

Question1

If $z = 3x + 4y$ be the objective function of a linear programming problem (LPP) and $(3, 1), (2, 4), (0, 4), (5, 0)$ be corner points of the bounded feasible region. Then the maximum value of objective function is

1. 13
2. 10
3. 22
4. 14

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

The maximum value of the objective function is the maximum value it obtains at the corner points.

$$z = 3x + 4y$$

$$\text{At } (x, y) = (3, 1) : z = 3 \times 3 + 4 \times 1 = 9 + 4 = 13$$

$$\text{At } (x, y) = (2, 4) : z = 3 \times 2 + 4 \times 4 = 6 + 16 = 22$$

$$\text{At } (x, y) = (0, 4) : z = 3 \times 0 + 4 \times 4 = 0 + 16 = 16$$

$$\text{At } (x, y) = (5, 0) : z = 3 \times 5 + 4 \times 0 = 15 + 0 = 15$$

Therefore the maximum value of the objective function is 22

Correct Option: (3)

Question2

If A be a square matrix of order 3 such that $|A| = 2$, then $|\text{adj}(2A)|$ is equal to

1. 16
2. 32
3. 64
4. 256

Options:

- A.

B.

2

C.

3

D.

4

Answer: D

Solution:

We know that if the order of a square matrix A is n , then

$$|\text{adj}(A)| = |A|^{n-1}$$

and

$$|cA| = c^n |A|$$

Here, the order of matrix A is 3.

$$\therefore |\text{adj}(2A)| = |2A|^{3-1} = |2A|^2 = (2^3 |A|)^2 = (8|A|)^2 = 64|A|^2$$

If $|A| = 2$, then

$$|\text{adj}(2A)| = 64 \times 2^2 = 64 \times 4 = 256$$

Correct Option: (4)

Question3

Match List-I with List-II

List I	List II
(A) Degree of this differential equation $\frac{d^4 y}{dx^4} + 2 \log_e \left(\frac{d^3 y}{dx^3} \right) = 0$	(I) 1
(B) Order of this differential equation $e^{\left(\frac{dy}{dx} \right)^3} + 3y \left(\frac{d^2 y}{dx^2} \right)^3 = 0$	(II) 4
(C) Degree of $\frac{d^4 y}{dx^4} + \left(\frac{dy}{dx} \right)^2 = 0$	(III) not defined
(D) Order of the differential equation $2 \frac{d^4 y}{dx^4} + \left(\frac{d^2 y}{dx^2} \right)^5 = 0$	(IV) 2

Choose the correct answer from the options given below:

- (A) - (II), (B) - (IV), (C) - (III), (D) - (I)
- (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
- (A) - (III), (B) - (IV), (C) - (II), (D) - (I)
- (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

We know that the order of a differential equation is the order of the highest derivative present,
and the degree of a differential equation (if defined) is the power of the highest order derivative after the equation is free from radicals and fractions.

$$(A) \frac{d^4 y}{dx^4} + 2 \log_e \left(\frac{d^3 y}{dx^3} \right) = 0$$

Since $\log_e \left(\frac{d^3 y}{dx^3} \right)$ involves a logarithm of a derivative,

the equation is not polynomial in its derivatives. Hence, the degree is not defined.

$$\Rightarrow (A) \rightarrow (III)$$

$$(B) e^{\left(\frac{dy}{dx}\right)^3} + 3y \left(\frac{d^2 y}{dx^2}\right)^3 = 0$$

The highest derivative is $\frac{d^2 y}{dx^2}$.

So the order is 2.

$$\Rightarrow (B) \rightarrow (IV)$$

$$(C) \frac{d^4 y}{dx^4} + \left(\frac{dy}{dx} \right)^2 = 0$$

The equation is polynomial and the highest order derivative is $\frac{d^4 y}{dx^4}$.

Degree is 1.

$$\Rightarrow (C) \rightarrow (I)$$

$$(D) 2 \frac{d^4 y}{dx^4} + \left(\frac{d^2 y}{dx^2} \right)^5 = 0$$

The highest order derivative is $\frac{d^4 y}{dx^4}$.

Order is 4.

$$\Rightarrow (D) \rightarrow (II)$$

Correct Option: (2)

Question4

If A and B are symmetric matrices, then $AB - BA$ is

1. Singular matrix
2. Zero matrix
3. Symmetric matrix
4. Skew symmetric matrix

Options:

A. 1

- B. 2
- C. 3
- D. 4

Answer: D

Solution:

We know that if A and B are symmetric matrices, then $A^T = A$ and $B^T = B$.

Consider the matrix $AB - BA$,

$$(AB - BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T = BA - AB = -(AB - BA)$$

Since the transpose of $(AB - BA)$ is the negative of itself, it is a skew-symmetric matrix.

Correct Option: (4)

Question5

If A is an invertible matrix, then which of the following statement(s) is/are TRUE?

- (A) $|A^{-1}| = |A|$
- (B) $(A^{-1})^{-1} = A$
- (C) $A^{-1} = \frac{\text{adj } A}{|A|}$
- (D) $(A^T)^{-1} = (A^{-1})^T$

Choose the **correct** answer from the options given below:

1. (A), (B) and (C) only
2. (B) and (C) only
3. (C) only
4. (B), (C) and (D) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

We know that if A is invertible, then $A^{-1} = \frac{\text{adj } A}{|A|}$ and $|A^{-1}| = \frac{1}{|A|}$.

