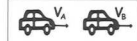
$$\beta = \tan^{-1}(y/x)$$



MOTION IN A PLANE - 01

Relative Motion

1) Velocity of A with respect to B $V_{A/B} = V_A - V_B$

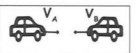


$$3) V_{A/Tree} = V_A - V_{Tree} = 60 - 0 = 60$$

$$V_{B/Tree} = V_B - V_{Tree} = -40$$

2) $V_{A/B} = V_A - V_B$

$$= V_A - (-V_B) = V_A + V_B$$



$$V_A = 60 \text{ km/hr}$$

$$V_B = 40 \text{ km/hr}$$

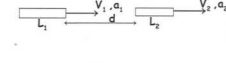
RELATIVE MOTION

Relative Motion in one dimension (overtaking & chasing)

$$1) t = \frac{d + L_1 + L_2}{V_1 + V_2}$$

$$2) t = \frac{d + L_1 + L_2}{V_1 - V_2}$$

$$3) d + L_1 + L_2 = (u_1 - u_2)t + \frac{1}{2}(a_1 - a_2)t^2$$



Minimum separation to avoid collision

$$u_{\text{rel}} = u_1 - u_2$$

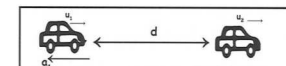
$$a_{\text{rel}} = -a_1$$

$$0 = u_{\text{rel}}^2 + 2a_{\text{rel}}s_{\text{rel}}$$

$$0 = (u_1 - u_2)^2 - 2a_1s$$

to avoid collision,
 $s \leq d$

$$\Rightarrow d \geq \frac{(u_1 - u_2)^2}{2a_1}$$



a_1 = retardation of car 1

RELATIVE MOTION

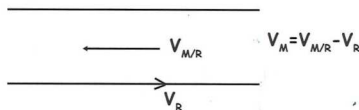
Man-river problem

1) V_{MR} or $V_{M/Still\ water}$ = velocity due to effort of man, OR velocity of man in still water

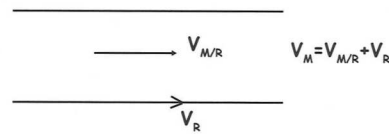
2) V_R = velocity of River

3) V_m = Resultant velocity of man with respect to ground

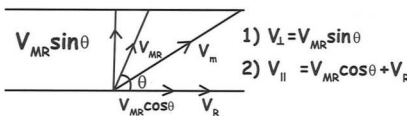
1) Upstream



2) Down stream



Swimming across the river



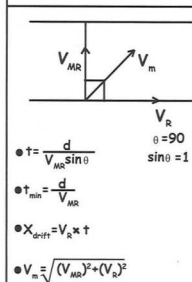
$$t_{cross} = \frac{d}{V_{MR} \cos \theta} = \frac{d}{V_{\perp}}$$

$$X_{drift} = (V_{MR} \sin \theta + V_R) \times t$$

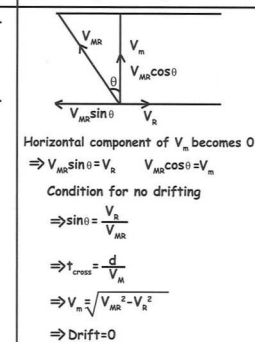
$= V_{MR} \sin \theta \times t + V_R t$

Due to effort of man Additional due to push of river

Shortest time



Shortest Path (No drift)



Escalator

$$t_3 = \frac{d}{V_E + V_{M/E}} = \frac{d}{\frac{d}{t_1} + \frac{d}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$$

t_1 = Time taken by a man to move distance d on a stationary escalator

t_2 = Time taken by a stationary man to move distance d along with moving escalator

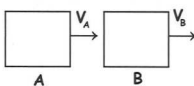
t_3 = Time taken by a man to move distance d while walking along a moving escalator

V_E = Velocity of escalator

$V_{M/E}$ = Velocity of man w.r.t escalator

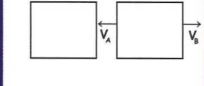
MAN RAIN PROBLEM

Man-rain problem

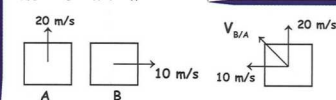


In order to find the relative velocity of B with respect to A we have to reverse the direction of vector A and add it with vector B

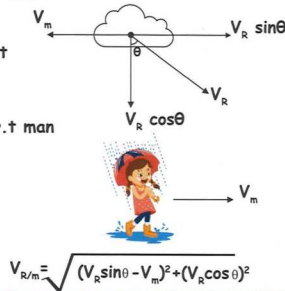
w.r.t. A



$V_{B/A} = V_B - V_A$, V_B w.r.t A
 $V_{A/B} = V_B - V_A$, V_A w.r.t B



Method



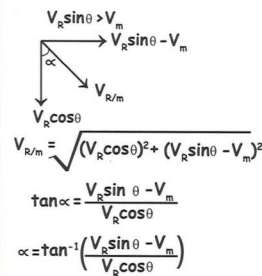
Terms

$V_R \Rightarrow$ Velocity of rain w.r.t stationary man

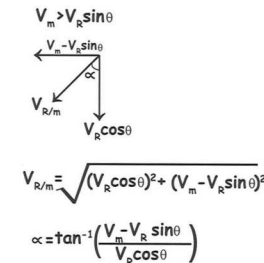
$V_m \Rightarrow$ Velocity of man

$V_{R/m} \Rightarrow$ Velocity of Rain w.r.t man

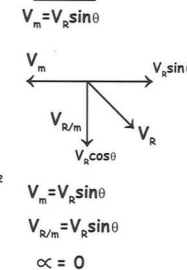
Case 1



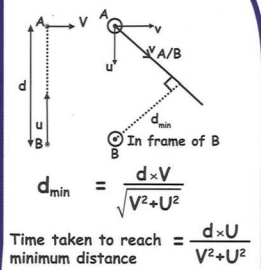
Case 2



Case 3



minimum distance



CIRCULAR MOTION

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

Angular velocity

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} \text{ (in uniform circular motion), } V = \omega r$$

$$\vec{V} = \vec{\omega} \times \vec{r}$$

Angular acceleration

$$\alpha = \frac{d\omega}{dt} \quad a_t = r\alpha$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

Equation of angular motion

- 1) Constant angular velocity : $\omega = \text{constant}$
- 2) Constant angular acceleration

$$\Rightarrow \omega = \omega_0 + \alpha t$$

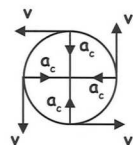
$$\Rightarrow \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Rightarrow \omega^2 = \omega_0^2 + 2\alpha(\theta)$$

Centripetal acceleration

Directed towards centre

Not a constant vector



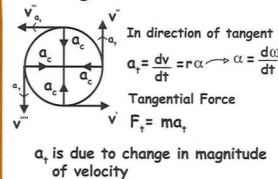
a_c is due to change in the direction of velocity

$$a_c = \frac{v^2}{R} = a_c = r\omega^2$$

$$\vec{a}_c \perp \vec{v}$$

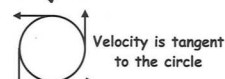
$$\vec{F}_c \perp \vec{s}$$

Tangential acceleration



Resultant acceleration

$$a_r = \sqrt{a_c^2 + a_t^2}$$



Circular Motion

Uniform Circular Motion

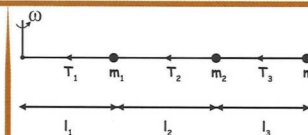
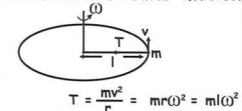
- Speed Constant
- Direction of velocity changes

- $\omega = \text{Constant}$
- $a_c = \frac{v^2}{r} = r\omega^2$
- $a_t = 0$

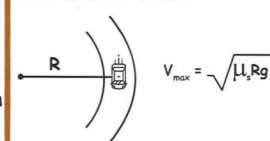
Non-uniform Circular Motion

- Speed not Constant
- Velocity changes in direction and magnitude
- a_c = Centripetal acceleration
- a_t = tangential acceleration
- $\frac{dv}{dt}$
- $\alpha = \frac{d\omega}{dt}$
- ω = Changes $\rightarrow \alpha$ angular acceleration

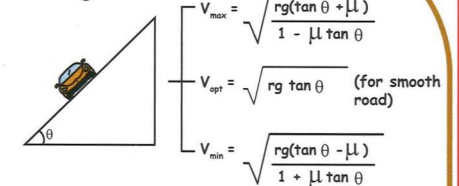
Horizontal circular motion



Flat circular track



Banking of Road



At Bottom

- a) $T_{max} = \frac{mv^2}{r} + mg$
- b) min velocity at bottom to complete circle $= \sqrt{5gr}$

At Top

- a) $T_{min} = \frac{mv^2}{r} - mg$
- b) min velocity at top to complete the circle $= \sqrt{gr}$

INERTIA

A body cannot change its state of rest or uniform motion along a straight line. This property is called inertia. Inertia has no unit and no dimension.

INERTIA

Inertia of Rest
Inertia of Motion
Inertia of Direction

- Inertia of Rest
Inability to change state of rest by itself.
- Inertia of Motion
Inability of a body to change its state of uniform motion by itself.
- Inertia of Direction
Inability of a body to change direction of motion by itself.

Newton's Second Law

F_{net} = Rate of change of linear momentum.

Instantaneous

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Average

$$\vec{F}_{av} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

MOMENTUM

$$\vec{p} = m\vec{v}$$

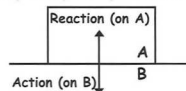
- It is a vector quantity having direction same as that of velocity
- Unit is kg m/s.

NEWTON'S THIRD LAW

- To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.
- Forces in nature always occur in pairs.
- A single isolated force is not possible.
- Counter force experienced by a body - reaction
- Action and reaction never act on the same body

- * Force exerted on body A by body B (action)
- * force exerted on body B by body A (reaction)

$$\vec{F}_{Action} = -\vec{F}_{Reaction}$$

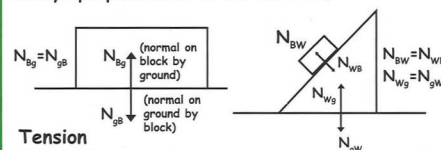


Some common forces

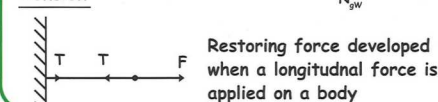
1. Normal Reaction

- Occurs when two surfaces are in contact with each other

Always perpendicular to the surface.

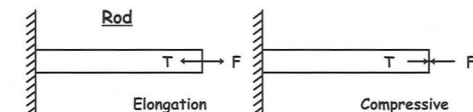


Tension



Ideal Rope

- * Massless
- * Tension is same everywhere
- * Opposes only elongation
- * On compression it becomes slack.
- * Tension always acts away from the object.



can support both elongation and compression

SINGLE BLOCK

Horizontal Force

Acceleration is along x-axis only

Along y-axis $a_y = 0$

$$N - Mg = Ma_y$$

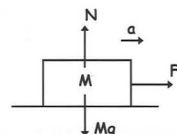
$$\therefore a_y = 0$$

$$N = Mg$$

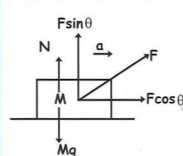
Along x-axis

$$F - 0 = Ma$$

$$a = \frac{F}{M}$$



Inclined Forces



- If, $F_{sin\theta} < Mg$
block remains in contact with ground & accelerates horizontally
- If, $F_{sin\theta} = Mg$
block just leaves contact with ground
- If, $F_{sin\theta} > Mg$
the block leaves contact with ground and it begins to accelerate obliquely.

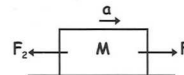
LAWS OF MOTION

MOTION OF CONNECTED BODIES

$$F_1 > F_2$$

$$F_1 - F_2 = Ma$$

$$a = \frac{F_1 - F_2}{M}$$



Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

here, f , f_1 and f_2 are normal reactions between blocks

LIQUID JETS

When jet is stopped at wall

$$F_{jet} = \frac{P_f - P_i}{\Delta t} = \frac{m \Delta v}{\Delta t} = \frac{m \Delta v}{\Delta t} = \rho A u^2$$

$$F_{wall} = \rho A u^2$$

When liquid jet bounce back

$$F_{jet} = \frac{P_f - P_i}{\Delta t} = \frac{-2mu}{\Delta t} = -2\rho A u^2$$

$$F_{wall} = 2\rho A u^2$$

When liquid jet strikes obliquely

$$F_{jet} = -2\rho A u^2 \cos\theta$$

$$F_{wall} = 2\rho A u^2 \cos\theta$$

Change in momentum = $-2mu \cos\theta$

THICK ROPE

Tension will be different at different points.

$$\text{Mass per unit length} = \frac{M}{L}$$

$$\text{Mass of } x \text{ length of rope} = \frac{M}{L} x$$

Note:

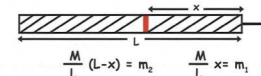
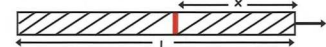
$$\text{Mass of given length} = \frac{\text{total mass}}{\text{total length}} \times \text{given length}$$

mass = constant

$$a_{rope} = \frac{F}{M}$$

For $(L-x)$ rope length,

$$\vec{F} = m\vec{a} \Rightarrow T = m_2 \frac{F}{M}$$



MOTION OF BLOCKS CONNECTED BY MASSLESS STRING

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \frac{(m_1 + m_2) F}{m_1 + m_2 + m_3}$

FRAME OF REFERENCE & PSEUDO FORCE

Frame of Reference

A frame in which observer is situated and makes his observation

Inertial frame of reference	Non-Inertial frame of reference
At rest or moving with uniform velocity along straight line i.e unaccelerated	Accelerated frame of reference.
Newton's law of motion hold's $\vec{F}_{\text{net}} = m\vec{a}$	Newton's law of motion not applicable. $\vec{F}_{\text{net}} + \vec{F}_{\text{pseudo}} = m\vec{a}$ $\vec{F}_{\text{pseudo}} = -m\vec{a}_0$

LIFT PROBLEMS

Apparent weight of body in a lift

Reading of weighing machine = reaction force exerted by weighing machine

Apparent weight, $(W_{\text{apparent}}) = \text{Reaction force (R)}$

Case 1: Lift is at rest

$R = mg$, $W_{\text{apparent}} = W_{\text{actual}} = mg$

Case 2: Lift moving up or down with constant velocity

$R = mg$, $W_{\text{apparent}} = W_{\text{actual}} = mg$

Case 3: Accelerated upward at a rate of 'a'

$R - mg = ma \Rightarrow R = m(g+a) = W_{\text{app}}$

$W_{\text{apparent}} > W_{\text{actual}} \rightarrow \text{Feels over weight}$

Accelerated upward at a rate of 'g'

$R - mg = mg$, $R = 2mg$, $W_{\text{app}} = 2 \times W_{\text{act}}$

Case 4: Accelerated downward at a rate of 'a'

$mg - R = ma$, $R = m(g-a) = W_{\text{app}}$, $W_{\text{app}} < W_{\text{act}}$

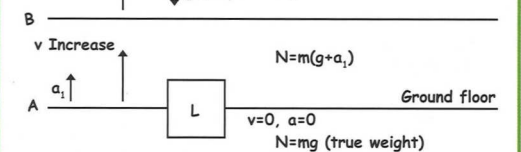
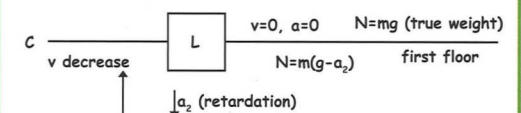
Accelerated downward at a rate of 'g' [Freefall]

$mg - R = mg$, $R = mg - mg = 0$, $W_{\text{app}} = 0$

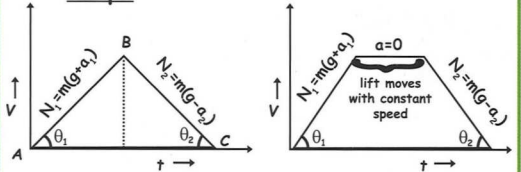
If $a > g$:

body leaves contact from ground and begins free-fall

Lift moves from ground floor to first floor



Graph



Acceleration, $a_1 = \tan \theta_1$

Retardation, $a_2 = \tan \theta_2$

LAWS OF MOTION

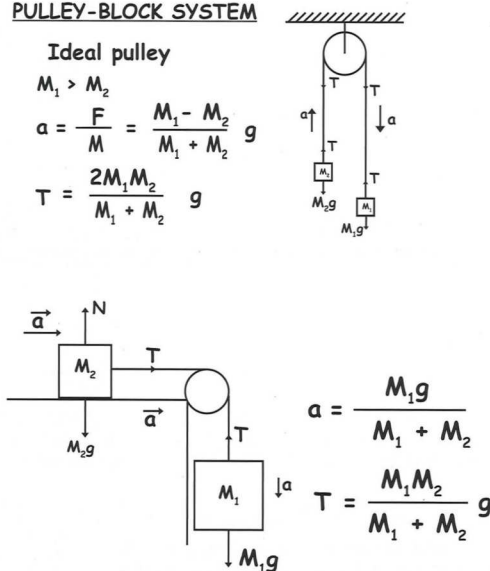
PULLEY-BLOCK SYSTEM

Ideal pulley

$M_1 > M_2$

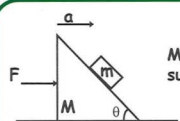
$$a = \frac{F}{M} = \frac{M_1 - M_2}{M_1 + M_2} g$$

$$T = \frac{2M_1 M_2}{M_1 + M_2} g$$



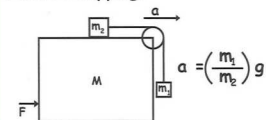
INCLINED PLANE + PULLEY

		$m_1 a = T - m_1 g \sin \theta$	$a = \left[\frac{m_2 - m_1 \sin \theta}{m_1 + m_2} \right] g$
		$m_2 a = m_2 g \sin \alpha - T$	$T = \frac{m_1 m_2 (1 + \sin \theta)}{m_1 + m_2} g$
		$T - m_1 g \sin \alpha = m_1 a$	$a = \frac{(m_2 \sin \beta - m_1 \sin \alpha)}{m_1 + m_2} g$
		$m_2 a = m_2 g \sin \beta - T$	$T = \frac{m_1 m_2 (\sin \alpha + \sin \beta)}{m_1 + m_2} g$
		$m_1 g \sin \theta - T = m_1 a$	$a = \frac{m_1 g \sin \theta}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{m_1 m_2 \sin \theta g}{m_1 + m_2}$

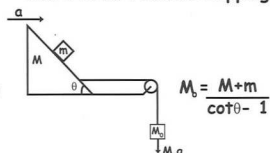
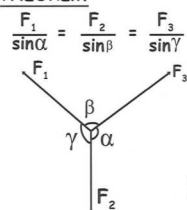
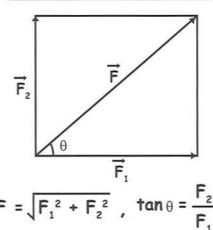
RELATIVE SLIPPING

Minimum force required to push the inclined plane such that "m" does not slip with respect to "M"
 $F = (m+M) g \tan \theta$, $a = g \tan \theta$

Minimum acceleration of "M" such that there is no relative slipping.



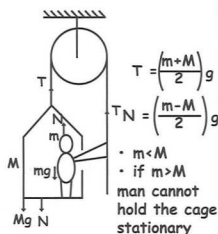
Minimum mass M_0 such that there is no relative slipping

**EQUILIBRIUM & LAMI'S THEOREM****PARALLELOGRAM LAW**

$$\vec{F} = \vec{F}_1 + \vec{F}_2, F = \sqrt{F_1^2 + F_2^2}, \tan \theta = \frac{F_2}{F_1}$$

MAN-CAGE PROBLEM

Man holds the cage stationary



$T = \frac{(m+M)}{2} g$
 $T_N = \frac{(m-M)}{2} g$
 • $m < M$
 • if $m > M$ man cannot hold the cage stationary

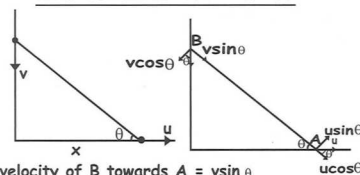
IMPULSE

If a large force acting for short period of time, there will be a sudden change in momentum

$$\vec{F}_{imp} = \frac{d\vec{p}}{dt}$$

$$\vec{p}_f - \vec{p}_i = \int_0^t \vec{F}_{imp} dt = \text{area under } F-t \text{ graph}$$

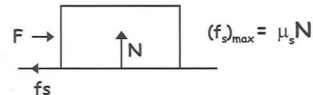
$$\vec{I} = \text{Impulse} = \vec{p}_f - \vec{p}_i = \int_0^t \vec{F}_{imp} dt = \text{area of } F-t \text{ graph}$$

ROD SLIDING ON A WALL

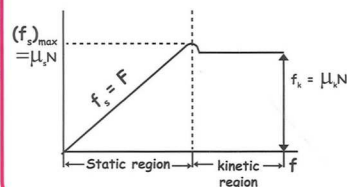
velocity of B towards A = $v \sin \theta$
 velocity of A away from B = $u \cos \theta$
 these velocities should be equal,
 $\Rightarrow v \sin \theta = u \cos \theta$
 $\Rightarrow v = u \cot \theta$

FRICTION**Static friction**

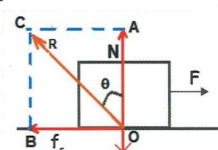
- It is a self adjusting force.
- The opposing force that comes into play, when object tends to slip over the surface of other object, but slipping has not yet started.
- As applied force increases static friction also increases.
- The body doesn't move until a maximum value of static friction is attained
- This value is called limiting friction or $(f_s)_{max}$

**Kinetic friction**

If the applied force is increased further and slipping between surfaces start, the friction opposing the slipping is called kinetic friction.