(A) $|A^{-1}| = |A|$ is false since $|A^{-1}| = \frac{1}{|A|}$.

(B) $(A^{-1})^{-1} = A$ is true.

(C) $A^{-1} = \frac{\text{adj } A}{|A|}$ is true.

(D) $(A^T)^{-1} = (A^{-1})^T$ is also true.

Correct Option: (4)

Question6

$\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$ is equal to

1. a

2. $\frac{a}{2}$

3. $\frac{a}{3}$

4. $2a$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$\text{Using } \int_0^a f(x) dx = \int_0^a f(a-x) dx,$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx$$

$$2I = \int_0^a \left[\frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} + \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} \right] dx$$

$$2I = \int_0^a 1 dx = a \Rightarrow I = \frac{a}{2}$$

Correct Option: (2)

Question7

Let X denotes the number of heads in a simultaneous toss of three coins, then $P(0 < X < 3)$ is

1. $\frac{1}{2}$

2. $\frac{3}{4}$

3. $\frac{7}{8}$

4. 1

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

$$P(0 < X < 3) = P(X = 1) + P(X = 2) = \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$$

Correct Option: (2)

Question8

The area (in sq. units) of the region bounded by $y = -1$, $y = 2$, $x = y^3$ and $x = 0$ is equal to

- 1. 4
- 2. 8
- 3. $\frac{17}{4}$
- 4. $\frac{15}{4}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Bounded by $y = -1$, $y = 2$, $x = y^3$, $x = 0$.

$$A = \int_0^2 (y^3 - 0) dy + \int_{-1}^0 (0 - y^3) dy$$

$$\int y^3 dy = \frac{y^4}{4}$$

$$\int_0^2 y^3 dy = \frac{16}{4} - 0 = 4$$

$$\int_{-1}^0 (-y^3) dy = -\left(0 - \frac{1}{4}\right) = \frac{1}{4}$$

$$A = 4 + \frac{1}{4} = \frac{17}{4}$$

Correct Option: (3)

Question9

The general solution of the differential equation $\frac{dy}{dx} = -4xy^2$ is given by

1. $2x^2 - y = C$: C is an arbitrary constant
2. $2x^2 - \frac{1}{y} = C$: C is an arbitrary constant
3. $2x^2 + \frac{1}{y^2} = C$: C is an arbitrary constant
4. $2x^2 + \frac{1}{y} = C$: C is an arbitrary constant

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

$$\frac{dy}{dx} = -4xy^2$$

$$\frac{dy}{y^2} = -4x \, dx$$

$$\int \frac{dy}{y^2} = \int -4x \, dx$$

$$-\frac{1}{y} = -2x^2 + C$$

$$\frac{1}{y} = 2x^2 - C$$

$$2x^2 - \frac{1}{y} = C$$

Correct Option: (2)

Question10

Assume A, B and C are matrices of order $m \times n$, $n \times 3$ and $3 \times q$ respectively. The restrictions on m, n and q so that $AB + BC$ is defined are

1. n is arbitrary, $q = 2$
2. $m = n, q = 3$
3. $q = 3, m = 3$
4. q is arbitrary, $n = 2$

Options:

- A. 1
- B. 2
- C. 3

D. 4

Answer: B

Solution:

Given $A : m \times n$, $B : n \times 3$, $C : 3 \times q$.

AB has order $m \times 3$, BC has order $n \times q$.

$AB + BC$ defined only if $m = n$ and $q = 3$.

Correct Option: (2)

Question11

Which of the following terms are associated with a linear programming problem?

- (A) Constraints
- (B) Independent events
- (C) Feasible region
- (D) Objective function

Choose the **correct** answer from the options given below:

1. (A) and (C) only
2. (A), (C) and (D) only
3. (B), (C) and (D) only
4. (A), (B) and (D) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

The terms associated with linear programming problems are: constraints, feasible region, objective function.

The term “independent events” is associated with probability.

Correct Option: (2)

Question12

Which of the following functions has a local minima at $x = 0$?

(A) $f(x) = x^3$

(B) $f(x) = |x|$

(C) $f(x) = x^2$

(D) $f(x) = x^{-2}$

Choose the **correct** answer from the options given below:

1. (A), (B) and (C) only
2. (D) only
3. (B), (C) and (D) only
4. (B) and (C) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

(A) $f(x) = x^3$: $f'(0) = 0, f''(0) = 0$ inflection point.

(B) $f(x) = |x|$ minimum at $x = 0$.

(C) $f(x) = x^2$: $f'(0) = 0, f''(0) = 2 > 0$ local minimum.

(D) $f(x) = x^{-2}$ not defined at $x = 0$.

Hence only (B) and (C).

Correct Option: (4)

Question13

$\int_0^8 \left(x^{\frac{2}{3}} + 1 \right) dx$ is equal to

1. $\frac{28}{3}$
2. $\frac{25}{3}$
3. $\frac{184}{3}$
4. $\frac{136}{5}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

$$\int_0^8 (x^{\frac{2}{3}} + 1) dx = \left. \frac{3}{5} x^{\frac{5}{3}} + x \right|_0^8 = \frac{3}{5} \times 32 + 8 = \frac{96}{5} + \frac{40}{5} = \frac{136}{5}$$

Correct Option: (4)

Question14

Function $f(x) = x^x$, $x > 0$ decreases on the interval

1. $(0, \frac{1}{e})$
2. $(0, e)$
3. $(\frac{1}{e}, \infty)$
4. (e, ∞)

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

$$f(x) = x^x = e^{x \ln x}, f'(x) = x^x (\ln x + 1)$$

$$f'(x) = 0 \Rightarrow \ln x = -1 \Rightarrow x = \frac{1}{e}$$

$$f'(x) < 0 \Rightarrow \ln x < -1 \Rightarrow x < \frac{1}{e}$$

Decreasing on $(0, \frac{1}{e})$

Correct Option: (1)

Question15

If $x = at^2$, $y = 2at$; then $\frac{d^2y}{dx^2}$ is equal to

1. 0
2. $\frac{1}{t}$
3. $-\frac{1}{t^2}$
4. $-\frac{1}{2at^3}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

$$x = at^2, y = 2at$$

$$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{t}$$

$$\frac{d(dy/dx)}{dt} = -\frac{1}{t^2}$$

$$\frac{d^2y}{dx^2} = \frac{-1/t^2}{dx/dt} = \frac{-1/t^2}{2at} = -\frac{1}{2at^3}$$

Correct Option: (4)

Core Mathematics

Question1

The number of arbitrary constants in the particular solution of a differential equation of order 4 and degree 3 is

1. 0
2. 1
3. 3
4. 4

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

The general solution of a differential equation of order n contains n arbitrary constants.

For a differential equation of order 4, the general solution contains 4 arbitrary constants.

The particular solution has no arbitrary constants.

Correct Option: (1)

Question2

If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$ and C are three matrices such that $3A + 5B + 2C = 0$, then the matrix C is equal to

1. $\begin{bmatrix} -48 & -20 \\ -56 & -76 \end{bmatrix}$
2. $\begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$
3. $\begin{bmatrix} -48 & 20 \\ 56 & -76 \end{bmatrix}$
4. $\begin{bmatrix} 24 & 10 \\ 28 & 38 \end{bmatrix}$

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: B

Solution:

$$A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}, \quad B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}, \quad 3A + 5B + 2C = 0$$

$$2C = -3A - 5B$$

$$-3A = \begin{bmatrix} -3 & -15 \\ -21 & -36 \end{bmatrix}, \quad -5B = \begin{bmatrix} -45 & -5 \\ -35 & -40 \end{bmatrix}$$

$$-3A - 5B = \begin{bmatrix} -48 & -20 \\ -56 & -76 \end{bmatrix}$$

$$C = \frac{1}{2} \begin{bmatrix} -48 & -20 \\ -56 & -76 \end{bmatrix} = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$$

Correct Option: (2)

Question3

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$. Then match List-I with List-II

List-I	List-II
(A) $\sin \theta$	(I) $\frac{\vec{a} \cdot \vec{b}}{ \vec{a} \vec{b} }$
(B) $\cos \theta$	(II) $ \vec{a} \times \vec{b} $
(C) Area of the parallelogram with adjacent sides represented by $\vec{a}$ and $\vec{b}$	(III) $\frac{\vec{a} \cdot \vec{b}}{ \vec{a} }$
(D) Projection of $\vec{a}$ on $\vec{b}$	(IV) $\frac{ \vec{a} \times \vec{b} }{ \vec{a} \vec{b} }$

Choose the correct answer from the options given below:

1. (A) - (IV), (B) - (I), (C) - (II), (D) - (III)
2. (A) - (I), (B) - (IV), (C) - (II), (D) - (III)
3. (A) - (IV), (B) - (I), (C) - (III), (D) - (II)
4. (A) - (I), (B) - (IV), (C) - (III), (D) - (II)

Options:

A.

1

B.

2

C.

3

D.

4

Answer: A

Solution:

$\theta = \text{angle between } \vec{a} \text{ and } \vec{b}$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta, \quad |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$$

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}, \quad \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\text{Area of parallelogram} = |\vec{a} \times \vec{b}|$$

$$\text{Scalar projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Correct option: (1)

Question4

If $\vec{a}$ is any vector, then $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is equal to

1. $|\vec{a}|^2$
2. $2|\vec{a}|^2$
3. $3|\vec{a}|^2$
4. $4|\vec{a}|^2$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{a} \times \hat{i} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} = -a_3\hat{j} + a_2\hat{k}$$

$$|\vec{a} \times \hat{i}|^2 = a_3^2 + a_2^2$$

$$\vec{a} \times \hat{j} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 0 & 1 & 0 \end{bmatrix} = -a_3\hat{i} + a_1\hat{k}$$

$$|\vec{a} \times \hat{j}|^2 = a_3^2 + a_1^2$$

$$\vec{a} \times \hat{k} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 0 & 0 & 1 \end{bmatrix} = a_2\hat{i} - a_1\hat{j}$$

$$|\vec{a} \times \hat{k}|^2 = a_2^2 + a_1^2$$

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2(a_1^2 + a_2^2 + a_3^2) = 2|\vec{a}|^2$$