**ANGLE OF FRICTION**

Angle θ made by resultant of normal (N) & frictional force (f_s) with normal



$$R = \sqrt{N^2 + f_s^2} \quad R \text{ is resultant}$$

$$\tan \theta = \frac{f_s}{N}$$

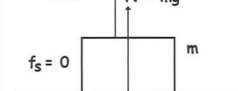
$$\text{When } f_s = (f_s)_{max} \quad R = N \sqrt{1 + \mu_s^2}$$

$\tan \theta = \mu_s$, μ_s is Coefficient of friction

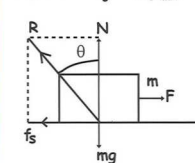
$$\text{i) } f = 0$$

$$\tan \theta = 0$$

$$\theta = 0$$



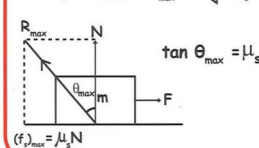
$$\text{ii) } 0 < f_s < (f_s)_{max}$$



$$0 \leq \tan \theta \leq \mu_s$$

$$N < R < N \sqrt{1 + \mu_s^2}$$

$$\text{iii) } f_s = (f_s)_{max} \quad R_{max} = N \sqrt{1 + \mu_s^2}$$

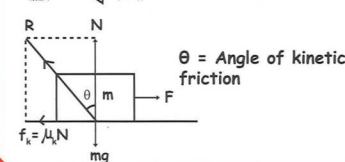


$$\tan \theta_{max} = \mu_s$$

iv) When slipping has started,

$$f = f_k$$

$$R_{max} = N \sqrt{1 + \mu_k^2}$$



θ = Angle of kinetic friction

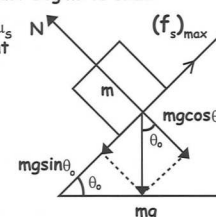
ANGLE OF REPOSE

Angle made by inclined plane such that a block kept on it just begins to slide

Depends only on μ_s and is independent of mass.

$$\tan \theta_0 = \mu_s$$

Variation of R

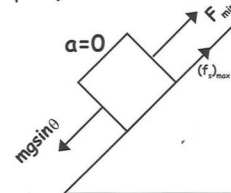


As angle of inclined plane increases, R remains constant (=mg) and when sliding starts R starts decreasing.

Variation of angle of friction.

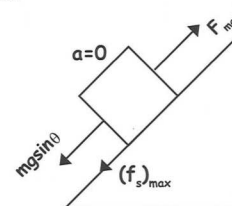
As angle of inclined plane increases, angle of friction will also increase and as sliding starts its value becomes constant and $\tan \theta = \mu_k$

Minimum & Maximum force (applied parallel to inclined plane)



$$F_{min} = mg (\sin \theta - \mu_s \cos \theta)$$

F_{min} is the minimum force required to keep the block stationary if the angle of inclined plane is greater than angle of repose



F_{max} is the maximum force required to keep the block stationary on the inclined plane if the angle of inclined plane is greater than angle of repose

$$F_{max} = mg (\sin \theta + \mu_s \cos \theta)$$

$$mg (\sin \theta - \mu_s \cos \theta) \leq F \leq mg (\sin \theta + \mu_s \cos \theta)$$

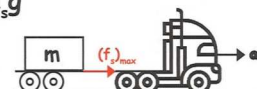
HORIZONTAL TRUCK BOX**Case-1**

Box does not slip.

$$f_s \leq \mu_s N$$

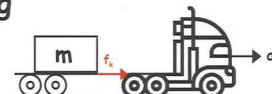
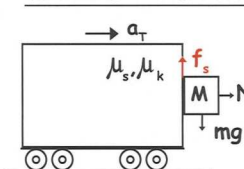
$$\Rightarrow ma_T \leq \mu_s mg$$

$$\Rightarrow a_T \leq \mu_s g$$

**Case-2**

Box slips

$$a_T > \mu_s g$$

**VERTICAL TRUCK BOX****Case-1**

Box does not slip.

$$f_s \leq \mu_s N$$

$$\Rightarrow mg \leq \mu_s ma_T$$

$$\Rightarrow \mu_s \geq \frac{g}{a_T}$$

Case-2

Box slips

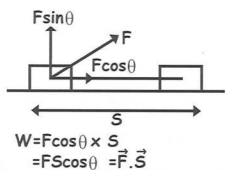
$$\mu_k \leq \frac{g}{a_T}$$

LAWS OF MOTION

WORK

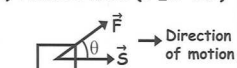
Work is said to be done when a force applied on the body displaces the body through a certain distance

WORK DONE BY CONSTANT FORCE

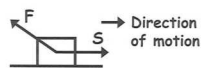


NATURE OF WORK DONE

1) Positive work ($0^\circ \leq \theta < 90^\circ$)



2) Negative work ($90^\circ < \theta \leq 180^\circ$)



2) Zero work

Work done becomes 0 for three conditions

1. Force is perpendicular to displacement
2. if there is no displacement
3. if there is no force acting on the body

WORK DONE BY VARIABLE FORCE

$$dW = \vec{F} \cdot d\vec{s}$$

$$W = \int \vec{F} \cdot d\vec{s} = \int F \cos(\theta) ds$$

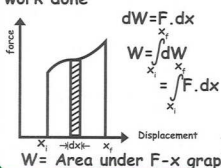
in terms of rectangular components

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

$$W = \int F_x dx + \int F_y dy + \int F_z dz$$

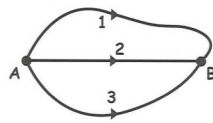
Graphical representation of work done



WORK DONE BY CONSERVATIVE & NON CONSERVATIVE FORCE

Conservative: work done does not depend on path followed

Non-conservative: work depends on the path followed



- $W_{A \rightarrow B}$ (Path 1) = $W_{A \rightarrow B}$ (Path 2) = $W_{A \rightarrow B}$ (Path 3) (for conservative force)
- $W_{A \rightarrow B}$ (Path 1) $\neq$ $W_{A \rightarrow B}$ (Path 2) $\neq$ $W_{A \rightarrow B}$ (Path 3) (for non conservative force)

Note :

Work done for a complete cycle for a conservative force is zero

WORK DONE IN PULLING THE CHAIN

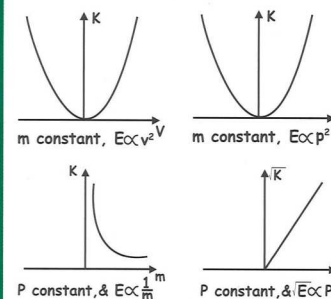


$L \rightarrow$ Total length
 $(1/n)^{th}$ part of length hanging
 $M \rightarrow$ Mass of chain
 Work done in pulling the hanging portion back on the table
 $W = \frac{MgL}{2n^2}$

RELATION OF KINETIC ENERGY WITH OTHER QUANTITIES

Linear momentum: $P = \sqrt{2mK}$

Variation of graph of kinetic Energy



WORK ENERGY & POWER

POTENTIAL ENERGY

- Defined only for conservative force
 - Energy possessed by a body by virtue of its position/configuration
 - Can either be positive, negative or zero according to point of reference
 - Force always acts from higher potential to lower potential
- Identifying forces with the help of potential energy

1) Force opposing the motion:-

On increasing x , if U increases

$$\frac{dU}{dx} = \text{positive}$$

(BC portion of graph)

2) Force supporting the motion:-

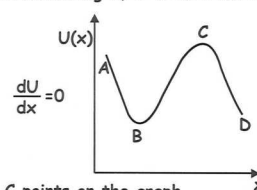
On increasing x , if U decreases

$$\frac{dU}{dx} = \text{negative}$$

(AB portion of graph)

3) Zero force:-

On increasing x , if U does not change



B, C points on the graph

Types of Potential Energy
 - Elastic Potential Energy
 - Electric Potential Energy
 - Gravitational Potential Energy

• Types of equilibrium

If net force acting on a particle is zero it is said to be in equilibrium

STABLE

- If particle displaced from equilibrium position, force acting on it tries to bring the particle back to the equilibrium position

- Potential energy is minimum at stable equilibrium

$$F = -\frac{dU}{dx} = 0$$

$$\frac{d^2U}{dx^2} = \text{positive}$$



UNSTABLE

- If particle displaced from equilibrium position, force acting on it tries to displace it further away from equilibrium position

- Potential energy is maximum at unstable equilibrium

$$F = -\frac{dU}{dx} = 0$$

$$\frac{d^2U}{dx^2} = \text{negative}$$



NEUTRAL

- If particle is slightly displaced from equilibrium, then it does not experience a force or continues to be in equilibrium

- Potential energy is constant

$$F = -\frac{dU}{dx} = 0$$

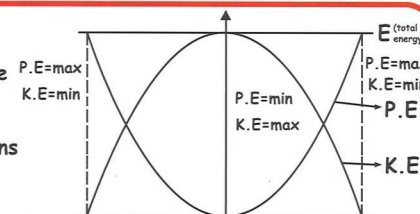
$$\frac{d^2U}{dx^2} = 0$$



CONSERVATION OF ENERGY

For an isolated system or body in the presence of only conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion

$$K.E + P.E = \text{constant}$$



POWER

- Rate at which body does work

$$\text{Average power } (P_{av}) = \frac{W}{t}$$

- Instantaneous power

$$(P_{inst}) = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

Relation between units:

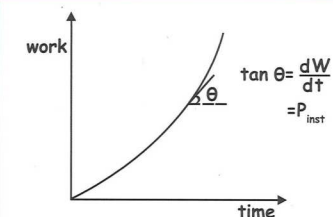
- 1 watt = 1 joule/sec = 10^7 erg/sec
- 1 HP = 746 watt, 1 MW = 10^6 watt
- 1 KW = 10^3 watt

- If work done by two bodies is same then power $\propto \frac{1}{\text{time}}$

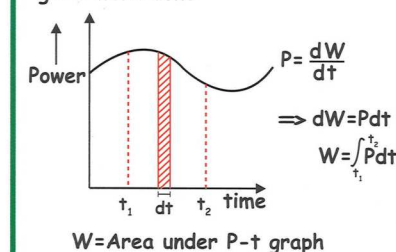
- Unit of power multiplied by time always gives work

$$1 \text{ KWh} = 3.6 \times 10^6 \text{ Joules}$$

- Slope of work-time curve gives instantaneous power



- Area under power-time graph gives work done



Position and velocity in terms of power ($P = \text{constant}$)

1) Velocity, $V = \left[\frac{2Pt}{m} \right]^{1/2}$

2) Position, $S = \left[\frac{8Pt^3}{9m} \right]^{1/2} t^{3/2}$

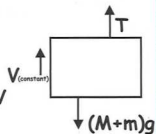
Power delivered by an elevator

$a=0$, $T = (M+m)g$

$P = \vec{T} \cdot \vec{V}$

$= TV$

Power, $P = (M+m)gV$



Power of a water drawing pump

• Power, $P = \frac{dW}{dt} = \frac{dm}{dt} \left[gh + \frac{V^2}{2} \right]$

• h = height of water level

$\frac{dm}{dt} \Rightarrow$ mass flow rate of pump

$V \rightarrow$ velocity of the water outlet

• Power required to just lift water, $V=0$

$P = gh \left(\frac{dm}{dt} \right)$

Efficiency of pump

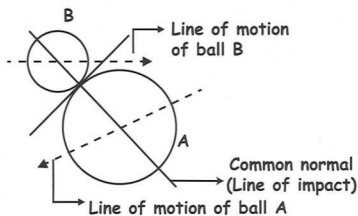
$\mu = \frac{\text{Output Power}}{\text{Input Power}}$

WORK ENERGY & POWER

Collision is the event in which impulsive force acts between two or more bodies which results in change of their velocities.

Line of impact

Line passing through common normal to surfaces in contact during impact



Coefficient of restitution (e)

$e = \frac{\text{Velocity of separation along the line of impact}}{\text{Velocity of approach along the line of impact}}$

$= \frac{\text{Relative velocity along the line of impact after collision}}{\text{Relative velocity along the line of impact before collision}}$

$= \frac{\text{Relative velocity along the line of impact after collision}}{\text{Relative velocity along the line of impact before collision}}$

Conditions

1. For elastic collision: $e=1$
2. For inelastic collision: $e<1$
3. For perfectly inelastic collision: $e=0$

Classification



Perfectly elastic collision

K.E before and after collision is same

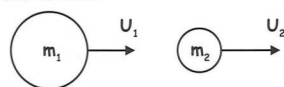
Inelastic collision

K.E before and after collision is not same

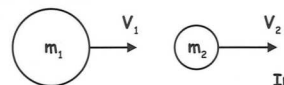
Head-on collision / One dimensional collision

Initial velocities of the bodies are along the line of impact.

Before collision



After collision

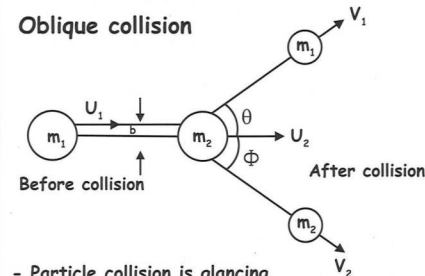


Impact parameter $b=0$

COLLISION

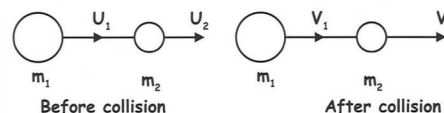
If initial velocities of the bodies are not along the line of impact.

Oblique collision



- Particle collision is glancing
- Directions of motion after collision are not along initial line of motion
- Impact parameter $0 < b < (r_1 + r_2)$ where r_1, r_2 are radii of colliding bodies

Perfectly elastic Head-on collision



Velocity after collision

$V_1 = U_1 \left[\frac{m_1 - m_2}{m_1 + m_2} \right] + \frac{2m_2 U_2}{m_1 + m_2}$

$V_2 = U_2 \left[\frac{m_2 - m_1}{m_1 + m_2} \right] + \frac{2m_1 U_1}{m_1 + m_2}$

Special cases:

1) Projectile and target having same mass $m_1 = m_2$, then $v_1 = u_2, v_2 = u_1$, the velocities get interchanged.

2) If massive projectile collides with a light target i.e. $m_1 \gg m_2$, then $v_1 = u_1, v_2 = -u_2 + 2u_1$

3) If a light projectile collides with a very heavy target, $m_1 \ll m_2$, then $v_1 = -u_1 + 2u_2, v_2 = u_2$

Energy transfer from projectile to target

1) Fractional decrease in kinetic energy of projectile (If target is at rest)

$\frac{\Delta K}{K} = \frac{4m_1 m_2}{(m_1 - m_2)^2 + 4m_1 m_2}$

Greater the difference in masses, less will be transfer of K.E and vice versa

Transfer of K.E will be maximum when difference in masses is maximum

If $m_2 = nm_1$, $\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$

Inelastic collision

$e = \frac{V_2 - V_1}{U_1 - U_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$

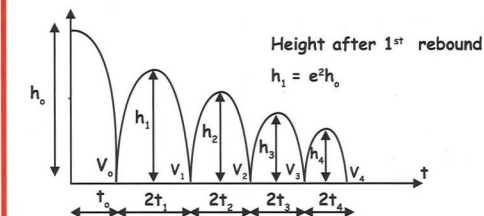
Velocity after collision $V_1 = \frac{(1+e)m_2 U_2}{m_1 + m_2} + \frac{(m_1 - e m_2)U_1}{m_1 + m_2}$

Ratio of velocities $V_2 = \frac{(1+e)m_1 U_1}{m_1 + m_2} + \frac{(m_2 - e m_1)U_2}{m_1 + m_2}$

$\frac{V_1}{V_2} = \frac{1-e}{1+e}$

Loss in kinetic energy $\Delta K = \frac{1}{2} \left[\frac{m_1 m_2}{m_1 + m_2} \right] (1-e^2) (U_1 - U_2)^2$

Rebounding of ball



Total height covered by the ball before it stops bouncing

$H = h_0 \left[\frac{1+e^2}{1-e^2} \right]$

Total time taken by the ball until it stops bouncing $T = \left(\frac{1+e}{1-e} \right) \frac{2h_0}{g}$

Perfectly inelastic collision

Colliding bodies stick together

After collision are moving in the same



Loss in kinetic energy $\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (U_1 - U_2)^2$

Colliding bodies are moving in the opposite direction

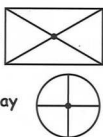
$V = \frac{m_1 U_1 - m_2 U_2}{m_1 + m_2}$, Change in kinetic energy $\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (U_1 - U_2)^2$

Centre of Mass

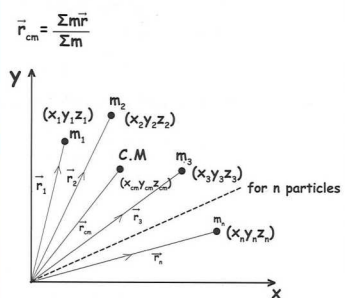
• Avg. position of all the parts of the system, weighted according to their mass

• For homogeneous objects, centre of mass lies at their geometric centre

• Centre of mass may or may not lie inside the object



Centre of Mass For System of n Particles



General Equation

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

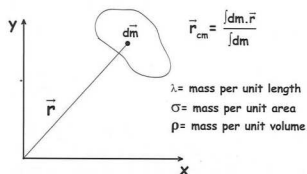
In terms of Cartesian co-ordinates

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_n y_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \dots + m_n z_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

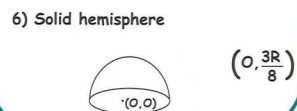
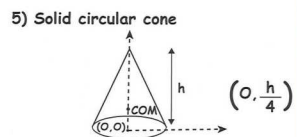
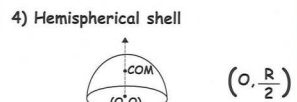
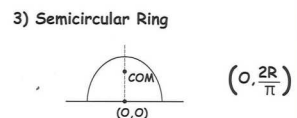
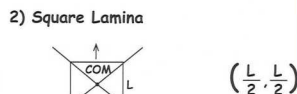
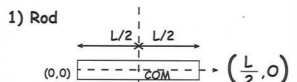
Centre Of Mass For Continuous Body



- 1) Mass distributed over length $\Rightarrow dm = \lambda dl$
- 2) Mass distributed over area $\Rightarrow dm = \sigma dA$
- 3) Mass distributed over volume $\Rightarrow dm = \rho dV$

Centre of mass for various shapes

Uniformly distributed mass centre of mass



Motion of centre of mass

velocity of centre of mass

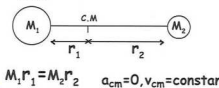
$$\vec{v}_{cm} = \frac{M_1 \vec{v}_1 + M_2 \vec{v}_2 + M_3 \vec{v}_3 + \dots}{M_1 + M_2 + M_3 + \dots}$$

Acceleration of centre of mass

$$\vec{a}_{cm} = \frac{M_1 \vec{a}_1 + M_2 \vec{a}_2 + M_3 \vec{a}_3 + \dots}{M_1 + M_2 + M_3 + \dots}$$

Isolated System

• No net external force acting on the system
• bodies within the system can have mutual force between them



Moment of Inertia (for a point object)

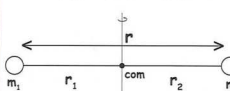
$$I = mr^2$$

m = Mass of body
r = Perpendicular distance of the body from the axis of rotation

Moment of Inertia
Tensor Quantity $I = mr^2$ Rotational analogous of mass

Two Point Masses

$$I_{com} = m_1 r_1^2 + m_2 r_2^2$$



$$r_1 = \frac{m_2 r}{m_1 + m_2}, \quad r_2 = \frac{m_1 r}{m_1 + m_2}$$

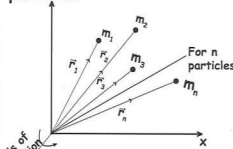
$$I_{com} = m_{red} r^2, \quad m_{red} = \frac{m_1 m_2}{m_1 + m_2}$$

Factors Affecting Moment of Inertia

Mass of the body
Axis of rotation
Mass distribution

Moment of Inertia

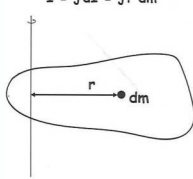
i) for discrete system of particles



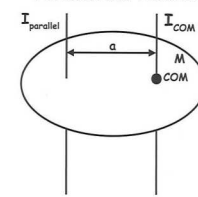
$$I = \frac{m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2}{m_1 + m_2 + m_3 + \dots + m_n}$$

ii) for continuous body

$$I = \int r^2 dm$$



Parallel Axis Theorem



Conditions:-

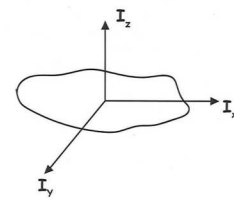
- 1) the two axes must be parallel to each other
- 2) One of the axis must pass through centre of mass