Correct option: (2)

Question5

A bag contains 4 red and 6 black balls. Two balls are drawn in succession without replacement. The probability that the first is red and the second is black is

1. $\frac{2}{5}$
2. $\frac{2}{3}$
3. $\frac{4}{15}$
4. $\frac{6}{25}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Total balls = 4 + 6 = 10

$$P(\text{first red}) = \frac{4}{10}$$

$$P(\text{second black}|\text{first red}) = \frac{6}{9}$$

$$P(\text{first red and second black}) = \frac{4}{10} \times \frac{6}{9} = \frac{24}{90} = \frac{4}{15}$$

Correct option: (3)

Question6

$\int e^{2x} \left(\sin x + \frac{1}{2} \cos x \right) dx$ is equal to

1. $-\frac{1}{2}e^{2x} \cos x + C$, C is an arbitrary constant

2. $\frac{1}{2}e^{2x} \sin x + C$, C is an arbitrary constant

3. $e^{2x} \sin x + C$, C is an arbitrary constant

4. $-e^{2x} \cos x + C$, C is an arbitrary constant

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

$$I = \int e^{2x} \left(\sin x + \frac{1}{2} \cos x \right) dx$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$$

$$a = 2, b = 1$$

$$I = e^{2x} \left[\frac{1}{5} (2 \sin x - \cos x) + \frac{1}{2} \cdot \frac{1}{5} (2 \cos x + \sin x) \right] + C$$

$$I = \frac{e^{2x}}{5} \left[(2 \sin x - \cos x) + \frac{1}{2} (2 \cos x + \sin x) \right] + C$$

$$I = \frac{e^{2x}}{5} \cdot \frac{5}{2} \sin x + C = \frac{1}{2} e^{2x} \sin x + C$$

Correct option: (2)

Question7

Let $f(x) = \begin{cases} |x| + 3 & \text{if } x \leq -3 \\ -2x & \text{if } -3 < x < 3 \\ 6x + 2 & \text{if } x \geq 3 \end{cases}$, then which of the following is true?

1. $f(x)$ is discontinuous at both $x = 3$ and $x = -3$

2. $f(x)$ is continuous at both $x = 3$ and $x = -3$

3. $f(x)$ is discontinuous at $x = -3$

4. $f(x)$ is discontinuous at $x = 3$

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

$$f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$$

$$f(-3) = 6, \quad \lim_{x \rightarrow -3^-} f(x) = 6, \quad \lim_{x \rightarrow -3^+} f(x) = 6$$

$$f(3) = 20, \quad \lim_{x \rightarrow 3^-} f(x) = -6, \quad \lim_{x \rightarrow 3^+} f(x) = 20$$

$f(x)$ continuous at $x = -3$, but discontinuous at $x = 3$.

Correct option: (4)

Question8

The system of equations

$$x + y + z = 4$$

$$x + 2y + 3z = 12$$

$$x + 3y + \lambda z = \mu$$

has a unique solution if

1. $\lambda = 5$, μ can be any real number
2. $\lambda = 5$, $\mu \neq 20$
3. $\lambda \neq 5$, μ can be any real number
4. $\lambda = 5$, $\mu = 20$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

Possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

$$P(X = 0) = \frac{1}{8}, \quad P(X = 1) = \frac{3}{8}, \quad P(X = 2) = \frac{3}{8}, \quad P(X = 3) = \frac{1}{8}$$

$$P(0 < X < 3) = P(X = 1) + P(X = 2) = \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$$

Correct Option: (2)

Question9

If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 4 & -8 \end{vmatrix}$, then value(s) of x is/are

1. 3
2. $2\sqrt{2}$
3. $\pm 2\sqrt{2}$
4. 8

Options:

- A. 1
- B. 2

C. 3

D. 4

Answer: C

Solution:

$$\det \begin{bmatrix} 2x & 5 \\ 8 & x \end{bmatrix} = 2x^2 - 40, \quad \det \begin{bmatrix} 3 & 0 \\ 4 & -8 \end{bmatrix} = -24$$

On equating the two sides we get:

$$2x^2 - 40 = -24$$

$$2x^2 = 16$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

Correct option: (3)

Question10

The probabilities of occurrence of two events E and F are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14.

The probability that neither E nor F occurs is

1. 0

2. 1

3. 0.39

4. 0.61

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

$$P(E) = 0.25, P(F) = 0.50, P(E \cap F) = 0.14$$

$$P(\text{neither } E \text{ nor } F) = 1 - P(E \cup F) = 1 - (P(E) + P(F) - P(E \cap F)) = 1 - 0.61 = 0.39$$

Correct option: (3)

Question11

The probability distribution of a random variable X is

X	0	1	2	3	4
$P(X)$	0.2	k	k	2k	k

Match List-I with List-II

List I	List II
(A) value of k	(I) $\frac{16}{25}$
(B) $P(X \geq 2)$	(II) $\frac{9}{25}$
(C) $P(X = 3)$	(III) $\frac{4}{25}$
(D) $P(X < 2)$	(IV) $\frac{8}{25}$

Choose the correct answer from the options given below:

1. (A) - (III), (B) - (II), (C) - (I), (D) - (IV)
2. (A) - (III), (B) - (I), (C) - (II), (D) - (IV)
3. (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
4. (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

Options:

A.