Perpendicular axis theorem

$$I_z = I_x + I_y \quad (\text{Only valid for laminar bodies})$$

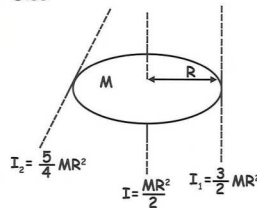
Note:-

X and Y axis must lie in the plane of body
Z- axis must be $\perp$ to the plane of the body
Axes need not pass through center of mass

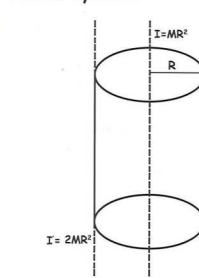


Moment Of Inertia For Various Objects

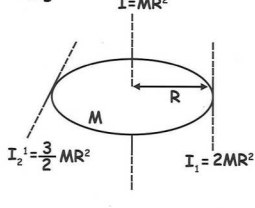
Disc



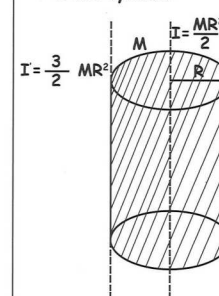
Hollow cylinder



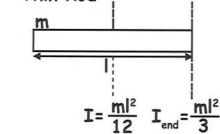
Ring



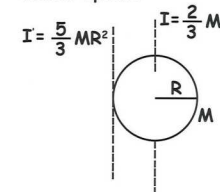
Solid cylinder



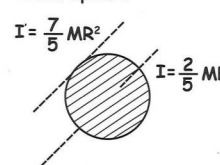
Thin Rod



Hollow sphere

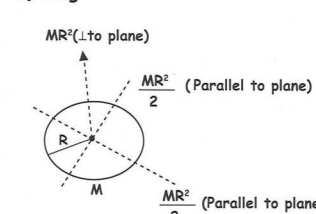


Solid sphere

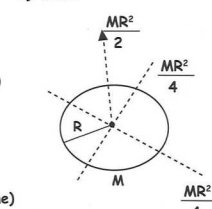


Moment of Inertia along the centre of mass and perpendicular to the plane surface

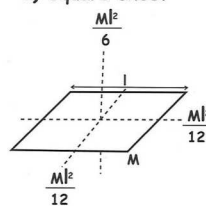
1) Ring



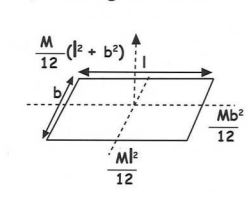
2) Disc



3) Square sheet



4) Rectangular sheet



01

ROTATIONAL MOTION

Radius of Gyration

Definition: The distance of a point mass from the axis whose mass is equal to the mass of whole body and whose moment of inertia is equal to moment of inertia of the body about that axis

$$I = \frac{mR^2}{2}$$

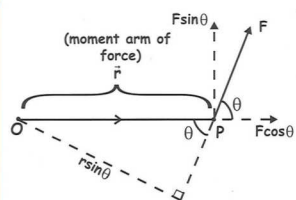
$$I = mk^2$$

$$\frac{mR^2}{2} = mk^2$$

$$k = \frac{R}{\sqrt{2}}$$

k is the radius of gyration

Torque



$$\text{Torque } \tau_o = r F \sin \theta$$

$$= F \sin \theta \times r = F_1 r \quad (1)$$

$$= F \times r \sin \theta = F r_1 \quad (2)$$

$$\vec{\tau}_o = \vec{r} \times \vec{F} \text{ (Vector form)}$$

If force is radial i.e. $\theta = 0^\circ$ or 180°

$$\text{Torque } \tau = 0$$

If force is tangential and $\perp$ to radius vector i.e. $\theta = 90^\circ$

$$\text{Torque } \tau = \tau_{\max} = rF$$

Equilibrium

For translational equilibrium

$$F_{\text{net}} = 0$$

τ_{net} may or maynot be zero

Rotational Equilibrium

$$\tau_{\text{net}} = 0$$

F_{net} may or maynot be zero

Static Equilibrium

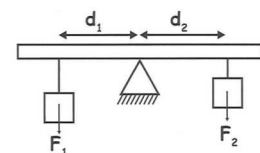
Combination of both translational and rotational equilibrium

$$F_{\text{net}} = 0 \Rightarrow \text{Forces are balanced}$$

$$\tau_{\text{net}} = 0 \Rightarrow \tau_{\text{clockwise}} = \tau_{\text{anticlockwise}}$$

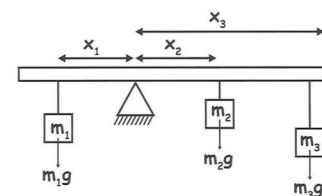
Principle of moments

When a body is in rotational equilibrium sum of clock wise moments about any point is equal to sum of anticlockwise moments about that point

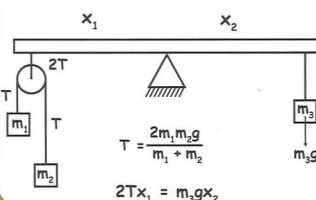


$$F_1 \times d_1 = F_2 \times d_2$$

Load $\times$ load arm = Effort $\times$ effort arm



$$m_1 g x_1 = m_2 g x_2 + m_3 g x_3$$



$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

$$2Tx_1 = m_3 g x_2$$

Angular acceleration

$$\tau = I\alpha$$

τ - torque

I moment of inertia

α angular acceleration

Initial angular acceleration when a rod is released

Initial angular acceleration when a body is released from an angle θ

$$\tau = I\alpha$$

$$(mg \sin \theta) r_{\text{cm}} = I\alpha$$

$$\alpha = \frac{(mg \sin \theta) r_{\text{cm}}}{I}$$

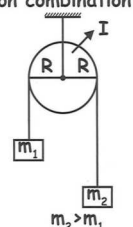
$$\text{For rod } r_{\text{cm}} = \frac{l}{2}$$

$$I = \frac{M l^2}{12}$$

Translation - rotation combination

$$\alpha = \frac{\tau_{\text{applied}} - \tau_{\text{opposition}}}{\text{Total } I}$$

$$\alpha = \frac{m_1 R g - m_2 R g}{m_1 R^2 + m_2 R^2 + I}$$



$$m_2 > m_1$$

Angular momentum & its conservation

Angular momentum of a point mass:-

Angular momentum about origin

$$\vec{L}_o = \vec{r} \times \vec{p}$$

$$= m(\vec{r} \times \vec{v})$$

$$\vec{p} = m\vec{v}$$

1) When $\theta = 0^\circ$ or 180°

$$L_o = mvr \sin 180^\circ = 0$$

$$\text{OR } mvr \sin 0^\circ = 0$$

Angular momentum is minimum

1) When $\theta = 90^\circ$

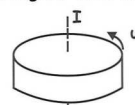
$$|\vec{L}| = r p \sin \theta$$

$$= r p \sin 90^\circ$$

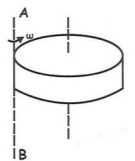
$$= r p$$

$$= L_{\text{max}}$$

Spin angular momentum



$$L_{\text{axis}} = I\omega$$



$$L_{AB} = I_{AB}\omega$$

Conservation of Angular momentum

If there is no external torque, angular momentum is conserved

$$\tau = \frac{dL}{dt}$$

$$\text{If } \tau = 0 \Rightarrow \frac{dL}{dt} = 0$$

$$L = \text{constant}$$

$$I\omega = \text{constant}$$

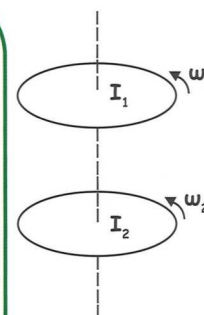
$$I_1 \omega_1 = I_2 \omega_2$$

If moment of inertia increases angular velocity decreases and if moment of inertia decreases angular velocity increases

Moment of inertia when two discs are joined

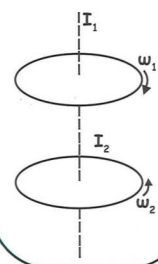
Discs initially rotating in same direction:-

$$\omega_f = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2}$$



Discs initially rotating in opposite direction:-

$$\omega_f = \frac{I_1 \omega_1 - I_2 \omega_2}{I_1 + I_2}$$



Work, Energy & Power in rotation

1) Work done by a torque,

$$W = \tau \theta \text{ (if torque is uniform)}$$

$$= \int \tau d\theta \text{ (if torque is non uniform)}$$

2) K.E for a rotating body = $\frac{1}{2} I \omega^2$

$$= \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} L \omega$$

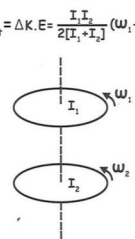
3) Work-Energy theorem

$$\Sigma W = \Delta K = \frac{1}{2} I (\omega_2^2 - \omega_1^2)$$

Energy loss when 2 discs are joined:-

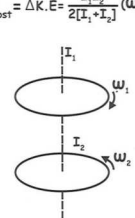
1) Same direction:-

$$E_{\text{lost}} = \Delta K.E = \frac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 - \omega_2)^2$$



2) Opposite direction:-

$$E_{\text{lost}} = \Delta K.E = \frac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 + \omega_2)^2$$



Acceleration

$$a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}}$$

$$a \propto \frac{1}{1 + \frac{k^2}{R^2}}$$

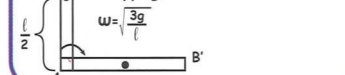
Time of descend:-

$$t = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g} \left(1 + \frac{k^2}{R^2}\right)} \propto \sqrt{1 + \frac{k^2}{R^2}}$$

Ring > Hollow sphere > Disc > solid sphere

Mechanical energy conservation

Angular velocity with which the rod hits the ground without slipping, released from rest



Rolling Motion

Translatory + Rotatory = Rolling

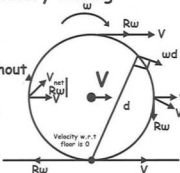
Velocity in rolling

Condition for rolling without slipping:- $V = R\omega$

Velocity of any point on rolling object, V_p

$$= \omega d = \frac{v d}{R}$$

d is the distance from point of contact



Energy in rolling motion

1) Translatory Motion

$$T_{K.E} = \frac{1}{2} m v^2$$

2) Spinning motion/rotational motion

$$R_{K.E} = \frac{1}{2} I \omega^2 = \frac{1}{2} m k^2 \frac{V^2}{R^2} = \frac{1}{2} m v^2 \times \left(\frac{k^2}{R^2}\right)$$

3) Rolling motion

$$T_{K.E} + R_{K.E} = \frac{1}{2} m v^2 + \frac{1}{2} m v^2 \times \frac{k^2}{R^2} = \frac{1}{2} m v^2 \left(1 + \frac{k^2}{R^2}\right)$$

$$\frac{k_{\text{total}}}{k_{\text{trans}}} = \left(1 + \frac{k^2}{R^2}\right)$$

Motion on an inclined plane

Velocity at bottom = $\sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$

$$v \propto \frac{1}{\sqrt{1 + \frac{k^2}{R^2}}}$$

$$\frac{k^2}{R^2} \uparrow \Rightarrow v \downarrow \Rightarrow \text{Time } \uparrow$$

$$v \propto \frac{1}{\left(1 + \frac{k^2}{R^2}\right)}$$

$$h$$

$$V \propto \frac{1}{\left(1 + \frac{k^2}{R^2}\right)}$$

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ROTATIONAL MOTION 02

NEWTON'S LAW OF GRAVITATION

$$F = \frac{Gm_1m_2}{r^2}$$

G - Universal gravitational constant
 Value of G
 $6.67 \times 10^{-11} \text{ Nm}^2\text{Kg}^{-2}$ (SI or MKS)
 $6.67 \times 10^{-8} \text{ dyne cm}^2\text{g}^{-2}$ (CGS)
 Dimensional formula G
 $[G] = [M^{-1}L^3T^{-2}]$

IMPORTANT POINTS ABOUT GRAVITATIONAL FORCE

1. Gravitational force is
 - * Always attractive in nature
 - * Independent of the nature of medium between masses
 - * Independent of presence or absence of other bodies
2. Is a central force, acts along the line joining centre of gravity of two bodies.
3. Conservative force

VECTOR FORM

$\vec{F}_{21}$ = force on 2 due to 1
 $\vec{r}_{12}$ = vector from m_1 to m_2
 $\vec{F}_{21} = \frac{Gm_1m_2}{r_{12}^2} (-\hat{r}_{12})$
 Similarly
 $\vec{F}_{12}$ = force on 1 due to 2
 $\vec{F}_{12} = \frac{Gm_1m_2}{r_{12}^2} (\hat{r}_{12})$ or $\vec{F}_{12} = \frac{Gm_1m_2}{r_{12}^3} \vec{r}_{12}$
 Clearly, Gravitational force follows:
 Newton's third law $\vec{F}_{21} = -\vec{F}_{12}$

Gravitational force is a two body interaction. Force between two particles does not depend on the presence or absence of other particles. The principle of superposition is valid here. "Force on a particle due to a no. of particles is the resultant of forces due to individual particles."

ROTATION OF 2 MASSES UNDER MUTUAL GRAVITATIONAL FORCE OF ATTRACTION

$$v_1 = \sqrt{\frac{Gm_2^2}{(m_1+m_2)r}}$$

$$v_2 = \sqrt{\frac{Gm_1^2}{(m_1+m_2)r}}$$

$$T^2 = 4\pi^2 \frac{r^3}{G(m_1+m_2)}$$
 or $T^2 \propto r^3$

THREE EQUAL MASSES REVOLVING UNDER MUTUAL GRAVITATIONAL FORCE

$$v_1 = \sqrt{\frac{GM}{l}} = \sqrt{\frac{GM}{R\sqrt{3}}}$$

$$\text{Time period } T^2 \propto l^3$$

FOUR EQUAL MASSES UNDER MUTUAL GRAVITATIONAL FORCE

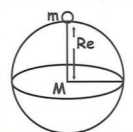
$$v = \sqrt{\frac{GM}{2\sqrt{2}l}} = \sqrt{\frac{GM}{2\sqrt{2}R}}$$

$$v = \frac{1}{2} \sqrt{\frac{GM}{R}} (2\sqrt{2}+1)$$

$$\text{Here } R = \frac{l}{\sqrt{2}}$$

GRAVITY

Acceleration due to gravity on the surface of earth, $g = \frac{GM_e}{R_e^2}$



M - mass of earth
 R - Radius of earth
 [Put $GM_e = gR_e^2$ to solve problems easily]

g IN TERMS OF DENSITY OF EARTH

$$g = \frac{4\pi G \rho R_e}{3} \quad g \propto \rho R_e$$

"If density is mentioned use the above equation"

VARIATION IN THE VALUE OF ACCELERATION DUE TO GRAVITY

● Variation due to height 'h'

$$g' = \frac{GM}{(R+h)^2}$$

$$g' = \frac{gR^2}{(R+h)^2}$$

General Equation

$$\text{Approximate equation } h \ll R \quad (h < 100 \text{ km})$$

$$\text{use, } g' = g \left(1 - \frac{2h}{R}\right)$$

Note the point

If $h \ll R$, then decrease in the value of g with height

$$\text{Absolute decrease} = \Delta g = g - g' = \frac{2hg}{R}$$

$$\text{Fractional decrease} = \frac{\Delta g}{g} = \frac{g - g'}{g} = \frac{2h}{R}$$

$$\text{Percentage decrease} = \frac{\Delta g}{g} \times 100 = \frac{g - g'}{g} \times 100$$

$$= \frac{2h}{R} \times 100$$

● Variation due to depth 'd'

$$g' = g \left[1 - \frac{d}{R}\right]$$

$$= \frac{gR}{R}$$

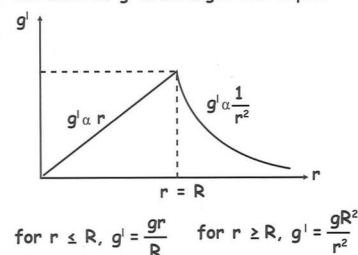
$$\text{Absolute decrease} = \frac{\Delta g}{g} = g - g' = \frac{dg}{R}$$

$$\text{Fractional decrease} = \frac{\Delta g}{g} = \frac{g - g'}{g} = \frac{d}{R}$$

$$\text{Percentage decrease} = \frac{\Delta g}{g} \times 100 = \frac{d}{R} \times 100$$

Very important graph

The graphical representation of change in the value of g' with height and depth



● Variation of g due to rotation of earth

Latitude - λ
 Angle which the line joining the point to the centre of earth makes with the equatorial plane

$$g' = g - (\omega^2 R \cos^2 \lambda)$$

Note ω value of $(\omega)^2 R = 0.034$

For poles $\lambda = 90^\circ$ $g' = g$

There is no effect of rotational motion of the earth on the value of g at poles.

For equator $\lambda = 0^\circ$ $g' = g - (\omega)^2 R$

The effect of rotational motion of the earth on the value of g at the equator is maximum.

When a body of mass m is moved from equator to the poles, weight increases by an amount $m(g_p - g_e) = m(\omega)^2 R$

WORK DONE IN MOVING OBJECT FROM ONE ORBIT TO ANOTHER

$$v_{01} = \sqrt{\frac{GM}{r_1}}$$

$$v_{02} = \sqrt{\frac{GM}{r_2}}$$

CONCEPT - WORK DONE BY EXTERNAL AGENT = CHANGE IN MECHANICAL ENERGY

$$W = E_2 - E_1 = \frac{GMm}{2} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

WORK DONE IN MOVING AN OBJECT FROM SURFACE OF EARTH TO HEIGHT h ABOVE SURFACE

$$W = (U_f - U_i) = \frac{-GMm}{R} - \left[\frac{-GMm}{R+h} \right]$$

Or, $W = mgh \frac{R}{R+h}$

Work done to move object to a height $h = R$

$$W = \frac{mgR}{2}$$

Work done to move object to a height $h = R/2$

$$W = \frac{mgR}{3}$$

ORBITAL VELOCITY OF A SATELLITE

Orbit at a height ' h ' from the surface

$$v_s = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}}$$

If orbit is closer to earth's surface (neglect ' h ') $v_s = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

(called first cosmic velocity)

Note - for easy calculations

$$\sqrt{gR} = 8 \text{ km/s or } \sqrt{\frac{GM}{R}} = 8 \text{ km/s} = 8 \times 10^3 \text{ m/s}$$

$$\text{or } \frac{GM}{R} = 64 \times 10^6$$

K.E. P.E AND T.E FOR AN ORBITING SATELLITE

$$KE = \frac{GMm}{2r}, U = -\frac{GMm}{r}$$

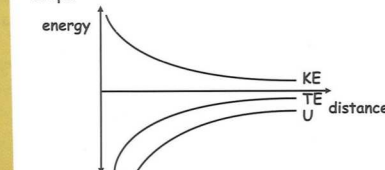
$$TE = -\frac{GMm}{2r}$$

Relation between K.E, U & T.E

$$U = 2 \times T.E$$

$$K.E = -T.E$$

Graph



WORK DONE IN MOVING OBJECT FROM SURFACE TO CIRCULAR ORBIT

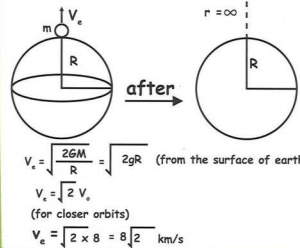
$$W = E_f - E_i$$

$$W = E_{\text{total}} - U_i = \frac{-GMm}{2(R+h)} + \frac{GMm}{R}$$

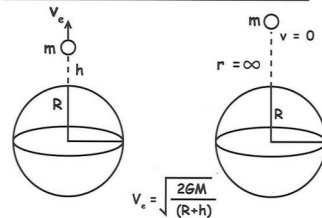
GRAVITATION

ESCAPE VELOCITY

"Minimum velocity given to an object such that it escapes out of Earth's gravitational field" $v=0$



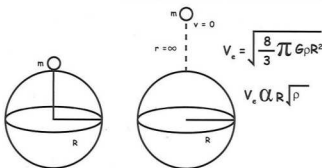
ESCAPE VELOCITY FROM A HEIGHT 'H'



ESCAPE ENERGY FOR ORBITING BODY

$$\Delta E = \text{Escape energy} = \frac{GMm}{2(R+h)}$$

ESCAPE VELOCITY FROM SURFACE OF EARTH AND RELATION WITH DENSITY



TRICK TO SOLVE PROBLEMS

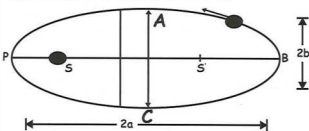
Given speed greater than escape speed (Hint) find the final speed after escaping (question) short trick
 if $V_{\text{given}} = nV_e$ (when $n > 1$)
 final speed, $V = V_e \sqrt{n^2 - 1}$

Given speed less than escape speed (Hint) find the maximum height it reached (question) short trick
 if $V_{\text{given}} = nV_e$ ($n < 1$)

maximum height,
 $h = \frac{n^2 R}{1 - n^2}$

KEPLER'S LAWS OF PLANETARY MOTION

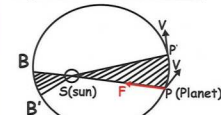
I) THE LAW OF ORBITS



Every planet moves around the sun in an elliptical orbit with sun at one of the foci.

P → Perihelion (perigee) (nearest point)
 B → apogee or aphelion (farthest point)

II) THE LAW OF AREAS



"The line joining the sun to the planet sweeps out equal areas in equal interval of time"

"i.e. areal velocity is constant"

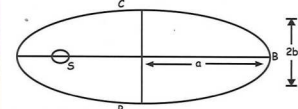
"According to this law, planet will move slowly when it is farthest from sun & rapidly when it is nearest to sun."

"Law of areas is due to law of conservation of angular momentum"

Areal velocity = $\frac{L}{2m}$

$\frac{\Delta A}{\Delta t} = \frac{L}{2m}$ L → Angular constant momentum
 → Areal velocity is constant

III) THE LAW OF PERIODS



$T^2 \propto a^3$ a → semi-major axis of elliptical orbit

$T^2 = \frac{4\pi^2}{GM_s} a^3$ M_s = Mass of sun

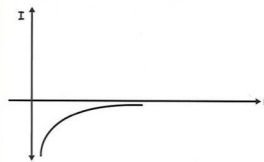
GRAVITATIONAL FIELD INTENSITY ($\vec{I}$)

$\vec{I} = \frac{\vec{F}}{m}$

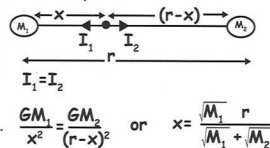
vector quantity
 direction: same as that of gravitational force
 SI unit - N/kg Dimensions - $M^0 L T^{-2}$
 Due to point mass

$\vec{I} = \frac{GM}{r^2} \hat{r}$ Magnitude, $I = \frac{GM}{r^2}$

Graph



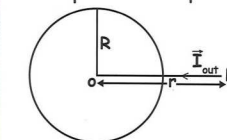
Neutral point



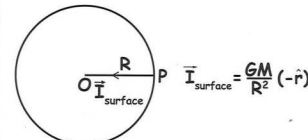
GRAVITATIONAL FIELD INTENSITY DUE TO A SPHERICAL SHELL

CASE-1 $r > R$

$\vec{I}_{\text{out}} = \frac{GM}{r^2} (-\hat{r}) \Rightarrow I \propto \frac{1}{r^2}$

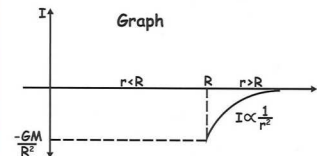


CASE-2 $r = R$



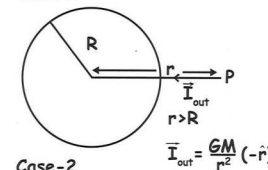
CASE-3 $r < R$

The point is inside then $I=0$

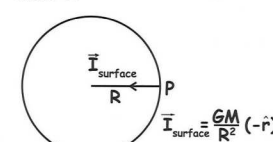


SOLID SPHERE

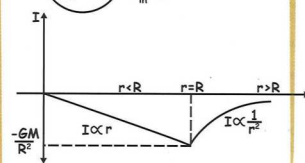
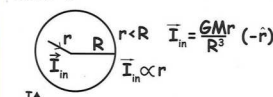
Case-1



Case-2



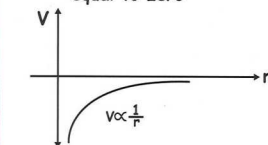
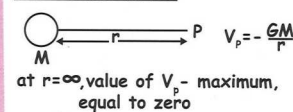
Case-3



GRAVITATIONAL POTENTIAL

$V = \frac{W_{\text{net}}}{m}$ W_{net} - Work done

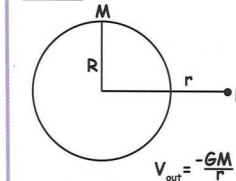
GRAVITATIONAL POTENTIAL FOR POINT MASS



GRAVITATIONAL POTENTIAL DUE TO OTHER BODIES

SOLID SPHERE

Case-1



Case-2

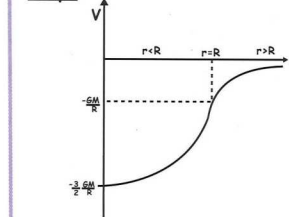
$r = R$
 $V_{\text{surface}} = -\frac{GM}{R}$

Case-3

$V_{\text{in}} = -\frac{GM}{2R^3} (3R^2 - r^2)$

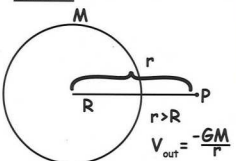
At centre, $V_{\text{centre}} = -\frac{3}{2} \frac{GM}{R} = -\frac{3}{2} V_{\text{surface}}$

Graph



HOLLOW SPHERE

Case-1



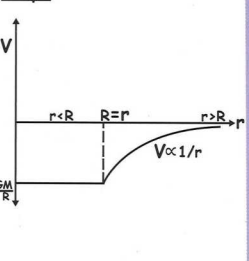
Case-2

$r = R$
 $V_{\text{surface}} = -\frac{GM}{R}$

Case-3

$V_{\text{in}} = V_{\text{surface}}$ = same everywhere

Graph



RELATION BETWEEN FIELD AND POTENTIAL

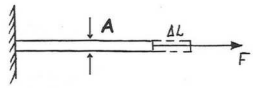
$I = -\frac{dV}{dr}$ & $\Delta V = \int \vec{I} \cdot d\vec{r}$

$\vec{I} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}$

GRAVITATION

LONGITUDINAL STRESS

Longitudinal/Tensile stress causes increase in length



$$\text{Longitudinal stress} = \frac{F}{A} \text{ unit N/m}^2$$

tensile stress in wire

$$T.S = \frac{Fm}{(M+m)/A}$$

$$T.S = T = \frac{m(g+a)}{A}$$

$$T.S_1 = \frac{T_1}{A} = \frac{6mg}{A}$$

$$T.S_2 = \frac{T_2}{A} = \frac{5mg}{A}$$

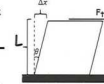
$$T.S_3 = \frac{T_3}{A} = \frac{3mg}{A}$$

VOLUME STRESS

- Same as pressure
 - Causes change in volume
- $$\text{volume stress} = \frac{F}{A} = \text{pressure}$$
- here, F = normal force (or thrust)

SHEARING STRESS

- Causes change in shape
- $$\text{shearing stress} = \frac{F_t}{A}$$
- F_t = tangential force



RATIO OF DENSITY OF BODY TO THAT OF LIQUID IN WHICH BODY IS IMMERSED

$$mg = Kl_a$$

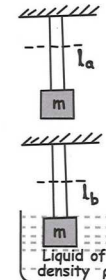
$$mg(1 - \frac{\rho}{\sigma}) = Kl_b$$

$$(1 - \frac{\rho}{\sigma}) = \frac{l_a}{l_b}$$

$$\frac{\rho}{\sigma} = \frac{l_a - l_b}{l_a}$$

$$\frac{\sigma}{\rho} = \frac{l_a}{l_a - l_b}$$

here, l_a and l_b are extensions in the rod in the two cases.



STRAIN

01

$$\text{Longitudinal strain} = \frac{\Delta L}{L} = \frac{\text{Change in length}}{\text{Original length}}$$

02

$$\text{Volumetric strain} = \frac{\Delta V}{V} = \frac{\text{Change in volume}}{\text{Original volume}}$$

03

$$\text{Shearing strain} = \theta = \frac{\Delta x}{L}$$

HOOKE'S LAW

$$E = \frac{\text{Stress}}{\text{Strain}}$$

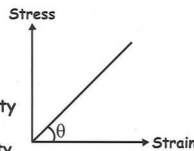
(E = modulus of elasticity)

$\tan \theta$ = modulus of elasticity

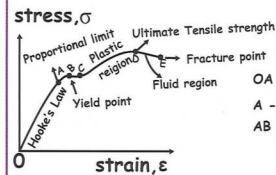
- For rigid body E = infinity
- Steel is more elastic than rubber
- Depends on :-
(a) Nature of metal
(b) Temperature
- Independent of dimensions

Young's modulus

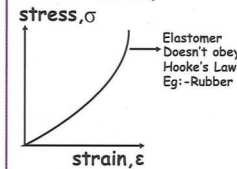
$$E = \frac{\text{Bulk Modulus}}{\text{Modulus of rigidity}}$$



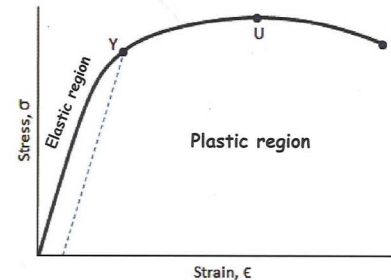
STRESS STRAIN CURVE



- OA - Hooke's law obeyed
- A - Proportional limit
- AB - stress is not proportional to strain but body regains its original shape and size regains load is removed
- B - Yield point
- B to D - Body doesn't regain its original dimension. Beyond B is plastic region.
- D - Ultimate stress point
- Beyond D - large strain is produced even for a small applied force.
- E - Fracture occurs

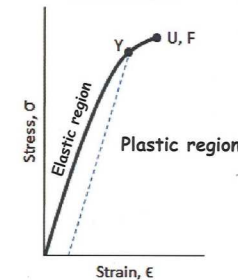


Ductile Material



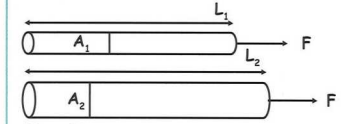
Plastic region is large for ductile materials and smaller for brittle materials

Brittle Material



BREAKING STRESS

Breaking Force = breaking stress x area, $B.F \propto A$



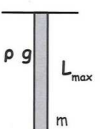
$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow (B.S)_1 = (B.S)_2$$

BREAKING OF WIRE UNDER IT'S OWN WEIGHT

$$B.S \times A = T = mg$$

$$B.S \times A = V \times \rho \times g = A L_{\max} \rho g$$

$$L_{\max} = \frac{B.S}{\rho g}$$



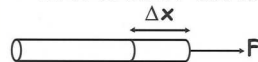
YOUNG'S MODULUS

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{FL}{A\Delta L}$$

Comparing with a spring of force constant K

$$Y = \frac{FL}{Ax} \quad F = \frac{AYx}{L} = kx \quad k = \frac{AY}{L}$$

ELASTIC POTENTIAL ENERGY AND ENERGY DENSITY



$$E.P.E = \frac{1}{2} kx^2 = \frac{F^2}{2k} = \frac{1}{2} Fx$$

$$= \frac{1}{2} \left(\frac{YA}{L} \right) x^2$$

$$= \frac{1}{2} \times \frac{\text{Stress}}{\text{Strain}} \times \text{volume} \times \text{strain}^2$$

$$= \frac{1}{2} \times \text{Stress} \times \text{strain} \times \text{volume}$$

$$\text{Energy Density} = \frac{E.P.E}{\text{volume}}$$

$$\Rightarrow e = \frac{1}{2} \times \text{Stress} \times \text{strain}$$

INCREASE IN LENGTH DUE TO IT'S OWN WEIGHT

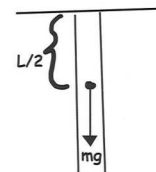
$$F = kx$$

$$mg = \frac{YA}{L} x$$

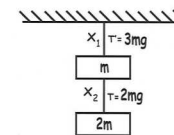
$$x = \frac{mgL}{2YA}$$

$$P.E = \frac{F^2}{2k}$$

$$= \frac{m^2 g^2}{2 \frac{YA}{L}} = \frac{1}{4} \frac{m^2 g^2 L}{YA}$$



RATIO OF EXTENSION



$$\frac{x_1}{x_2} = \frac{\frac{F_1}{k_1}}{\frac{F_2}{k_2}} = \frac{\frac{3mg \times L_1}{Y_1 \times A_1}}{\frac{2mg \times L_2}{Y_2 \times A_2}} = \frac{3l}{2yd^2}$$

$$\text{where } l = \frac{L_1}{L_2} \quad y = \frac{Y_1}{Y_2} \quad d = \frac{D_1}{D_2}$$

BULK MODULUS

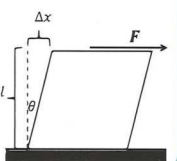
$$\text{Bulk modulus, } B = \frac{\text{Volumetric stress}}{\text{Volumetric strain}} = \frac{\Delta P}{-\frac{\Delta V}{V}}$$

$$K = \frac{1}{B} = \text{compressibility}$$

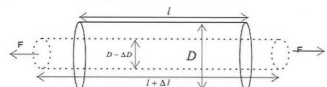
$$B_{\text{isothermal}} = P, \quad B_{\text{adiabatic}} = \gamma P, \quad B_{\text{isotropic}} = \frac{2}{3}P$$

MODULUS OF RIGIDITY

$$\eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F}{A\theta} = \frac{Fl}{A\Delta x}$$



POISSON'S RATIO



$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{(-dr/r)}{(\frac{\Delta L}{L})} = \frac{-\frac{\Delta r}{r}}{\frac{\Delta L}{L}} = -\frac{\Delta r}{\Delta L} \frac{L}{r}$$

Theoretical value :- $-1 < \sigma < 0.5$

Practical value :- $0 < \sigma < 0.5$

MECHANICS OF SOLIDS

- Density (ρ) = $\frac{\text{mass (m)}}{\text{volume (v)}}$

- unit = kg/m^3
- density of water = $1000 \text{ kg/m}^3 = 1 \text{ g/cc}$
- for two bodies of same mass
 $\rho_1 V_1 = \rho_2 V_2$
 if $\rho_1 > \rho_2$
 $V_1 < V_2$

Mixing of liquid

Calculation of resultant/final density

- 1) If volumes of the liquids are equal

$$d = \frac{d_1 + d_2}{2}$$

- 2) If masses of two liquids are equal

$$\text{For 2- liquids} \Rightarrow d = \frac{2d_1 d_2}{d_1 + d_2}$$

$$\text{For 3- liquids} \Rightarrow d = \frac{3d_1 d_2 d_3}{d_1 d_2 + d_2 d_3 + d_1 d_3}$$

$$\text{For n-liquids, } n = \frac{1}{\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \dots + \frac{1}{d_n}}$$

- 3) If masses and volumes of two liquids are different

$$d = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1 + m_2}{V_1 + V_2}$$

$$m_1 = \rho_1 V_1 \text{ \& } m_2 = \rho_2 V_2$$

$$V_1 = \frac{m_1}{\rho_1} \text{ \& } V_2 = \frac{m_2}{\rho_2}$$

Relative density (R.D)/Specific gravity

- 1) Relative density of a body

$$(R.D)_{\text{solid}} = \frac{d_{\text{solid}}}{d_{\text{water}}} = \frac{w_a}{w_a - w_w}$$

- 2) Relative density of liquid

$$(R.D)_{\text{liquid}} = \frac{d_{\text{liquid}}}{d_{\text{water}}} = \frac{w_a - w_L}{w_a - w_w}$$

- 3) Relative density of a solid to that of liquid

$$\frac{(R.D)_{\text{solid}}}{(R.D)_{\text{liquid}}} = \frac{w_a}{w_a - w_L}$$

weight of object where

W_a = weight of object when in air

W_w = weight of object when dipped in water

W_L = weight of object when dipped in liquid

Pressure

Normal force or thrust exerted by liquid per unit area

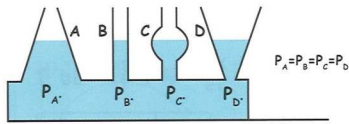
$$P = \frac{F}{A}$$

Pressure-depth relation

$$P = h \rho g$$

Hydrostatic paradox

Whatever the shape or width of vessel the pressure at any particular depth is same



- Gauge pressure = $P - P_{\text{atm}} = h \rho g$
- $P_{\text{atm}} = 1.01325 \times 10^5 \text{ Pa}$

Inclined barometer

if θ = angle with horizontal

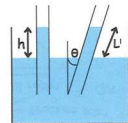
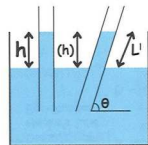
$$\sin \theta = \frac{h}{L'}$$

$$L' = \frac{h}{\sin \theta}$$

if θ = angle with vertical

$$\cos \theta = \frac{h}{L'}$$

$$L' = \frac{h}{\cos \theta}$$

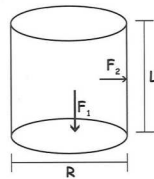


Bubble rising up at constant temperature

radius 'r' becomes 'nr' when bubble rises in liquid from bottom to the surface

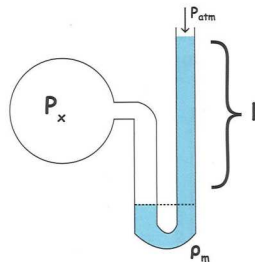
$$\rho g h = p_{\text{atm}} [n^3 - 1]$$

Conditions for equal forces on wall and bottom of a cylinder



If $L = R$
then, $F_1 = F_2$

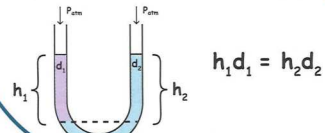
Manometer



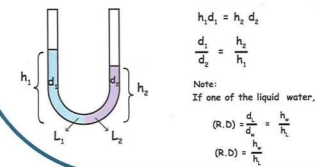
$$P_x = P_{\text{atm}} + h \rho_m g$$

$$h \rho_m g = P_x - P_{\text{atm}}$$

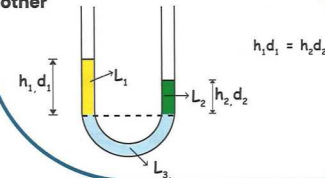
1) U-Tube manometer



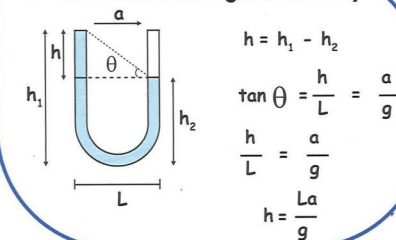
2) U-Tube type



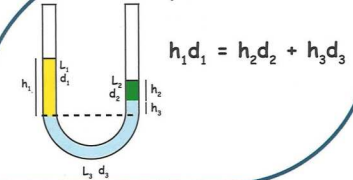
3) The third liquid is in level with other



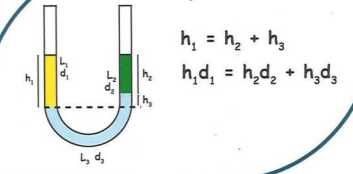
U - tube accelerating horizontally



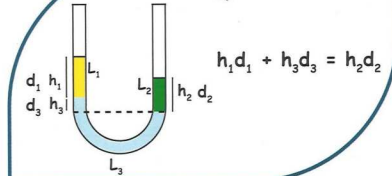
4)



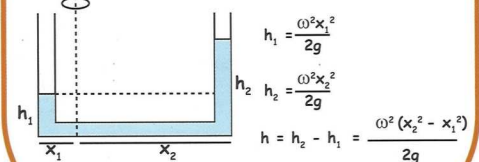
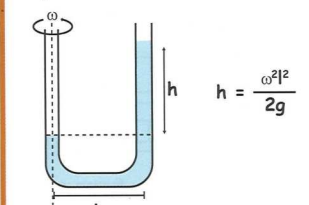
5)



6)



Special case: U - tube rotating

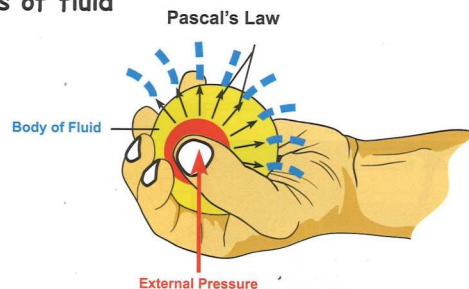


FLUID MECHANICS

01

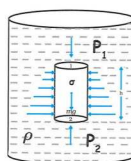
Pascal's Law

Any change in pressure at a point of an enclosed incompressible fluid is equally transmitted at all other points of fluid



Pascal's Law States that Pressure Applied on One Point of Liquid Transmits Equally in All Direction

Archimede's principle



liquid applies net upward force on an immersed body, called as upthrust or buoyant force
upthrust = weight of the liquid displaced = $V\rho g$
Apparent weight = Actual weight - upthrust

$$W_{app} = W_{air} - U$$

$$= W_{air} \left[1 - \frac{\sigma}{\rho} \right]$$

$$P_2 > P_1, P_2 - P_1 = \text{Upthrust}(U)$$

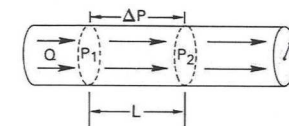
Unit of Coefficient of viscosity

- 1) The CGS Unit of η is dyne s cm^{-2} and is called poise.
 - 2) The SI unit of η is Nsm^{-2} called decapoise or poiseuille
- 1 poiseuille = 10 poise

Poiseuille's formula

$$Q = \frac{\pi \Delta P r^4}{8 \eta L}$$

where, Q = rate of flow



Stoke's law

$$F = 6 \eta \pi r v$$

F_{net} = Apparent weight - viscous force

Terminal velocity

$$V_t = \frac{2r^2}{9\eta} (\rho - \sigma) g$$

- 1) If $\rho > \sigma$, the body will attain terminal velocity in the downward direction.
- 2) If $\rho < \sigma$ the terminal velocity will be negative and the body will move in the upward direction.
- 3) $\rho = \sigma$, the body remain suspended in the fluid.