1

B.

2

C.

3

D.

4

Answer: D

Solution:

Since Sum of all probabilities is 1:

$$0.2 + k + k + 2k + k = 1$$

$$0.2 + 5k = 1 \Rightarrow k = 0.16 = \frac{4}{25}$$

$$P(X \geq 2) = k + 2k + k = 4k = \frac{16}{25}$$

$$P(X = 3) = 2k = \frac{8}{25}, \quad P(X < 2) = 0.2 + k = \frac{9}{25}$$

Correct option: (4)

Question12

The relation R on the set of real numbers defined by $R = \{(a, b) : a \leq b^2\}$ is

1. neither reflexive nor symmetric nor transitive
2. reflexive but not symmetric
3. symmetric but not transitive
4. transitive but neither reflexive nor symmetric

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

$$R = \{(a, b) : a \leq b^2\}$$

Reflexive? $a \leq a^2$ not true for all a , so not reflexive

Symmetric? $(1, 2) \in R, (2, 1) \notin R$, so not symmetric

Transitive? Fails generally

Take a counterexample:

Let $a = 3, b = 2, c = 1.5$

$$a \leq b^2 \Rightarrow 3 \leq 4$$

$$b \leq c^2 \Rightarrow 2 \leq 2.25$$

$$a \leq c^2 \Rightarrow 3 \leq 2.25$$

So, the relation is not transitive.

Correct option: (1)

Question13

The angle at which the line $\frac{x-1}{0} = \frac{2-y}{-1} = \frac{2z-3}{-2}$ is inclined with the positive direction of z-axis is

1. $\frac{\pi}{4}$
2. $\frac{\pi}{2}$
3. $\frac{2\pi}{3}$
4. $\frac{3\pi}{4}$

Options:

- A. 1
- B. 2
- C. 3

D. 4

Answer: D

Solution:

$$\frac{x-1}{0} = \frac{2-y}{-1} = \frac{2z-3}{-2} \implies l:m:n = 0:-1:-2$$

$$\cos \theta = \frac{n}{\sqrt{l^2 + m^2 + n^2}} = \frac{-2}{\sqrt{0+1+4}} = \frac{-2}{\sqrt{5}}$$

$$\theta = \pi - \cos^{-1} \frac{2}{\sqrt{5}} = \frac{3\pi}{4}$$

Correct option: (4)

Question14

Which of the following statement is/are correct?

- (A) A square matrix $A = [a_{ij}]$ is called a symmetric matrix if $a_{ij} = a_{ji}$ for all i, j .
- (B) $A = [a_{ij}]_{m \times m}$ is a diagonal matrix if $a_{ij} = 0$ when $i \neq j$.
- (C) A square matrix $A = [a_{ij}]$ is called a skew symmetric matrix if $a_{ij} = -a_{ji}$ for all i, j .
- (D) The multiplication of diagonal matrices of same order is commutative.

Choose the **correct** answer from the options given below:

- 1. (A), (B) and (C) only
- 2. (B), (C) and (D) only
- 3. (A), (C) and (D) only
- 4. (C) and (D) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

- (A) Correct: symmetric matrix has $a_{ij} = a_{ji}$
- (B) Incorrect: diagonal matrix has $a_{ij} = 0$ for $i \neq j$
- (C) Correct: skew symmetric has $a_{ij} = -a_{ji}$
- (D) Correct: diagonal matrix multiplication commutes

Correct option: (3)

Question15

If A and B are two square matrices of same order such that $AB = A$ and $BA = B$, then the value of $A^{2024} + B^{2024}$ is equal to

1. $2024A + 2024B$
2. O
3. $A + B$
4. $A - B$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

From $AB = A$ and $BA = B$:

$$A^2 = AA = (AB)A = A(BA) = AB = A,$$

$$\text{so } A^2 = A.$$

By induction, $A^n = A$ for every $n \geq 1$.

$$\text{Similarly, } B^2 = BB = B(BA) = (BA)B = BB = B,$$

$$\text{so } B^n = B \text{ for every } n \geq 1.$$

$$\text{Hence, } A^{2024} + B^{2024} = A + B.$$

Correct option: (3)

Question16

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ be three vectors. If $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, then the value of λ is

1. 3
2. 5
3. 2
4. -3

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Given: $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$

and the condition: $(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$

Step 1: Expand the expression

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \lambda(\vec{b} \cdot \vec{c})$$

Step 2: Compute each dot product

$$\vec{a} \cdot \vec{c} = (1)(3) + (2)(1) + (3)(0) = 3 + 2 + 0 = 5$$

$$\vec{b} \cdot \vec{c} = (-1)(3) + (2)(1) + (1)(0) = -3 + 2 + 0 = -1$$

Step 3: Substitute in the condition

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0 \Rightarrow 5 + \lambda(-1) = 0 \Rightarrow 5 - \lambda = 0$$

$$5 - \lambda = 0 \Rightarrow \lambda = 5$$

Correct option: (2)

Question17

The sides of an equilateral triangle are increasing at the rate of 5 cm/sec. The rate at which the area increases when the side is 20 cm, is