Application

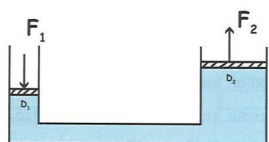
Hydraulic Lift



$$\frac{F}{A} = \frac{f}{a}$$

As $A > a$ therefore

If the cylinders are connected



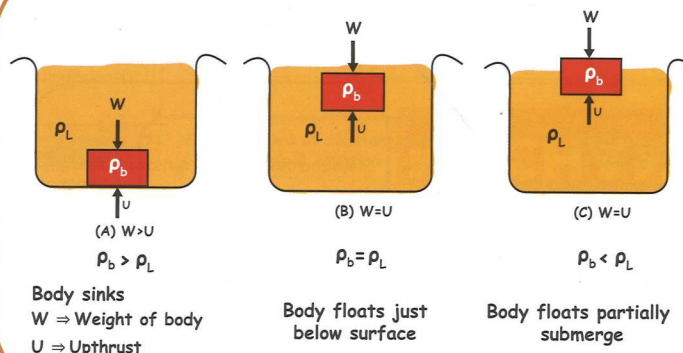
$$\frac{F_1}{A} = \frac{F_2}{A}$$

$$\frac{F_1}{R_1^2} = \frac{F_2}{R_2^2}$$

$$\frac{F_1}{D_1^2} = \frac{F_2}{D_2^2}$$

FLUID MECHANICS

Law of floatation



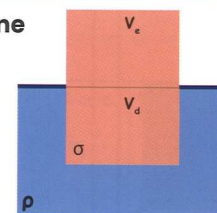
Fractional submerged volume

$$\frac{\text{Displaced volume}(V_d)}{\text{Total volume}(V)} = \frac{\sigma}{\rho} \text{ (submerged fraction)}$$

$$\frac{\text{Exposed volume}(V_e)}{\text{Total volume}(V)} = 1 - \frac{\sigma}{\rho} \text{ (Exposed fraction)}$$

$$\text{Relative density of a solid} = \frac{\text{weight of solid in air}}{\text{Loss of weight in water}} = \frac{W_a}{W_a - W_w} = \frac{\rho_b}{\rho_w}$$

$$\text{Relative density of a liquid} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight of an object dipped in water}} = \frac{W_a - W_l}{W_a - W_w} = \frac{\rho_l}{\rho_w}$$



Critical velocity

Reynold's number

$$R_e = \frac{\rho v D}{\eta}$$

Significance of Reynold's number:

- If R_e lies between 0 and 2000 the flow is streamlined or laminar.
- If $R_e > 3000$, the flow is turbulent.
- If R_e lies between 2000 & 3000 the flow of liquid is unstable. It may change from laminar to turbulent and vice versa.

Newton's Law of viscosity

$$F \propto A \frac{dv}{dx} \Rightarrow F = \eta A \frac{dv}{dx}$$

Where, Velocity gradient = $\frac{dv}{dx}$

$$\eta = \frac{F}{A \frac{dv}{dx}} \Rightarrow \text{coefficient of viscosity} \Rightarrow \eta = \frac{F/A}{\frac{dv}{dx}} = \frac{F/A}{\frac{v/l}{\frac{dx}{dt}}} = \frac{F/A}{\frac{v}{l} \frac{dx}{dt}} = \frac{F/A}{\frac{d}{dt} \left(\frac{x}{l} \right)}$$

$$\Rightarrow \eta = \frac{\text{shearing stress}}{\text{strain rate}}$$

Equation of continuity

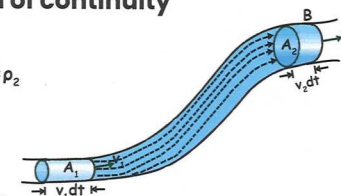
$$v_1 A_1 \Delta t \rho_1 = v_2 A_2 \Delta t \rho_2$$

since the liquid is incompressible $\rho_1 = \rho_2$

$$v_1 A_1 = v_2 A_2$$

$Av = \text{constant}$.

$$Av = \frac{dV}{dt} = Q \Rightarrow \text{Volume rate of flow}$$



Energy of fluid in a study flow

$$\text{kinetic Energy} = \frac{1}{2} mv^2$$

$$\text{kinetic energy per unit mass} = \frac{1}{2} v^2$$

$$\text{kinetic energy per unit volume} = \frac{1}{2} \rho v^2$$

$$\text{Potential Energy} = mgh$$

$$\text{Potential energy per unit mass} = gh$$

$$\text{Potential energy per unit volume} = \rho gh$$

$$\text{Pressure energy} = PV$$

$$\text{Pressure energy per unit mass} = \frac{P}{\rho}$$

$$\text{Pressure energy per unit volume} = P$$

BERNOULLI'S PRINCIPLE

$$P_1 V_1 + \frac{1}{2} \rho V_1 v_1^2 + \rho g h_1 = P_2 V_2 + \frac{1}{2} \rho V_2 v_2^2 + \rho g h_2$$

$$(P_1 - P_2) V = \frac{1}{2} \rho V (v_2^2 - v_1^2) + \rho g (h_1 - h_2)$$

$$(P_1 - P_2) V = \frac{1}{2} \rho V (v_2^2 - v_1^2) + \frac{\rho g}{V} (h_1 - h_2)$$

$$\Rightarrow P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$$

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}, \quad \frac{P}{\rho g} = \text{pressure head}$$

$$\frac{v^2}{2g} = \text{velocity head} \quad h = \text{Gravitational head}$$

Conditions:

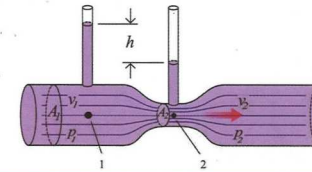
1. Flow should be streamlined.
2. Non-viscous and incompressible fluid.
3. Friction is absent everywhere.

Note: It is based on conservation of energy.

VENTURIMETER

Device to measure the flow of speed of incompressible fluid

$$v_1 = \sqrt{\frac{2hg}{(A_1^2/A_2^2) - 1}}$$

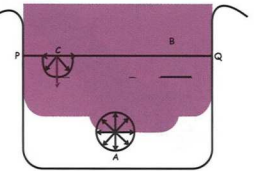


SURFACE TENSION

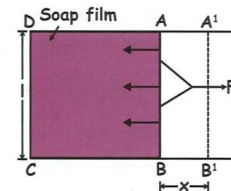
$$\text{Surface tension } T = \frac{\text{Force}}{\text{Length}} = \frac{F}{l}$$

$$\text{Unit in SI system} = \frac{\text{N}}{\text{m}}$$

$$\text{Unit in CGS system} = \text{dyne} / \text{cm}$$



SURFACE ENERGY



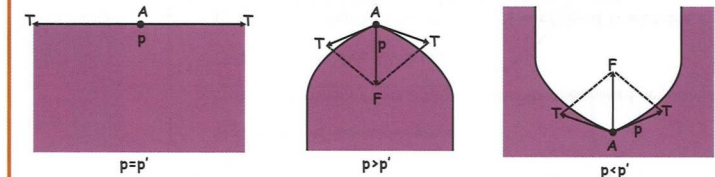
$$\text{Work done } W = F \times x$$

$$\text{But } F = 2TL$$

$$\Rightarrow W = 2TL \times x$$

$$\text{Energy of the additional surface} = W = 2TLx = T \Delta A$$

PRESSURE DIFFERENCE ACROSS A CURVED LIQUID SURFACE

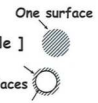


Pressure on concave side > pressure on convex side

$$P_{\text{concave}} - P_{\text{convex}} = \frac{2T}{R}$$

$$P_{\text{inside}} - P_{\text{outside}} = \frac{2T}{R} \quad [\text{Liquid drop or air bubble}]$$

$$P_{\text{inside}} - P_{\text{outside}} = \frac{4T}{R} \quad [\text{Soap bubble}]$$



APPLICATIONS OF BERNOULLI'S PRINCIPLE

Torricelli's Law of Efflux

$$v = \sqrt{\frac{2(P - P_a)}{\rho} + 2gh}$$

- It is assumed that size of hole << size of top of tank

If tank is open, $P = P_a$

$$\text{Then } v = \sqrt{2gh}$$

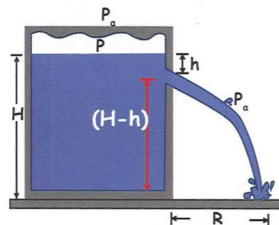
$$\text{Time of fall, } t = \sqrt{\frac{2(H-h)}{g}}$$

$$\text{Range } R = vt$$

$$= \sqrt{2gh} \times \sqrt{2(H-h)/g}$$

$$\Rightarrow R = 2\sqrt{h(H-h)}$$

$$R \text{ is max, when } h = \frac{H}{2}$$



Excess pressure inside a liquid drop

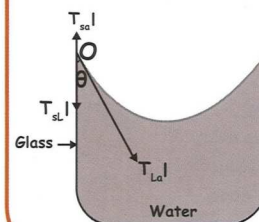
$$P_i - P_o = \frac{2T}{R}$$

Excess pressure inside a soap bubble

$$P_i - P_o = \frac{4T}{R}$$

FLUID MECHANICS 03

Shape of liquid meniscus



Consider the equilibrium along the surface at line of contact

$$T_{sa} = T_{sl} + T_{la} \cos \theta$$

$$T_{sa} = T_{sl} + T_{la} \cos \theta$$

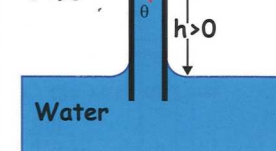
$$\cos \theta = \frac{T_{sa} - T_{sl}}{T_{la}}$$

θ = Angle of contact.

Capillarity

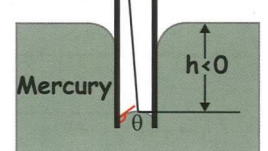
Meniscus

$$\theta < 90^\circ$$



Meniscus

$$\theta > 90^\circ$$



Ascent/descent formula:

$$h = \frac{2T}{R\rho g}, \quad h > 0 (\theta < 90^\circ) \quad h = \frac{2T \cos \theta}{r\rho g}, \quad h < 0 (\theta > 90^\circ)$$

where, R = radius of meniscus
 r = radius of the tube

TEMPERATURE SCALE

$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} \quad (\text{celcius-fahrenheit-kelvin conversion})$$

any scale conversion formula

$$\frac{\text{Reading on any scale} - \text{lower fixed point}}{\text{Upper fixed point} - \text{lower fixed point}} = \text{constant}$$

LINEAR THERMAL EXPANSION

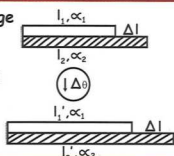
$$1. \Delta l = \alpha l \Delta \theta \quad \text{heat} \quad \frac{l}{\Delta \theta} \quad l' = l + \Delta l$$

$$2. l' = l(1 + \alpha \Delta \theta)$$

$$3. \alpha = \frac{\Delta l}{l \Delta \theta} \rightarrow \text{unit} \rightarrow /^{\circ}\text{C or } /^{\circ}\text{K, dimension-}K^{-1}$$

Whatever be the change in temperature, if the difference in length remains constant, then

$$l_1 \alpha_1 = l_2 \alpha_2$$



APPLICATIONS OF LINEAR EXPANSION

Pendulum clock

Fact $\rightarrow$ When temperature increases, time period increases, clock runs slow
 $\rightarrow$ When temperature decreases, time period decreases, clock runs fast

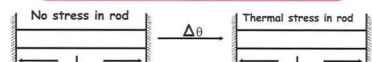
1) Loss of time in any given time interval t ,

$$\Delta t = \frac{1}{2} \alpha \Delta \theta t$$

2) Time lost by clock in a day

$$\Delta t = \frac{1}{2} \alpha \Delta \theta t = \frac{1}{2} \alpha \Delta \theta 86400 = 43200 \alpha \Delta \theta$$

Thermal Stress in a rigidly fixed rod



$$\text{Thermal Stress} = Y \alpha \Delta \theta$$

$$\text{Thermal Force} = Y \alpha A \Delta \theta$$

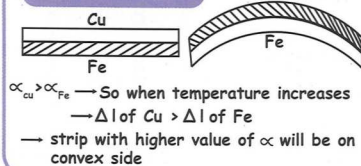
Y - Young's Modulus
 α - coefficient of linear expansion
 $\Delta \theta$ - temperature change
 A - Area of rod

ERROR IN SCALE READING DUE TO EXPANSION OR CONTRACTION

Result: At $\theta' > \theta$ True value $>$ Scale reading
 At $\theta' < \theta$ True value $<$ Scale reading

$$\text{True value} = \text{Scale reading} (1 + \alpha \Delta \theta)$$

BIMETALLIC STRIP



$\alpha_{Cu} > \alpha_{Fe} \rightarrow$ So when temperature increases
 $\rightarrow \Delta l \text{ of Cu} > \Delta l \text{ of Fe}$
 $\rightarrow$ strip with higher value of α will be on convex side

EXPANSION OF CAVITY

Area of hole increases. Body expands on heating. Expansion of area of body is independent of shape and size of hole

SUPERFICIAL/AREAL EXPANSION

$$1. \Delta A = A \beta \Delta \theta \quad \beta - \text{coefficient of areal expansion}$$

$$2. A' = A(1 + \beta \Delta \theta)$$

$$3. \beta = \frac{\Delta A}{A \Delta T} \rightarrow \text{unit} \rightarrow /^{\circ}\text{C or } /^{\circ}\text{K, dimension-}[K^{-1}]$$

$$4. \beta = 2\alpha$$

CUBICAL EXPANSION/VOLUME EXPANSION

$$1. \Delta V = V \gamma \Delta \theta \quad \gamma = \text{coefficient of volumetric expansion}$$

$$2. V' = V(1 + \gamma \Delta \theta)$$

$$3. \gamma = \frac{\Delta V}{V \Delta \theta} \rightarrow \text{unit} \rightarrow /^{\circ}\text{C or } /^{\circ}\text{K}$$

$$k, \text{ dimension-}[K^{-1}] \quad \alpha : \beta : \gamma = 1 : 2 : 3$$

$$4. \gamma = 3\alpha$$

Variation of density with temperature

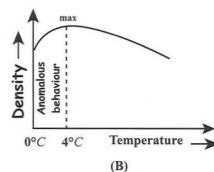
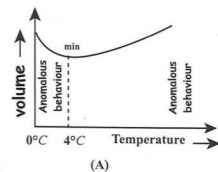
$$\text{Density} \propto \frac{1}{\text{Volume}}$$

$$V' = V(1 + \gamma \Delta \theta)$$

$$\text{then } \rho' = \rho(1 - \gamma \Delta \theta)$$

ANOMALOUS EXPANSION OF WATER

1. Water has maximum density at 4°C (minimum volume)
2. On heating, $0^{\circ}\text{C} \rightarrow 4^{\circ}\text{C}$, water contracts
 $4^{\circ}\text{C} \rightarrow$ above, water expands



CALORIMETRY

$$1 \text{ calorie} = 4.2 \text{ J}$$

Heat Supplied (ΔQ)

change in temperature of body

$$1. \Delta Q = ms \Delta T$$

s - specific heat capacity

$$\text{SI unit} - \frac{\text{Joule}}{\text{kg Kelvin}} \rightarrow \text{J kg}^{-1} \text{K}^{-1}$$

$$2. s_{\text{water}} = 1 \frac{\text{cal}}{\text{g}^{\circ}\text{C}} = 4.2 \frac{\text{J}}{\text{g}^{\circ}\text{C}} = 4200 \frac{\text{J}}{\text{kg}^{\circ}\text{C}}$$

$$s_{\text{ice}} = \frac{1}{2} \frac{\text{cal}}{\text{g}^{\circ}\text{C}} = 2.1 \frac{\text{J}}{\text{g}^{\circ}\text{C}} = 2100 \frac{\text{J}}{\text{kg}^{\circ}\text{C}}$$

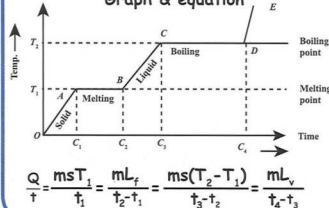
change of state of body

Melting
 $\Delta Q = mL_f$
 L_f - Latent heat of fusion

Boiling
 $\Delta Q = mL_v$
 L_v - Latent heat of vaporization

$$L_f = L_{\text{ice}} = 80 \frac{\text{cal}}{\text{g}} = 80 \times 4.2 \frac{\text{J}}{\text{g}} = 80 \times 4200 \frac{\text{J}}{\text{kg}}$$

$$L_v = L_{\text{steam}} = 540 \frac{\text{cal}}{\text{g}} = 540 \times 4.2 \frac{\text{J}}{\text{g}} = 540 \times 4200 \frac{\text{J}}{\text{kg}}$$

Heat supplied at constant rate
Graph & equation

$$\frac{Q}{t} = \frac{msT_1}{t_1} = \frac{mL_f}{t_2 - t_1} = \frac{ms(T_2 - T_1)}{t_3 - t_2} = \frac{mL_v}{t_4 - t_3}$$

if specific heat is variable

$$S = f(T) \quad T_1 \rightarrow T_2 \quad \Delta Q = \int_{T_1}^{T_2} msdT$$

HEAT CAPACITY

Heat capacity = mass $\times$ specific heat capacity

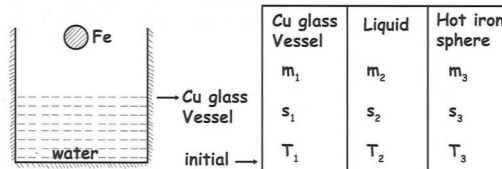
$$\text{Unit} = \frac{\text{cal}}{^{\circ}\text{C}}, \text{ SI unit is } \frac{\text{J}}{\text{K}}$$

WATER EQUIVALENT

The mass of water that will absorb or lose the same quantity of heat as a given substance will do for same change in temperature

$$m_w s_w = m_b s_b \quad w = \text{water} \quad b = \text{body}$$

PRINCIPLE OF CALORIMETRY



Heat lost by the hotter bodies = Heat gained by colder bodies

$$Q_s = Q_c + Q_2$$

Final equilibrium temperature,

$$T_{\text{eq}} = \frac{m_1 s_1 T_1 + m_2 s_2 T_2 + m_3 s_3 T_3}{m_1 s_1 + m_2 s_2 + m_3 s_3} = \frac{\sum msT}{\sum ms}$$

REAL AND APPARENT EXPANSION OF LIQUID

1. Apparent expansion of liquid $\rightarrow$ (Real expansion of liquid - expansion of solid in which liquid is contained)
2. Apparent change in volume

$$\Delta V_{\text{apparent}} = V_0 \gamma_{\text{apparent}} \Delta \theta$$

$$\Rightarrow \Delta V_{\text{apparent}} = V_0 (\gamma_l - \gamma_s) \Delta \theta$$

$$\Rightarrow \Delta V_{\text{apparent}} = V_0 (\gamma_l - 3\alpha_s) \Delta \theta$$

$$\Rightarrow \gamma_{\text{apparent}} = \gamma_l - 3\alpha_s$$

γ_l - Real expansion of liquid

α_s - coefficient of linear expansion of solid

ICE-WATER SYSTEM

Problem solving methodology

1. m_1 gram ice [$- \theta_1^{\circ}\text{C}$] mixed with m_2 gram water [$\theta_2^{\circ}\text{C}$]

2. Convert $- \theta_1^{\circ}\text{C}$ ice $\rightarrow 0^{\circ}\text{C}$ ice

$$\Delta Q_1 = m_1 s_{\text{ice}} \theta_1$$

3. Convert 0°C ice $\rightarrow 0^{\circ}\text{C}$ water

$$\Delta Q_2 = m_1 L_f$$

4. Convert $\theta_2^{\circ}\text{C}$ water $\rightarrow 0^{\circ}\text{C}$ water

$$\Delta Q_3 = m_2 s_{\text{water}} \theta_2$$

check

$$\Delta Q_3 > < \text{ or } \Delta Q_1 + \Delta Q_2$$

$$\Delta Q_3 > \Delta Q_1 + \Delta Q_2$$

1. Whole ice melts into water

2. Additional heat [$\Delta Q = \Delta Q_3 - (\Delta Q_1 + \Delta Q_2)$] is used to increase the temperature of system from 0°C

3. Final temperature can be found out by

$$\Delta Q = M_{\text{total}} s_{\text{water}} T$$

$$\Delta Q_3 < \Delta Q_1 + \Delta Q_2$$

1. Only m' g of ice melts

2. Mass of ice melted can be found by [m = mass of ice melted]
 $m L_f = Q_{\text{melted}}$

3. Final temperature is 0°C

CONVERSION OF MECHANICAL ENERGY TO HEAT ENERGY

1. Potential energy to heat energy

$$\Delta U = mgh \xrightarrow{\text{converts to heat}} \Delta Q = m' L_f \quad [m' = \text{mass of substance melted/vaporized}]$$

When equating, multiply ΔQ with 4200 J if L_f is in cal/g
 i.e., $mgh = m' L_f \times 4200$

2. Kinetic energy to Heat energy

$$K.E. = \frac{1}{2} mv^2 \xrightarrow{\text{converts to heat}} \Delta Q = m' L_f \quad [m' = \text{mass of substance melted/vaporized}]$$

If L_f is in $\frac{\text{calorie}}{\text{g}}$

then

$$\frac{1}{2} mv^2 = m' L_f \times 4200$$

HEAT TRANSFER

1. Conduction:

Heat flows from hot end to cold end. Medium is necessary.

Slow process.

$$\frac{dQ}{dt} = K A \frac{d\theta}{dx}$$

Unit of 'K' = $\frac{\text{watt}}{\text{metre}^{\circ}\text{C}}$ or $\frac{\text{watt}}{\text{metre K}}$

'K' depends on the nature of material

$$\frac{dQ}{dt} = \text{Rate of flow of heat}$$

A = Area of cross section

$$\frac{d\theta}{dx} = \text{Temperature gradient}$$

K = coefficient of thermal conductivity

Facts :
 Calorimeter - A device for measurement of amount of heat involved in a process.

THERMAL PROPERTIES OF MATTER

OHM'S LAW OF CONDUCTION

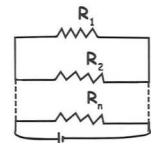
Electrical Conduction

- 1) current, $I = \frac{dq}{dt}$
- 2) $I = \frac{\Delta V}{R}$ ($\Delta V = V_{\text{high}} - V_{\text{low}}$)
- 3) electrical resistance, $R = \frac{\rho l}{A}$
- 4) $I = \frac{V_1 - V_2}{R} = \frac{(V_1 - V_2)}{\frac{\rho l}{A}} = \frac{\sigma A (V_1 - V_2)}{l}$
- 5) Combination of resistors
 - i) Series Combination

$$R_{\text{eq}} = R_1 + R_2 + \dots + R_n$$

Here 'I' is same in all resistors

- ii) Parallel Combination

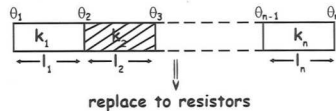


$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Here $(V_1 - V_2)$ is same for all resistors

Thermal Conduction

- 1) Heat current, $H = \frac{dQ}{dt}$
- 2) $H = \frac{\theta_1 - \theta_2}{R} = \frac{\Delta \theta}{R}$ ($\theta_1 > \theta_2$)
- 3) Thermal resistance, $R = \frac{l}{KA}$
- 4) $H = \frac{\theta_1 - \theta_2}{R} = \frac{\theta_1 - \theta_2}{\frac{l}{KA}} = \frac{KA}{l} (\theta_1 - \theta_2)$
- 5) Combination of conductors
 - i) Series Combination



replace to resistors

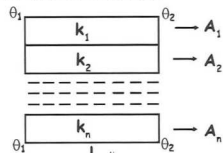
$$R_1 = \frac{l_1}{K_1 A}, R_2 = \frac{l_2}{K_2 A}$$

Find $R_{\text{eq}} = R_1 + R_2 + \dots$

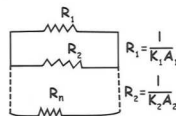
From that find ' K_{eq} '

Here, heat current, H is same in all conductors

- ii) Parallel Combination



replace with resistors



$$\text{Find } \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Here, Temp Difference is same for all conductors

CONVECTION

Requires a medium. Actual movement of fluid. Occurs naturally or forced.

Natural convection takes place due to the effect of gravity

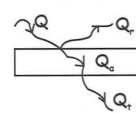
Applications:

Sea Breeze
Wind blows from sea to land during day time

Land Breeze
Wind blows from land to sea during night

RADIATION

Absorptive, reflective and Transmittive power



$$a + r + t = 1$$

$$\text{Absorptive power}(a) = \frac{Q_a}{Q} = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

$$\text{Reflective power}(r) = \frac{Q_r}{Q} = \frac{\text{Energy reflected}}{\text{Energy incident}}$$

$$\text{Transmittive power}(t) = \frac{Q_t}{Q} = \frac{\text{Energy transmitted}}{\text{Energy incident}}$$

EMISSIVE POWER/INTENSITY OF THERMAL RADIATION

$$\text{Emissive power}(E) = \frac{\text{Energy radiated}}{\text{area} \times \text{time}} \quad \text{unit} \rightarrow \frac{\text{Watt}}{\text{m}^2}$$

$$\text{Spectral emissive power}(E_\lambda) = \frac{\text{Energy radiated}}{\text{area} \times \text{time} \times \text{wavelength}} \quad \text{unit} \rightarrow \frac{\text{Watt}}{\text{m}^3}$$

$$\text{Relation between } E \text{ \& } E_\lambda \Rightarrow E = \int_0^\infty E_\lambda d\lambda$$

EMISSIVITY (e)

$$e = \frac{\text{Energy radiated by a general body}}{\text{Energy radiated by a black body}}$$

$$\text{value of } e \Rightarrow 0 < e < 1$$

If $e = 0$, the body radiates no energy

If $e = 1$, the body is a perfect black body

KIRCHHOFF'S LAW

Ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

$$\frac{E_1}{a_1} = \frac{E_2}{a_2} = \dots = E_b$$

STEFAN'S LAW

Emissive power of a black body $\propto$ fourth power of absolute temperature and surface area of the body

$$E = \sigma AT^4 \text{ OR } \frac{\Delta Q}{\Delta t} = \sigma AT^4$$

$$\sigma \rightarrow \text{Stefan's constant} \quad \frac{\Delta Q}{\Delta t} \rightarrow \text{Radiant power}$$

$$\text{value of } \sigma \rightarrow 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

$$\text{Dimension} \rightarrow [\sigma] = \text{MT}^{-3} \text{K}^{-4}$$

$$\text{For ordinary body } E = e \sigma T^4$$

$$\frac{\Delta Q}{\Delta t} = e \sigma A T^4 \quad e = \text{emissivity}$$

In the presence of a surrounding.
(T_0 = Surrounding temperature)
For black body,

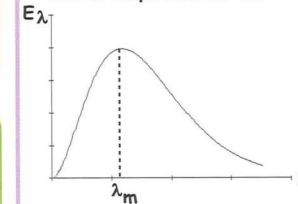
$$E = \sigma A (T^4 - T_0^4)$$

In the presence of a surrounding.
(T_0 = Surrounding temperature)
For general body,

$$E = e \sigma A (T^4 - T_0^4)$$

WIEN'S LAW

Wien's displacement law



$$\lambda_m \propto \frac{1}{T}$$

$$\lambda_m T = b$$

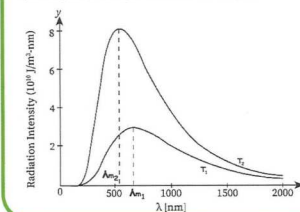
b = Wien's constant

$$\text{Hence } \frac{A_1}{A_2} = \left[\frac{T_1}{T_2} \right]^4$$

$$b = 2.89 \times 10^{-3} \text{ mK}$$

$$\lambda_m T_1 = \lambda_m T_2 \quad \text{Area under the graph, } A = \int_0^\infty E_\lambda d\lambda = E \sigma T^4 \quad [\text{Dimensions}] = [b] = [\text{LK}]$$

"As the temperature of the body increases, the wavelength at which the spectral intensity (E_λ) is maximum shift towards left."



$$T_1 < T_2$$

$$\lambda_{m1} > \lambda_{m2}$$

NEWTON'S LAW OF COOLING

Rate of cooling $\propto$ excess temperature of the body over the surrounding.

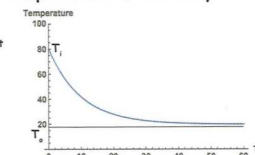
$$\frac{dT}{dt} \propto (T - T_0)$$

T = Temperature of body

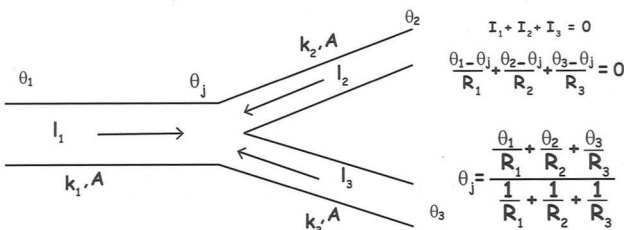
T_0 = Temperature of surrounding

T_i = initial temperature of the body

$$\frac{T - T_0}{T_i - T_0} = e^{-kt}$$



TEMPERATURE OF INTERMEDIATE JUNCTION



$$I_1 + I_2 + I_3 = 0$$

$$\frac{\theta_1 - \theta_j}{R_1} + \frac{\theta_2 - \theta_j}{R_2} + \frac{\theta_3 - \theta_j}{R_3} = 0$$

$$\theta_j = \frac{\frac{\theta_1}{R_1} + \frac{\theta_2}{R_2} + \frac{\theta_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

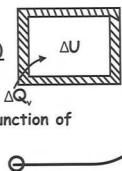
Internal Energy(U)

$$(1) \Delta U = \Delta Q_v = nC_v \Delta T = n \frac{f}{2} R \Delta T$$

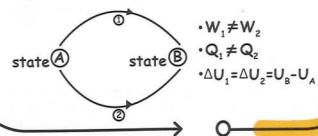
$$= \frac{nR \Delta T}{\gamma - 1} = \frac{\Delta(PV)}{\gamma - 1}$$

$$= \frac{P_2 V_2 - P_1 V_1}{\gamma - 1}$$

(Internal Energy is only the function of temperature of the gas)



- First law of T.D $\Rightarrow Q = \Delta U + W$
- $Q, W \Rightarrow$ path functions
- $\Delta U \Rightarrow$ state function



Isothermal process $\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$

Cyclic process

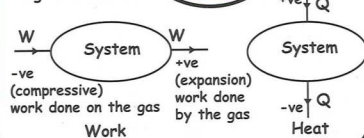
$A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$

$$\Delta U = U_A - U_A = 0$$

WORK

Work done:
path function
 $W = \int P dv$
Unit: Joule(J)

Sign Convention

**HEAT**

Heat: path function
 $\Rightarrow$ unit: calorie/Joule

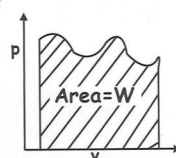
For any process,
 $Q = nC \Delta T$

where, $C =$ Specific heat capacity for the process

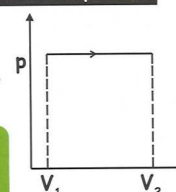
Adiabatic process $\Rightarrow \Delta Q = 0$ [No heat transfer]

At constant volume $\Rightarrow Q_v = \Delta U = nC_v \Delta T$

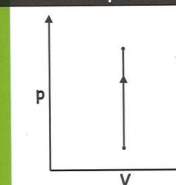
At constant pressure $\Rightarrow Q_p = \Delta U + W = nC_p \Delta T$

Work done from P-V Graph

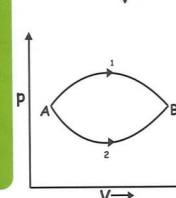
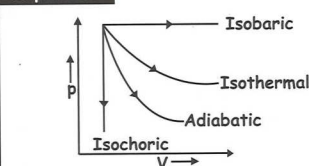
Area under P-V diagram gives work done by the gas

Isobaric process

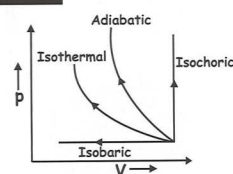
$$W = P(V_2 - V_1)$$

Isochoric process

$$W = P \Delta V = 0$$

**Expansion**

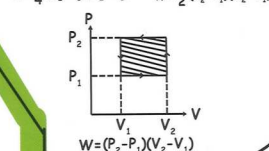
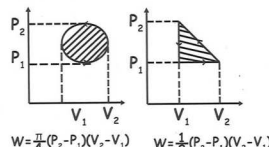
$$W_{\text{isobaric}} > W_{\text{isothermal}} > W_{\text{adiabatic}} > W_{\text{isochoric}}$$

Compression

$$W_{\text{adiabatic}} > W_{\text{isothermal}} > W_{\text{isobaric}} > W_{\text{isochoric}}$$

Cyclic process

- W are inside the graph
- For clockwise process, $W = +ve$
- For anti-clockwise process, $W = -ve$

**Thermodynamic processes****01****Adiabatic process**

$Q = 0$ [no exchange of heat]

Rapid or spontaneous process/insulated vessel

$$Q = \Delta U + W$$

$$Q = 0 \Rightarrow \Delta U = -W$$

Compression

$$W = +ve \Rightarrow \Delta U = -ve$$

$\Delta U \uparrow \Rightarrow$ Temperature $\uparrow$
 $\Rightarrow$ Pressure $\uparrow$

Expansion

$$W = -ve \Rightarrow \Delta U = +ve$$

$\Delta U \downarrow \Rightarrow$ Temperature $\downarrow$
 $\Rightarrow$ Pressure $\downarrow$

Equation of state

$$PV^\gamma = \text{constant}$$

$$TV^{\gamma-1} = \text{constant}$$

$$PT^{\frac{\gamma}{\gamma-1}} = \text{constant}$$

Work done by the gas

$$W = -\Delta U = nC_v(T_1 - T_2)$$

$$= n \frac{f}{2} R(T_1 - T_2)$$

$$W = \frac{nR}{\gamma-1}(T_1 - T_2)$$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma-1}$$

Slope of adiabatic process

$= \gamma \times$ slope of isothermal process

specific heat of gas $\Rightarrow C = 0$

$$C = \frac{Q}{\Delta T} \Rightarrow Q = 0$$

$$C = 0$$

02**Isothermal process**

$$\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$$

eg:- perfectly conducting slow process

$$Q = \Delta U + W$$

$$Q = W$$

equation of states $\Rightarrow PV = \text{Constant}$
 $P_1 V_1 = P_2 V_2$

Workdone by the gas
 $W = 2.303 nRT \log \left(\frac{V_2}{V_1} \right)$

$$W = 2.303 nRT \log \left(\frac{P_1}{P_2} \right)$$

Slope:

Slope of adiabatic process
 $= \gamma \times$ slope of isothermal process
specific heat $C = \infty$

03**Isochoric process**

$$\Delta P = 0$$

$$Q = \Delta U + W$$

equation of state $\Rightarrow V \propto T$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Work done by the gas

$$W = P \Delta V = P(V_2 - V_1) = nR(T_2 - T_1)$$

specific heat

$$\Rightarrow C_p = \left(1 + \frac{f}{2} \right) R$$

$$= \frac{\gamma R}{\gamma - 1}$$

04**Isochoric process**

$\Delta V = 0$ or $V = \text{constant}$

equation of state $\Rightarrow P \propto T \Rightarrow \frac{P_1}{P_2} = \frac{T_1}{T_2}$

Work done by the gas

$$\Delta V = 0 \Rightarrow W = 0$$

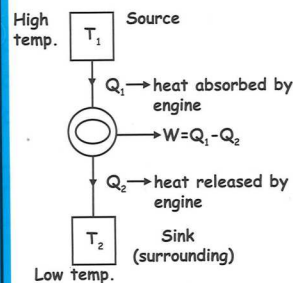
specific heat

$$\Rightarrow C_v = \frac{f}{2} R$$

$$= \frac{R}{\gamma - 1}$$

Heat Engine

'Device that converts heat into work'



Low temp.

efficiency(η)
 $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

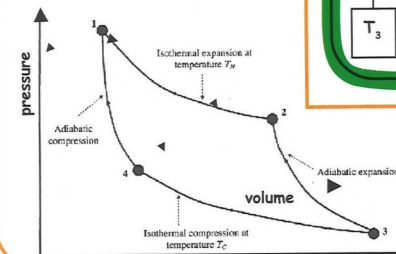
$$\eta_{\text{Carnot}} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

$\eta_{\text{max}} \Rightarrow$ When $Q_2 = 0$ or $T_2 = 0K$ (not possible)

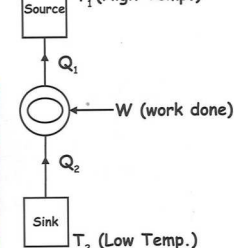
Carnot Engine

$\rightarrow$ Ideal engine

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

**Refrigerator and heat pump**

Source T_1 (High Temp.)



Coefficient of performance of refrigerator(β)

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \quad \beta = \frac{T_2}{T_1 - T_2}$$

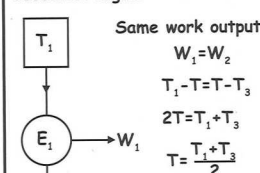
Coefficient of performance of heat pump

$$\beta = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

Relationship between

$$\beta = \frac{1}{1 - \eta}$$

$$(COP)_{\text{heat pump}} = 1 + (COP)_{\text{refrigerator}}$$

Cascaded engine

Same work output

$$W_1 = W_2$$

$$T_1 - T_2 = T_2 - T_3$$

$$2T_2 = T_1 + T_3$$

$$T_2 = \frac{T_1 + T_3}{2}$$

Same efficiency

$$1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$$

$$T_2^2 = T_1 T_3$$

$$T_2 = \sqrt{T_1 T_3}$$

THERMODYNAMICS

01

DEGREES OF FREEDOM

- For monoatomic gas, $f = 3$
- For diatomic gas,
 - (a) at room temperature, $f = 5$
 - (b) at high temperature, $f = 7$
- For triatomic gas,
 - (a) Linear $f = 5$
 - (b) Non-linear $f = 6$
- For each vibrational mode, $f = 2$

Q1

Ideal gas is composed of polyatomic molecule that has 4 vibrational modes. Total degrees of freedom is
a) 12 b) 14 c) 8 d) 6

02

SPECIFIC HEAT CAPACITY

- a) $C_p - C_v = R$
- b) $C_p - C_v = \frac{R}{M}$ (specific heat per unit mass) Mono- $\gamma = \frac{5}{3}$
- c) $C_v = \frac{R}{\gamma - 1} = \frac{f}{2} R$ Dia- $\gamma = \frac{7}{5}$
- d) $C_p = \frac{\gamma R}{\gamma - 1} = \left(1 + \frac{f}{2}\right) R$ Tri- $\gamma = \frac{4}{3}$
- e) $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

Q2

If C_p and C_v denote the specific heats of unit mass of nitrogen at constant pressure and volume respectively, then

- a) $C_p - C_v = \frac{R}{28}$ b) $C_p - C_v = \frac{R}{14}$ c) $C_p - C_v = \frac{R}{7}$ d) $C_p - C_v = R$

03

MIXING OF GASES

$$C_{V_{mix}} = \frac{n_1 C_{v1} + n_2 C_{v2} + \dots}{n_1 + n_2 + \dots}$$

$$C_{P_{mix}} = \frac{n_1 C_{p1} + n_2 C_{p2} + \dots}{n_1 + n_2 + \dots}$$

$$\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}}$$

Q3

Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_p/C_v value will be:

- a) 19/13 b) 67/45 c) 40/27 d) 23/15

04

LAW OF EQUIPARTITION OF ENERGY

Energy for each molecule per $f = \frac{1}{2} K_B T$
 Total energy for molecule = $\frac{f}{2} K_B T$
 Monoatomic Molecule = $\frac{3}{2} K_B T$
 Total energy for a mole = $\frac{f}{2} R T$
 Total energy for n moles = $n \frac{f}{2} R T$
 Monoatomic = $\frac{3}{2} R T$ (1 mole)
 Diatomic = $\frac{5}{2} R T$ (1 mole)
 Translatory Kinetic energy = $\frac{3}{2} R T$ (1 mole, $f = 3$)

Q4

A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at temperature T . Neglecting all vibrational modes, the total internal energy of the system is

- a) $4RT$ b) $15RT$ c) $9RT$ d) $11RT$

06

VELOCITY OF GAS

$$V_{mp} : V_{avg} : V_{rms} = 1 : 1.13 : 1.225$$

07

MEAN FREE PATH

Average distance travelled by molecules between two successive collisions
 $\lambda \propto \frac{1}{d^2}$
 $\lambda \propto \frac{1}{r^2}$
 $\lambda \propto \frac{1}{P}$
 d = diameter of molecules.
 n = no. of molecules per unit volume

05

FIRST LAW OF THERMODYNAMICS

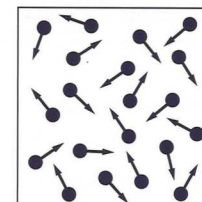
$$Q_p = \Delta U + W$$

$$\Delta U = n C_v \Delta T$$

$$W = \int P dv$$

$$\frac{\Delta U}{Q_p} = \frac{1}{\gamma}$$

$$\frac{W}{Q_p} = 1 - \frac{1}{\gamma}$$



Root Mean square speed:

Square root of mean of square of speeds of different molecules.