1. $25\sqrt{3} \text{ cm}^2/\text{sec}$
2. $50\sqrt{3} \text{ cm}^2/\text{sec}$
3. $100\sqrt{3} \text{ cm}^2/\text{sec}$
4. $15\sqrt{3} \text{ cm}^2/\text{sec}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

$$A = \frac{\sqrt{3}}{4} s^2, \quad \frac{dA}{dt} = \frac{\sqrt{3}}{2} s \frac{ds}{dt}$$

$$s = 20, \frac{ds}{dt} = 5 \Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \times 20 \times 5 = 50\sqrt{3}$$

Correct option: (2)

Question18

The region represented by the system of inequalities $x, y \geq 0$, $y \leq 6$, $x + y \leq 3$

1. is a bounded region in the first quadrant
2. is an unbounded region in the first and second quadrant
3. is an unbounded region in the first quadrant
4. has no point which satisfies all the inequalities

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

$$x \geq 0, y \geq 0, y \leq 6, x + y \leq 3$$

Feasible region is triangle with vertices $(0, 0), (3, 0), (0, 3)$

Correct option: (1)

Question19

The vector equation of line passing through $(-1, 3, -2)$ and perpendicular to the lines

$$\frac{x+4}{1} = \frac{y}{2} = \frac{z-3}{3} \text{ and } \frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-6}{5} \text{ is}$$

1. $\vec{r} = (-3\hat{i} + 4\hat{j} + 15\hat{k}) + \lambda(-\hat{i} + 3\hat{j} - 2\hat{k})$
2. $\vec{r} = (-\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(-3\hat{i} + 4\hat{j} + 15\hat{k})$
3. $\vec{r} = 2\hat{i} - 7\hat{j} + 4\hat{k} + \lambda(-\hat{i} + 3\hat{j} - 2\hat{k})$
4. $\vec{r} = (-\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

$$\text{Given: } \vec{d}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{d}_2 = -3\hat{i} + 2\hat{j} + 5\hat{k}$$

We are told that $\vec{d} = \vec{d}_1 \times \vec{d}_2$

$$\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & 2 & 5 \end{vmatrix}$$

Step 1: Expand the determinant (cross product)

$$\vec{d} = \hat{i}(2 \times 5 - 3 \times 2) - \hat{j}(1 \times 5 - 3 \times (-3)) + \hat{k}(1 \times 2 - 2 \times (-3))$$

Simplify each term:

$$\vec{d} = \hat{i}(10 - 6) - \hat{j}(5 - (-9)) + \hat{k}(2 - (-6))$$

$$\vec{d} = 4\hat{i} - 14\hat{j} + 8\hat{k}$$

$$\text{or simplified, } \vec{d} = 2(2\hat{i} - 7\hat{j} + 4\hat{k})$$

$$\vec{r} = (-1)\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

Question20

Let $f(x) = x^3 - 6x^2 + 9x - 8$ be a function, then which of the following statements are TRUE?

- (A) $f'(x) = 3(x - 1)(x - 3)$
- (B) The critical points of the function are $x = 1$ and $x = 3$
- (C) $x = 1$ is the point of local minimum
- (D) The local maximum value is -4

Choose the correct answer from the options below:

1. (A), (B) and (C) only
2. (A), (B) and (D) only
3. (B), (C) and (D) only
4. (C) and (D) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

$$f(x) = x^3 - 6x^2 + 9x - 8$$

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3)$$

(A) is TRUE

$$3(x - 1)(x - 3) = 0 \implies x = 1, 3$$

(B) is TRUE

$$f''(x) = 6x - 12$$

$$f''(1) = 6 - 12 = -6 < 0 \implies \text{Local maximum at } x = 1$$

(C) is FALSE

$$f''(3) = 18 - 12 = 6 > 0 \implies \text{Local minimum at } x = 3$$

$$f(1) = 1 - 6 + 9 - 8 = -4$$

(D) is TRUE

Correct answer: (2)

Question21

If $x^m y^n = (x + y)^{m+n}$, then $\frac{d^2 y}{dx^2}$ is equal to:

1. $\frac{y^2}{x^2}$
2. $-\frac{y}{x}$
3. $\frac{y}{x}$
4. 0

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

$$x^m y^n = (x + y)^{m+n}$$

$$m \ln x + n \ln y = (m + n) \ln(x + y)$$

$$\frac{m}{x} + \frac{n}{y} \frac{dy}{dx} = \frac{m+n}{x+y} \left(1 + \frac{dy}{dx} \right)$$

$$\frac{n}{y} \frac{dy}{dx} - \frac{m+n}{x+y} \frac{dy}{dx} = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\frac{dy}{dx} = \frac{\frac{m+n}{x+y} - \frac{m}{x}}{\frac{n}{y} - \frac{m+n}{x+y}}$$

$$\frac{m+n}{x+y} - \frac{m}{x} = \frac{(m+n)x - m(x+y)}{x(x+y)} = \frac{nx - my}{x(x+y)}$$

$$\frac{n}{y} - \frac{m+n}{x+y} = \frac{n(x+y) - (m+n)y}{y(x+y)} = \frac{nx - my}{y(x+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{nx-my}{x(x+y)}}{\frac{nx-my}{y(x+y)}} = \frac{y}{x}, \quad \text{if } nx - my \neq 0$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{y}{x} \right) = \frac{x \frac{dy}{dx} - y}{x^2} = \frac{x \frac{y}{x} - y}{x^2} = 0$$

Correct answer: (4)

Question22

$\int \frac{\sqrt{1-x}}{\sqrt{1+x}} dx = \alpha \sqrt{1-x^2} + \beta \sin^{-1} x + C$, where C is an arbitrary constant, then which of the following are TRUE?