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + \dots + V_n^2}{n}}$$

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3K_B T}{m}}$$

Average Speed:

Arithmetic mean of speed of molecules of gas at given temperature.

$$V_{avg} = \frac{|V_1| + |V_2| + \dots + |V_n|}{n}$$

$$V_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8P}{\pi \rho}}$$

Most probable speed:

Speed possessed by maximum number of molecules of gas.

$$V_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2K_B T}{m}}$$

Q5

Consider a gas of triatomic molecules. The molecules are assumed to be triangular, made up of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is:

- a) $\frac{5}{2} RT$ b) $\frac{3}{2} RT$ c) $\frac{9}{2} RT$ d) $3RT$

Q6

The rms speeds of the molecules of Hydrogen, Oxygen & Carbon dioxide at the same temperature are V_H , V_O and V_C respectively, then:

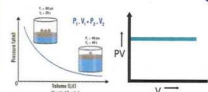
- a) $V_H > V_O > V_C$ b) $V_C > V_O > V_H$
- c) $V_H = V_O > V_C$ d) $V_H = V_O = V_C$

Q7

The mean free path of molecules of gas, (radius r) is inversely proportional to

- a) r^3 b) r^2
- c) r d) $\sqrt{r}$

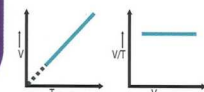
BOYLE'S LAW



$PV = \text{constant}$, if $T = \text{constant}$

$P_1 V_1 = P_2 V_2$, when gas changes its state under constant temperature.

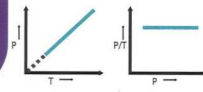
CHARLE'S LAW



$V \propto T$; $\frac{V}{T} = \text{constant}$; $P = \text{constant}$.

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$, when gas change its state under constant pressure.

GAY LUSSAC'S LAW



$P \propto T$; $\frac{P}{T} = \text{constant}$; $V = \text{constant}$.

$\frac{P_1}{T_1} = \frac{P_2}{T_2}$, when gas changes its state under constant Volume.

PRESSURE OF GAS

$$PV = \frac{1}{3} mn V_{rms}^2 = \frac{1}{3} mn \overline{v^2}$$

Relation between pressure and Kinetic Energy.

$$E = \frac{3}{2} PV$$

IDEAL GAS LAW

$$PV = nRT$$

$$R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$$

$$\rho = \frac{PM}{RT}$$

Specific heat of Solids = $3R$
 WATER = $9R$

Position and Velocity of SHM

$$x = A \sin(\omega t + \Phi_0)$$

$$v = A \omega \cos(\omega t + \Phi_0)$$

$$v = \omega \sqrt{A^2 - x^2}$$

a) $x = 0 \rightarrow v_{\max} = A\omega$

b) $x = \frac{A}{2} \rightarrow v = \frac{\sqrt{3}A\omega}{2}$

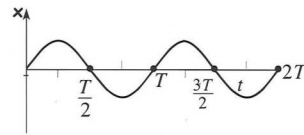
c) $x = \frac{A}{\sqrt{2}} \rightarrow v = \frac{A\omega}{\sqrt{2}}$

d) $x = A \rightarrow v = 0$

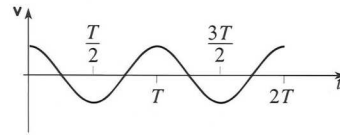
Graphical Representation of Position and Velocity

Start from mean position, $\Phi_0 = 0$

$$x = A \sin(\omega t)$$

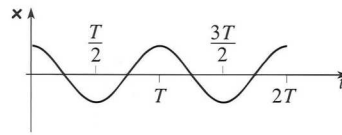


$$v = A \omega \cos(\omega t)$$

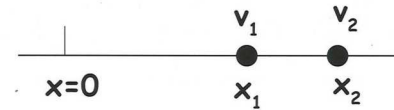
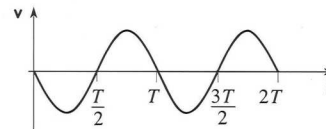


Start from extreme position $\Phi_0 = \frac{\pi}{2}$

$$x = A \cos(\omega t)$$



$$v = -A \omega \sin(\omega t)$$



$$A = \sqrt{\frac{V_1^2 x_2^2 - V_2^2 x_1^2}{V_1^2 - V_2^2}} \quad \omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}}$$

Calculation of Time period and amplitude

$$v_{\max} = A\omega$$

$$a_{\max} = A\omega^2$$

$$\omega = \frac{a_{\max}}{v_{\max}}$$

$$A = \frac{v_{\max}^2}{a_{\max}}$$

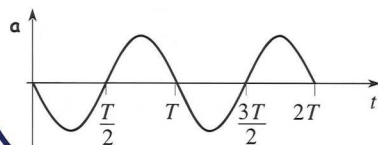
$$\frac{2\pi}{T} = \frac{a_{\max}}{v_{\max}}$$

$$T = 2\pi \frac{v_{\max}}{a_{\max}}$$

Graphical Representation of Acceleration

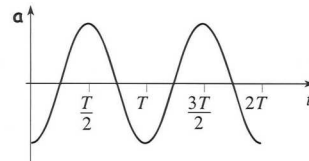
Start from mean position, $\Phi_0 = 0$

$$a = -A\omega^2 \sin(\omega t)$$



Start from extreme position, $\Phi_0 = \frac{\pi}{2}$

$$a = -A\omega^2 \cos(\omega t)$$



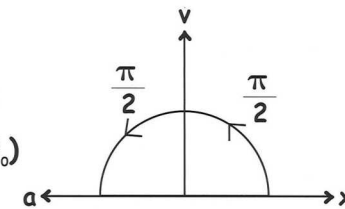
Acceleration of SHM

$$x = A \sin(\omega t + \Phi_0)$$

$$v = A \omega \cos(\omega t + \Phi_0)$$

$$a = -A \omega^2 \sin(\omega t + \Phi_0)$$

$$a_{\max} = -A \omega^2$$



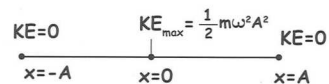
Phase difference between x and $v = \frac{\pi}{2}$

Phase difference between v and $a = \frac{\pi}{2}$

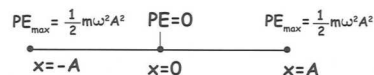
Phase difference between x and $a = \pi$

Energy of SHM

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$



$$P.E = \frac{1}{2}m\omega^2x^2$$



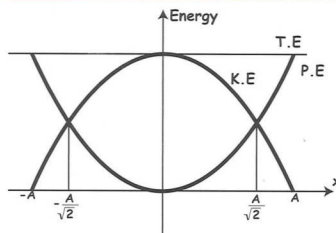
$$\text{Total mechanical energy, } E = \frac{1}{2}m\omega^2A^2 \\ = \text{Constant}$$

$$1) x = \frac{A}{\sqrt{2}} \\ K.E = P.E = \frac{E}{2}$$

$$2) x = \frac{A}{2} \\ K.E = \frac{3E}{4}, P.E = \frac{E}{4}$$

$$3) x = 0 \\ K.E = E \quad P.E = 0$$

$$4) x = A \\ K.E = 0 \quad P.E = E$$



$$\text{Total mechanical energy (E)} = \frac{1}{2}m\omega^2A^2$$

Note: In SHM, if particle oscillates with frequency ω , then the K.E & P.E oscillate with 2ω

Two particles executing SHM meet at $x = \frac{\sqrt{3}A}{2}$

for particle A

$$x = \frac{\sqrt{3}A}{2}, t=0 \Rightarrow \frac{\sqrt{3}A}{2} = A \sin \Phi_A$$

$$\Phi_A = 60^\circ = \frac{\pi}{3}$$

for particle B

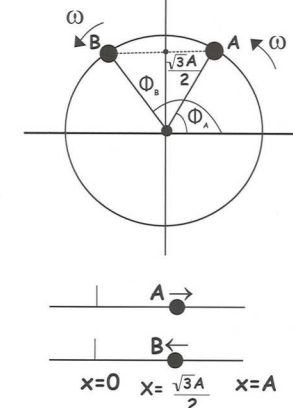
$$x = \frac{\sqrt{3}A}{2}, t=0 \Rightarrow \frac{\sqrt{3}A}{2} = A \sin \Phi_B$$

$$\Phi_B = 90^\circ + 30^\circ$$

$$\frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

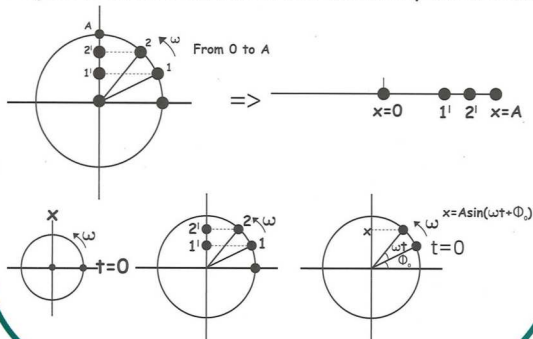
Phase difference between particles

$$\Phi = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3} = 60^\circ$$

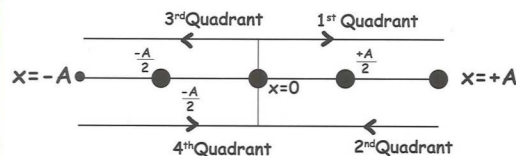


PROJECTION OF CIRCULAR MOTION

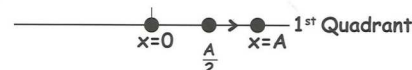
Projection/shadow of uniform circular motion on y axis is SHM



INITIAL PHASE FROM POSITION & DIRECTION



Eg: Particle is at $x = \frac{A}{2}$ [$t=0$] and moves towards A



1st Quadrant

1) Particle is at $x = \frac{A}{2}$ [$t=0$] and moves towards A

$$x = \frac{A}{2}, t=0 \Rightarrow \frac{A}{2} = A \sin \Phi_0$$

$$\Phi_0 = 30^\circ = \frac{\pi}{6}$$

$$x = A \sin(\omega t + \frac{\pi}{6})$$

2) Particle is at $x = \frac{A}{\sqrt{2}}$ [$t=0$] and moves towards A

$$x = \frac{A}{\sqrt{2}}, t=0 \Rightarrow \frac{A}{\sqrt{2}} = A \sin \Phi_0$$

$$\Phi_0 = 45^\circ = \frac{\pi}{4}$$

$$x = A \sin(\omega t + \frac{\pi}{4})$$

3) Particle is at $x = \frac{\sqrt{3}A}{2}$ [$t=0$] and moves towards A

$$x = \frac{\sqrt{3}A}{2}, t=0 \Rightarrow \frac{\sqrt{3}A}{2} = A \sin \Phi_0$$

$$\Phi_0 = 60^\circ = \frac{\pi}{3}$$

$$x = A \sin(\omega t + \frac{\pi}{3})$$

2nd Quadrant

1) Particle is at $x = \frac{A}{2}$ [$t=0$] and moves towards O

$$x = \frac{A}{2}, t=0 \Rightarrow \frac{A}{2} = A \cos \Phi_0$$

$$\Phi_0 = 90^\circ + 60^\circ = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$$

$$x = A \cos(\omega t + \frac{5\pi}{6})$$

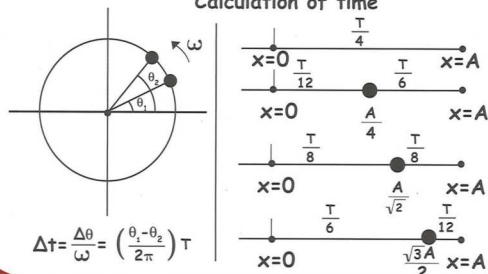
2) Particle is at $x = \frac{A}{\sqrt{2}}$ [$t=0$] and moves towards O

$$x = \frac{A}{\sqrt{2}}, t=0 \Rightarrow \frac{A}{\sqrt{2}} = A \cos \Phi_0$$

$$\Phi_0 = 90^\circ + 45^\circ = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

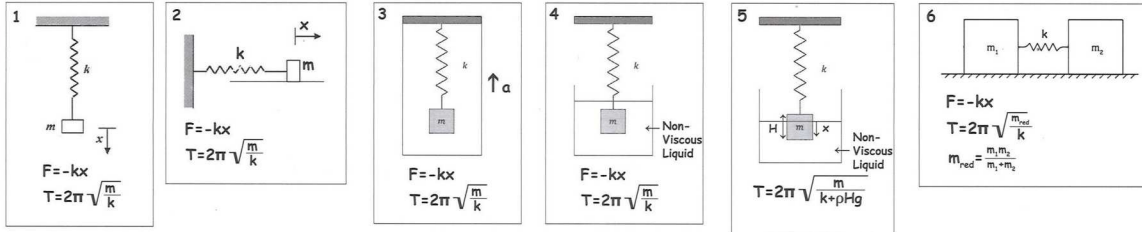
$$x = A \cos(\omega t + \frac{3\pi}{4})$$

Calculation of time

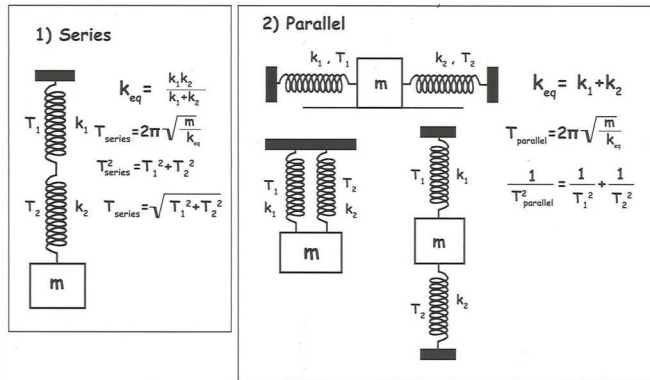


OSCILLATIONS 02

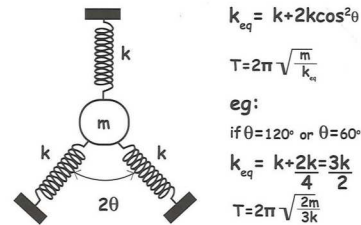
TIME PERIOD OF S.H.M



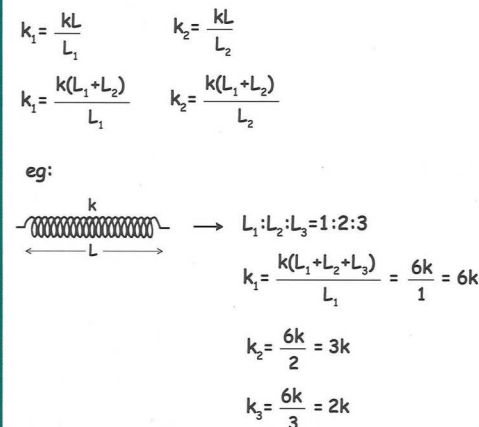
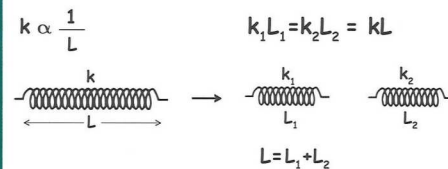
COMBINATIONS OF SPRINGS



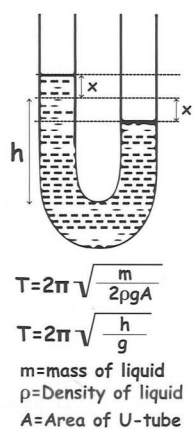
SPECIAL CASE



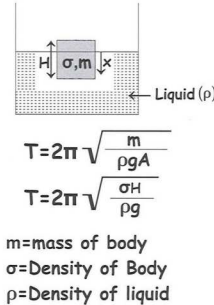
CUTTING OF SPRINGS



OSCILLATION OF LIQUID COLUMN



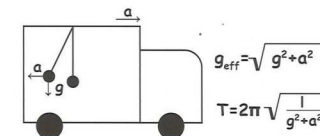
OSCILLATION OF FLOATING BODY



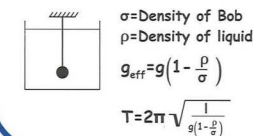
DIFFERENTIAL EQUATION OF S.H.M

$ma = -kx$
 $m \frac{d^2x}{dt^2} = -kx$
 $\frac{d^2x}{dt^2} = \frac{-k}{m} x$
 $\frac{d^2x}{dt^2} + \frac{k}{m} x = 0$
 $\frac{d^2x}{dt^2} + \omega^2 x = 0$
 where, $\omega^2 = \frac{k}{m}$ $\omega = \sqrt{\frac{k}{m}}$
 Solving,
 $x = A \sin(\omega t + \Phi_0)$
 $A = \text{Amplitude of SHM}$
 $\Phi_0 = \text{Initial phase angle}$
 Eg: $\begin{cases} m = 4 \text{ Kg} \\ k = 320 \text{ N/m} \end{cases}$
 $4 \frac{d^2x}{dt^2} + 320 x = 0$
 $\frac{d^2x}{dt^2} + 80 x = 0$
 $\omega^2 = 80$ $\omega = \sqrt{80}$
 $\frac{2\pi}{T} = \sqrt{80}$
 $T = \frac{2\pi}{\sqrt{80}}$

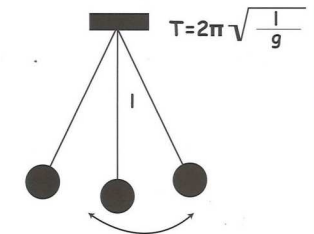
Pendulum in a truck moving with constant acceleration



Pendulum in Water



SIMPLE PENDULUM



• Second's pendulum

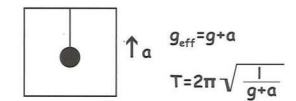
$T = 2 \text{ second}$
 $l = 1 \text{ meter}$

Concept of $g_{effective}$

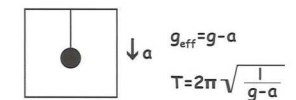
$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$

Pendulum in lift

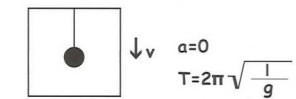
Case 1-Moving with constant upward acceleration ' a '



Case 2-Moving with constant downward acceleration ' a '



Case 3-Moving with constant velocity



Case 4-Free fall

$a = g$
 $g_{eff} = g - a = g - g = 0$ $T \rightarrow \infty$

OSCILLATIONS 03

Waves

Wave is a disturbance which carries energy and momentum from one place to another without the transport of medium.

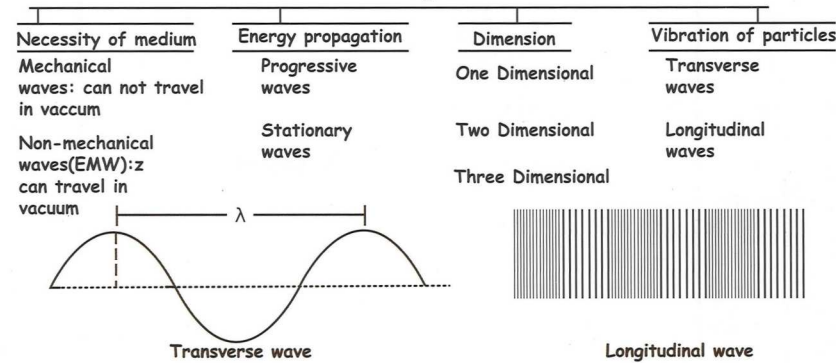
⇒ The medium should have elasticity and inertia

Characteristics of Wave

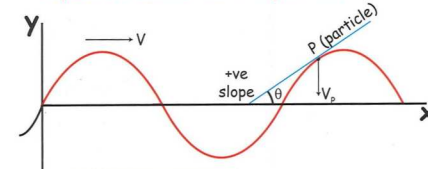
- ⇒ The particles of the medium are executing simple harmonic motion.
- ⇒ The phase of vibration of the particle keeps on changing.
- ⇒ Wave carries energy and momentum.
- ⇒ The velocity of the particle is not equal to velocity of wave.

Classification of waves

According to



All travelling waves satisfy a differential equation called wave equation.



$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2} \frac{\partial^2 y}{\partial t^2}$$

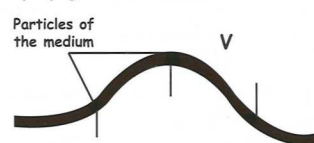
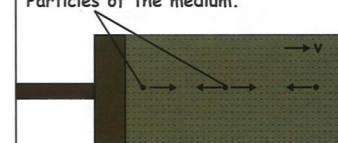
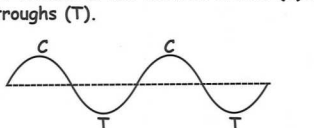
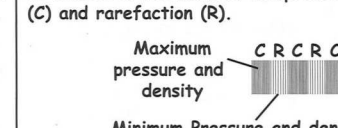
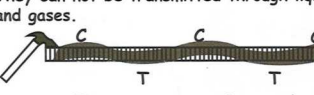
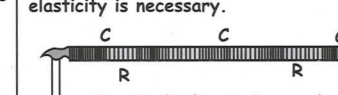
$$\frac{\partial y}{\partial t} = -\frac{\omega}{k} \times \frac{\partial y}{\partial x}$$

Velocity of particle = -Velocity of wave × Slope of the waveform

Equation of progressive wave

- (i) $y = A \sin(\omega t - kx)$
- (ii) $y = A \sin(\omega t - \frac{2\pi}{\lambda} x)$
- (iii) $y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda} \right]$
- (iv) $y = A \sin \frac{2\pi}{\lambda} (vt - x)$
- (v) $y = A \sin \omega \left(t - \frac{x}{v} \right)$
- The general equation of a plane progressive wave with initial phase is
- $$y(x, t) = A \sin(\omega t \pm kx + \phi_0)$$
- Labels in the diagram: Displacement, Amplitude, Oscillating term, phase, Initial phase, Direction, wave number, Angular frequency.