- (A) $\alpha = 1$
 (B) $\alpha = -1$
 (C) $\beta = 1$
 (D) $\beta = -1$

Choose the correct answer from the options given below:

1. (B) and (D) only
 2. (A) and (C) only
 3. (A) and (D) only
 4. (B) and (C) only

Options:

- A. 1
 B. 2
 C. 3
 D. 4

Answer: B

Solution:

$$\int \sqrt{\frac{1-x}{1+x}} dx$$

$$x = \cos 2t$$

$$1-x = 1 - \cos 2t = 2 \sin^2 t, \quad 1+x = 1 + \cos 2t = 2 \cos^2 t$$

$$\sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2 \sin^2 t}{2 \cos^2 t}} = \tan t$$

$$dx = -2 \sin 2t dt = -4 \sin t \cos t dt$$

$$\sqrt{\frac{1-x}{1+x}} dx = \tan t \cdot (-4 \sin t \cos t dt) = -4 \sin^2 t dt$$

$$\int -4 \sin^2 t dt = -4 \int \frac{1 - \cos 2t}{2} dt = \int (-2 + 2 \cos 2t) dt = -2t + \sin 2t + C$$

$$\sin 2t = \sqrt{1 - \cos^2 2t} = \sqrt{1 - x^2}, \quad t = \frac{1}{2} \cos^{-1} x$$

$$\int \sqrt{\frac{1-x}{1+x}} dx = -2 \cdot \frac{1}{2} \cos^{-1} x + \sqrt{1-x^2} + C = -\cos^{-1} x + \sqrt{1-x^2} + C$$

$$-\cos^{-1} x = -\frac{\pi}{2} + \sin^{-1} x + \text{constant}$$

$$\therefore \int \sqrt{\frac{1-x}{1+x}} dx = \sqrt{1-x^2} + \sin^{-1} x + C$$

$$\alpha = 1, \quad \beta = 1$$

Correct answer: (2)

Question23

Let $f(x) = \log_e(\sin x)$, $x \in (0, \pi)$, then which of the following statements is/are TRUE?

- (A) $f(x)$ is increasing on $(0, \pi/2)$
- (B) $f(x)$ is decreasing on $(\pi/2, \pi)$
- (C) $f(x)$ is increasing on $(0, \pi)$
- (D) $f(x)$ is decreasing on $(0, \pi)$

Choose the **correct** answer from the options given below:

- 1. (C) and (D) only
- 2. (A) and (B) only
- 3. (A) only
- 4. (B) only

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

$$f(x) = \log_8(\sin x), \quad x \in (0, \pi)$$

$$f'(x) = \frac{1}{\ln 8} \cdot \frac{\cos x}{\sin x} = \frac{\cot x}{\ln 8}$$

$\ln 8 > 0$ hence sign of $f'(x)$ same as $\cot x$

$\cot x > 0$ on $(0, \pi/2) \implies f'(x) > 0 \implies f$ increasing

$\cot x < 0$ on $(\pi/2, \pi) \implies f'(x) < 0 \implies f$ decreasing

Correct answer: (2)

Question24

The value of $\tan^{-1}(2) + \tan^{-1}(3)$ is equal to

- 1. $\frac{\pi}{4}$
- 2. $-\frac{\pi}{4}$
- 3. $\frac{3\pi}{4}$
- 4. $-\frac{3\pi}{4}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: C

Solution:

$$\tan^{-1}(2) + \tan^{-1}(3) = ?$$

$$\tan^{-1} a + \tan^{-1} b = \tan^{-1} \left(\frac{a+b}{1-ab} \right) \text{ if } ab \neq 1$$

$$a = 2, b = 3, \quad \frac{a+b}{1-ab} = \frac{5}{1-6} = \frac{5}{-5} = -1$$

$$\tan^{-1}(-1) = -\frac{\pi}{4}$$

Since $a, b > 0$, sum is in second quadrant

$$\tan^{-1} 2 + \tan^{-1} 3 = \pi + \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Correct answer: (3)

Question25

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then $|\vec{a} \times \vec{b}|$ is equal to

1. 16
2. 256
3. 20
4. 240

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

$$|\vec{a}| = 10, \quad |\vec{b}| = 2, \quad \vec{a} \cdot \vec{b} = 12$$

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$|\vec{a} \times \vec{b}|^2 + 144 = 100 \times 4 = 400$$

$$|\vec{a} \times \vec{b}|^2 = 256 \Rightarrow |\vec{a} \times \vec{b}| = 16$$

Correct answer: (1)

Question26

If the area of a triangle whose vertices are $(-2, 4)$, $(2, -6)$ and $(k, 4)$, $k > 0$, is 35 sq.units, then the value of k is