Classification of waves based on vibration of particles

Transverse waves	Longitudinal waves
Particles of the medium vibrate in a direction perpendicular to the direction of propagation of wave.	Particles of the medium vibrate in the direction of wave motion.
 Transverse wave on a string	 Longitudinal wave in a fluid
It travels in the form of crests (C) and troughs (T).	It travels in the form of compression (C) and rarefaction (R).
	
Transverse waves can be transmitted through solids. They can be setup on the surface of liquids but not inside the liquids. They can not be transmitted through liquids and gases.	These waves can be transmitted through solids, liquids and gases because for propagation of these waves, Only volume elasticity is necessary.
 Transverse-wave in a rod	 Longitudinal wave in a rod
Medium should possess the property of rigidity	Medium should possess the property of volume elasticity
Transverse waves can be polarised	Longitudinal waves can not be polarised
Movement of string of a sitar or violin, movement of the membrane of a tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.	Sound waves travelling through air, vibration of air column in organ pipes, vibration of air column above the surface of water in the tube of resonance apparatus.

Important Terms

- A = Amplitude f = Frequency
 λ = wave length T = Time period
 v = wave velocity
 ω = Angular frequency K = Wave number
 $\omega = \frac{2\pi}{T}$ or $2\pi f$ $K = \frac{2\pi}{\lambda}$

Intensity of wave

$$I = \frac{\text{Energy}}{\text{time} \times \text{Area}} = 2\pi^2 f^2 A^2 \rho v$$

$$I = \frac{P}{4\pi r^2} \quad I \propto f^2 A^2$$

velocity of particle

$$V = \omega \sqrt{A^2 - y^2}$$

$$V_{\max} = \omega A$$

If $v_{\max}$ of the particle = $n \times v_{\text{wave}}$

$$\omega A = n \times f \lambda$$

$$2\pi f A = n \times f \lambda$$

Acceleration of particle

$$a = \omega^2 y$$

$$a_{\max} = \omega^2 A$$

$$n = \frac{2\pi A}{\lambda}$$

$$\lambda = \frac{2\pi A}{n}$$

$$A = \frac{n\lambda}{2\pi}$$

Rate of Energy Transmission

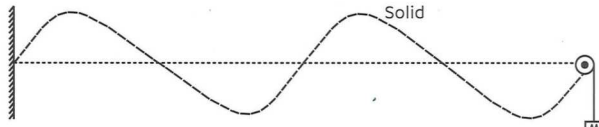
Power transmitted by a wave

$$\frac{dK}{dt} = \frac{dU}{dt} = \frac{1}{4} \mu V \omega^2 A^2$$

$$\frac{1}{2} \mu V \omega^2 A^2$$

 μ = mass per unit length of string

Velocity of Transmission of wave in a string



$$V = \sqrt{\frac{T}{\mu}}$$

$$T = Mg$$

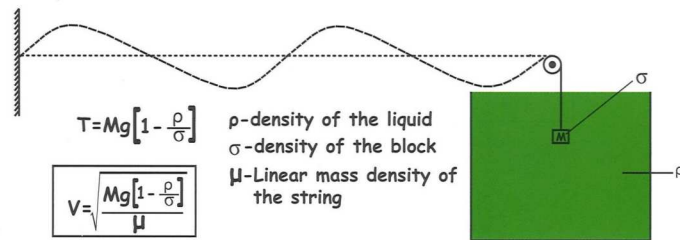
M - Mass of block
 m - mass of string
 ρ - density of material of string

$$V = \sqrt{\frac{T}{\rho \pi r^2}}$$

$$\mu = \frac{m}{l}$$

$$\mu = \rho A$$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} \times \frac{r_2}{r_1}$$

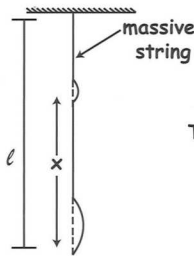
Case 1

$$T = Mg \left[1 - \frac{\rho}{\sigma} \right]$$

ρ - density of the liquid
 σ - density of the block

 μ - Linear mass density of the string

$$V = \sqrt{\frac{Mg \left[1 - \frac{\rho}{\sigma} \right]}{\mu}}$$

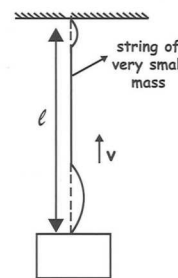
Case 2

Velocity at any Point

$$V = \sqrt{gx}$$

Time taken by the pulse to reach the top

$$t = 2\sqrt{\frac{l}{g}}$$

Case 3Velocity, $v = \sqrt{\frac{T}{\mu}}$

Time taken by the pulse to reach the top

$$t = \frac{l}{v} = l \sqrt{\frac{\mu}{T}}$$

 T = Tension in the string μ = Linear mass density of the rope

Factors affecting velocity of sound

→ Pressure

• Velocity of sound in air is independent of pressure

→ Density

$$V \propto \frac{1}{\sqrt{\rho}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

→ Temp

$$V = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

Temp Coefficient(α)

Increase in velocity of sound for 1°C
 or 1K rise in temperature of gas

$$\text{Value of } \alpha = 0.608 \frac{\text{m/s}}{^\circ\text{C}} = 0.61$$

Humidity

Humidity ↑

Speed of sound ↑

Sound travels faster in moist air
 than in dry air

Relation between Δx and $\Delta \Phi$

$$\Delta \Phi = \frac{2\pi}{\lambda} \Delta x$$

Velocity of Longitudinal Wave

$$V = \sqrt{\frac{E}{\rho}} \quad (E = \text{Elasticity of the medium; } \rho = \text{density of the medium})$$

(1) As solids are most elastic while gases are least, i.e. $E_s > E_L > E_g$;
 so the velocity of sound is maximum in solids and minimum in gases

Velocity Sound (air)

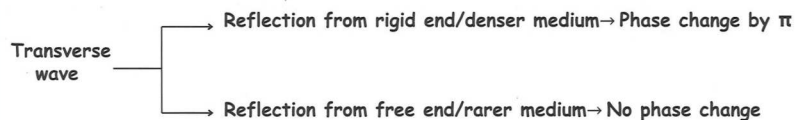
$$V = \sqrt{\frac{P}{\rho}} \quad \text{— Newton}$$

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \text{— Laplace}$$

$$\gamma = \frac{C_p}{C_v}$$

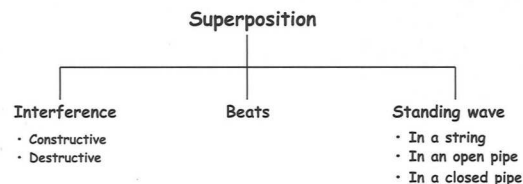
Mono atomic $\gamma = 5/3$ diatomic $\gamma = 7/5$

Reflection of Mechanical waves



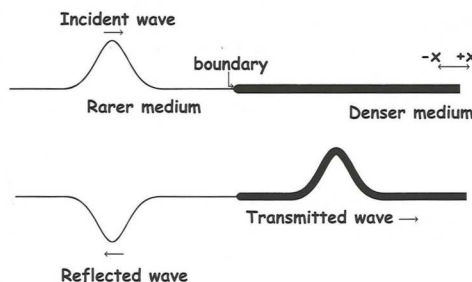
Principle of superposition

The displacement at any time due to number of waves meeting simultaneously at a point in a medium is the vector sum of individual displacements due each one of the waves at that point at same time



Waves on combination of strings

1) From rarer to denser medium

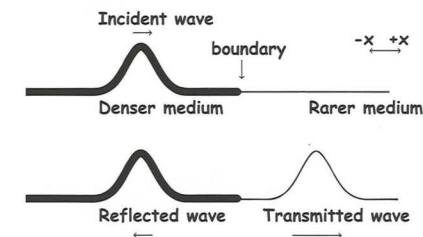


$$\text{Incident wave } y_1 = a_1 \sin(\omega t - k_1 x)$$

$$\begin{aligned} \text{Reflected wave } y_r &= a_r \sin(\omega t - k_1(-x) + \pi) \\ &= -a_r \sin(\omega t + k_1 x) \end{aligned}$$

$$\text{Transmitted wave } y_t = a_t \sin(\omega t - k_2 x)$$

2) From denser to rarer medium



$$\text{Incident wave } y_1 = a_1 \sin(\omega t - k_1 x)$$

$$\begin{aligned} \text{Reflected wave } y_r &= a_r \sin(\omega t - k_1(-x) + 0) \\ &= a_r \sin(\omega t + k_1 x) \end{aligned}$$

$$\text{Transmitted wave } y_t = a_t \sin(\omega t - k_2 x)$$

Echo



Source at distance "d" from screen

$$t = \frac{d}{v} + \frac{d}{v} = \frac{2d}{v}$$

Persistence of hearing for human ear is 0.1 sec

Conditions for echo:

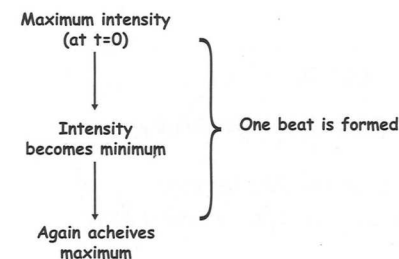
$$\text{if } t > 0.1 \Rightarrow \frac{2d}{v} > 0.1 \Rightarrow d > \frac{v}{20}$$

Beats:-

- sound waves travelling in same medium with slightly different frequencies superimpose on each other.
- The intensity of resultant sound at particular position rises and falls regularly with time.
- The phenomenon of variation of intensity of sound with time at a particular position is called beats.

Point to remember:-

1) One beat:-



Beat period:-

Time interval between two successive beats (ie. two successive maximum of sound) is called beat period.

Beat frequency:-

No. of beats produced per second

$$\text{Beat frequency:- } n = |n_1 - n_2|$$

$$\text{Beat period:- } T = \frac{1}{\text{Beat frequency}} = \frac{1}{|n_1 - n_2|}$$

Interference of sound wave

Condition:-

- Two waves of same frequency, same wavelength, same velocity
- Resultant intensities will be different from the sum of intensities of each wave separately
- This is due to the interference of waves

$$y_1 = a_1 \sin \omega t, y_2 = a_2 \sin(\omega t + \phi)$$

ϕ - Phase difference between two waves

$$\vec{y} = \vec{y}_1 + \vec{y}_2 = y = A \sin(\omega t + \theta)$$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi}$$

$$\tan \theta = \frac{a_2 \sin \phi}{a_1 + a_2 \cos \phi}$$

$$\text{Intensity} \propto A^2$$

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2, \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

i) For Constructive interference:-

$$\phi = 0, 2\pi, 4\pi, \dots \text{ OR } \phi = 2n\pi; \text{ where } n = 0, 1, 2, \dots$$

$$\Delta x = 0, \lambda, 2\lambda, \dots \text{ OR } \Delta x = n\lambda; \text{ where } n = 0, 1, 2, \dots$$

$$\begin{aligned} I_{\max} &= I_1 + I_2 + 2\sqrt{I_1 I_2} \\ &= (\sqrt{I_1} + \sqrt{I_2})^2 \propto (A_1 + A_2)^2 \end{aligned}$$

ii) For Destructive interference:-

$$\phi = \pi, 3\pi, 5\pi, \dots \text{ OR } \phi = (2n-1)\pi; \text{ where } n = 1, 2, 3, \dots$$

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \dots \text{ OR } \Delta x = (2n-1)\frac{\lambda}{2}; \text{ where } n = 1, 2, 3, \dots$$

$$\begin{aligned} I_{\min} &= I_1 + I_2 - 2\sqrt{I_1 I_2} \\ \Rightarrow I_{\min} &= (\sqrt{I_1} - \sqrt{I_2})^2 \propto (A_1 - A_2)^2 \end{aligned}$$

Determination of Unknown Frequency

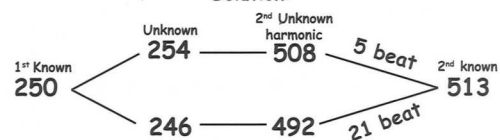
Let n_2 is the unknown frequency of tuning fork B, and this tuning fork B produce x beats per second with another tuning fork of known frequency n_1 .
As number of beat/sec is equal to the difference in frequencies of two sources, therefore $n_2 = n_1 \pm x$

By waxing	By filing
If B is loaded with wax, its frequency decreases	If B is filed, its frequency increases

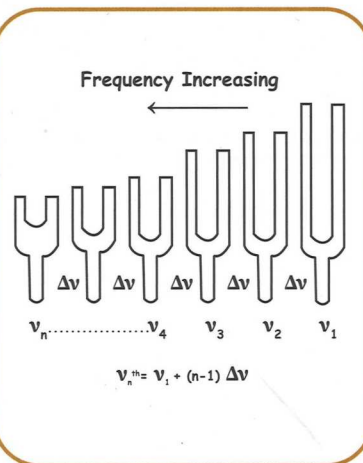
Q) A source of unknown frequency produces 4 beats/s when sounded with a source of known frequency 250 Hz. The second harmonic of the source of unknown frequency gives 5 beat/s when sounded with a source of frequency 513 Hz. The unknown frequency is?

- a) 254 Hz b) 246 Hz c) 240 Hz d) 260 Hz

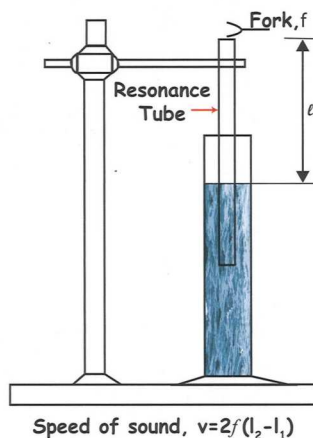
Solution:



Hence unknown frequency is 254 Hz

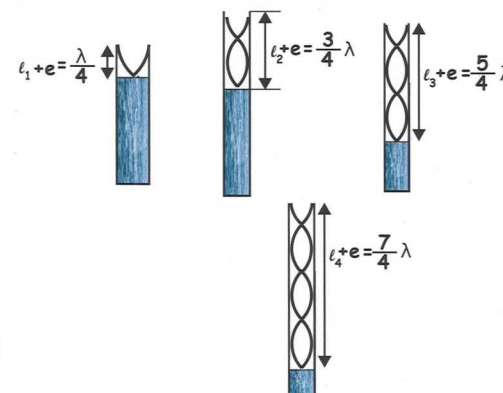


Resonance tube experiment



End correction:-

$$e = \frac{1}{2}(l_2 - 3l_1)$$



Standing Waves:

When two progressive waves (both longitudinal and transverse) having same amplitude, time period, frequency moving along a straight line in opposite direction superpose, a new wave is formed. It is called stationary Or standing wave.

Both end fixed String
 $y = 2a \sin(kx) \cos(\omega t)$
 $v_1 : v_2 : v_3 = 1 : 2 : 3$
 $n = 1, 2, 3, \dots$
 No. of nodes = $n+1$
 No. of Antinodes = n

One end fixed String
 $y = 2a \sin(kx) \cos(\omega t)$
 $v_1 : v_2 : v_3 = 1 : 3 : 5$
 $n = 1, 3, 5, \dots$
 • l = length of string or air column

Open pipe
 $y = 2a \cos(kx) \sin(\omega t)$
 $v_1 : v_2 : v_3 = 1 : 2 : 3$
 $n = 1, 2, 3, \dots$
 No. of nodes = $n+1$
 No. of Antinodes = n

Closed pipe
 $y = 2a \sin(kx) \cos(\omega t)$
 $v_1 : v_2 : v_3 = 1 : 3 : 5$
 $n = 1, 3, 5, \dots$

Note
 Distance between an adjacent node & antinode is $\frac{\lambda}{4}$
 Phase difference between 2 particles at the nodes is 180° or π
 Strain and pressure is maximum at node and minimum at antinode

Both end fixed string/open pipe	One end fixed string/closed pipe
$n=1 \Rightarrow$ Fundamental frequency / 1 st harmonic	$n=1 \Rightarrow$ Fundamental frequency / 1 st harmonic
$n=2 \Rightarrow$ First overtone / 2 nd harmonic	$n=3 \Rightarrow$ First overtone / 2 nd harmonic
$n=3 \Rightarrow$ Second overtone / 3 rd harmonic	$n=5 \Rightarrow$ Second overtone / 3 rd harmonic

Octave: The tone whose frequency is double the fundamental frequency is called as Octave.

(i) If $n_2 = 2n_1$ it means n_2 is an octave higher than n_1 or n_1 is an octave lower than n_2 .

(ii) If $n_2 = 2^3 n_1$ it means n_2 is 3-octave higher than n_1 or n_1 is 3-octave lower than n_2 .

(iii) Similarly if $n_2 = 2^n n_1$, it means n_2 is n -octave higher than n_1 or n_1 is n octave lower.

Unison: If the two frequencies are equal then vibrating bodies are said to be in unison.

Resonance: The phenomenon of making a body vibrate with its natural frequency under the influence of another vibrating body having same frequency is called resonance.

Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S. NO	Parameter	Stretched string	Open organ pipe	Closed organ pipe
1	Fundamental frequency or 1 st harmonic	Both ends fixed $n_1 = \frac{v}{2l}$	one ends fixed $n_1 = \frac{v}{4l}$	$n_1 = \frac{v}{4l}$
2	Frequency of or 2 nd harmonic	$n_2 = 2n_1$ 1 st overtone	$n_2 = 3n_1$ 1 st overtone	$n_2 = 2n_1$ 1 st overtone
3	Frequency of or 3 rd harmonic	$n_3 = 3n_1$ 2 nd overtone	$n_3 = 5n_1$ 2 nd overtone	$n_3 = 3n_1$ 2 nd overtone
4	Frequency ratio of overtones	2:3:4.....	3:5:7.....	2:3:4.....
5	Frequency ratio of harmonics	1:2:3:4.....	1:3:5:7.....	1:2:3:4.....
6	Nature of waves	Transverse stationary	Transverse stationary	Longitudinal stationary

WAVES 03

Relation between loudness and intensity

$$L \propto \log_{10} (\text{Intensity})$$

$$\downarrow \text{unit(dB)} \quad \downarrow \text{unit W/m}^2$$

$$I_0 = 10^{-12} \text{ W/m}^2$$

$$\text{dB} = 10 \times \log_{10} \frac{I}{I_0}$$

$$I_0 = \text{Threshold intensity}$$

$$L_1 = 10 \times \log_{10} \frac{I_1}{I_0}$$

$$I_1 \rightarrow L_1$$

$$L_2 = 10 \times \log_{10} \frac{I_2}{I_0}$$

$$I_2 \rightarrow L_2$$

$$L_2 - L_1 = 10 \left[\log_{10} \left(\frac{I_2}{I_0} \right) - \log_{10} \left(\frac{I_1}{I_0} \right) \right] \Delta L = \text{change in loudness}$$

$$\Delta L = 10 \log_{10} \left(\frac{I_2}{I_1} \right)$$

Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

General equation
(when both source & listener are moving)

$$v' = \left(\frac{V \pm V_L}{V \mp V_s} \right) v$$


Case 1
(listener is stationary & source is approaching the listener)



$$v' = \left(\frac{V}{V - V_s} \right) v$$

Case 2
(The source is stationary & listener is approaching the source)



$$v' = \left(\frac{V + V_L}{V} \right) v$$

Case 3
(source & listener are approaching each other)



$$v' = \left(\frac{V + V_L}{V - V_s} \right) v$$

Case 4
(source is stationary, listener is moving away from the source)



$$v' = \left(\frac{V - V_L}{V} \right) v$$

Case 5
(source is moving away from the listener, listener is stationary)



$$v' = \left(\frac{V}{V + V_s} \right) v$$

Case 6
(source and listener moving in same direction)



$$v' = v \frac{V + V_L}{V + V_s}$$

Case 7
(source and listener moving away from each other)



$$v' = \left(\frac{V - V_L}{V + V_s} \right) v$$

Case 8
(source approaching a stationary wall)



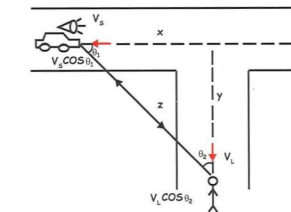
$$v' = \frac{V + V_s}{V - V_s} v$$

Case 9
(source is moving away from stationary wall)



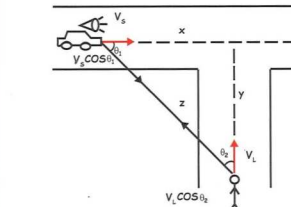
$$v' = \left(\frac{V - V_s}{V + V_s} \right) v$$

Case 10



$$v' = \left(\frac{V - V_L \cos \theta_2}{V + V_s \cos \theta_1} \right) v$$

Case 11

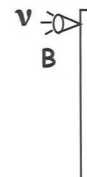
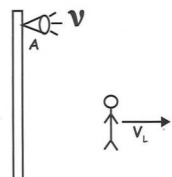


$$v' = \left(\frac{V + V_L \cos \theta_2}{V - V_s \cos \theta_1} \right) v$$

$$\cos \theta_1 = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos \theta_2 = \frac{y}{\sqrt{x^2 + y^2}}$$

Note:



$$v'_A = \left(\frac{V - V_L}{V} \right) v$$

$$v'_B = \left(\frac{V + V_L}{V} \right) v$$

$$\text{Beat frequency } (\Delta v) = v'_B - v'_A$$

$$= \frac{v}{V} [V + V_L - V - V_L]$$

$$\Delta v = \frac{2V_L v}{V}$$

WAVES 04