1. 3
2. 5
3. 6
4. 7

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Area of triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|.$$

Substitute $(-2, 4), (2, -6), (k, 4)$:

$$\text{Area} = \frac{1}{2} |(-2)((-6) - 4) + 2(4 - 4) + k(4 - (-6))|.$$

$$= \frac{1}{2} |(-2)(-10) + 2(0) + k(10)| = \frac{1}{2} |20 + 10k|.$$

$$\text{Given that Area} = 35, \quad \frac{1}{2} |20 + 10k| = 35 \Rightarrow |20 + 10k| = 70.$$

This gives two cases:

$$\begin{cases} 20 + 10k = 70 & \Rightarrow k = 5, \\ 20 + 10k = -70 & \Rightarrow k = -9. \end{cases}$$

Since $k > 0$, reject $k = -9$.

$$\boxed{k = 5}$$

Question 27

The corner points of the feasible region of a LPP with the constraints $x + 2y \leq 40, 3x + y \geq 30, 4x + 3y \geq 60, x, y \geq 0$ are

1. $(0, 0), (10, 0), (6, 12), (0, 20)$
2. $(0, 20), (0, 30), (4, 18)$
3. $(0, 20), (6, 12), (4, 18)$
4. $(15, 0), (40, 0), (4, 18), (6, 12)$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

Given the lines:

$$x + 2y = 40, \quad 3x + y = 30, \quad 4x + 3y = 60$$

(a) Intersection of $x + 2y = 40$ and $3x + y = 30$:

$$3x + y = 30 \Rightarrow y = 30 - 3x$$

$$x + 2(30 - 3x) = 40 \Rightarrow x + 60 - 6x = 40 \Rightarrow -5x = -20 \Rightarrow x = 4$$

$$y = 30 - 3(4) = 18$$

Point of intersection: (4, 18)

(b) Intersection of $3x + y = 30$ and $4x + 3y = 60$:

$$3x + y = 30 \Rightarrow y = 30 - 3x$$

$$4x + 3(30 - 3x) = 60 \Rightarrow 4x + 90 - 9x = 60 \Rightarrow -5x = -30 \Rightarrow x = 6$$

$$y = 30 - 3(6) = 12$$

Point of intersection: (6, 12)

(c) Intersection of $x + 2y = 40$ and $4x + 3y = 60$:

$$x = 40 - 2y$$

$$4(40 - 2y) + 3y = 60 \Rightarrow 160 - 8y + 3y = 60 \Rightarrow -5y = -100 \Rightarrow y = 20$$

$$x = 40 - 2(20) = 0$$

Point of intersection: (0, 20)

(d) Intersections with the x-axis ($y = 0$) :

$$x + 2y = 40 \Rightarrow x = 40 \Rightarrow (40, 0)$$

$$4x + 3y = 60 \Rightarrow 4x = 60 \Rightarrow x = 15 \Rightarrow (15, 0)$$

$$3x + y = 30 \Rightarrow 3x = 30 \Rightarrow x = 10 \Rightarrow (10, 0)$$

(2) Checking which points satisfy all constraints:

$$(4, 18) : \quad x + 2y = 40 \leq 40, \quad 3x + y = 30 \geq 30, \quad 4x + 3y = 70 \geq 60 \Rightarrow \text{Feasible}$$

$$(6, 12) : \quad x + 2y = 30 \leq 40, \quad 3x + y = 30 \geq 30, \quad 4x + 3y = 60 \geq 60 \Rightarrow \text{Feasible}$$

$$(0, 20) : \quad x + 2y = 40 \leq 40, \quad 3x + y = 20 \not\geq 30 \Rightarrow \text{Not feasible}$$

$$(40, 0) : \quad x + 2y = 40 \leq 40, \quad 3x + y = 120 \geq 30, \quad 4x + 3y = 160 \geq 60 \Rightarrow \text{Feasible}$$

$$(15, 0) : \quad x + 2y = 15 \leq 40, \quad 3x + y = 45 \geq 30, \quad 4x + 3y = 60 \geq 60 \Rightarrow \text{Feasible}$$

$$(10, 0) : \quad 4x + 3y = 40 < 60 \Rightarrow \text{Not feasible.}$$

(3) Feasible corner points: (40, 0), (15, 0), (6, 12), (4, 18)

Correct answer: (4)

The function $f : [-1, 1] \rightarrow \mathbb{R}$ is given by $f(x) = \frac{x}{x+2}$

1. onto only
2. both one-one and onto
3. one-one only
4. neither one-one nor onto

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: C

Solution:

$$f(x) = \frac{x}{x+2}, \quad x \in [-1, 1]$$

$$f'(x) = \frac{(1)(x+2) - x(1)}{(x+2)^2} = \frac{2}{(x+2)^2} > 0 \text{ for all } x \in [-1, 1]$$

f is strictly increasing, so one-one

$$f(-1) = -\frac{1}{1} = -1, \quad f(1) = \frac{1}{3}$$

Range is $[-1, \frac{1}{3}] \neq \mathbb{R} \Rightarrow$ not onto

Correct answer: (3)

Question29

The area (in sq.units) of region bounded by $y^2 = 9x, x = 2, x = 4$ and the x-axis in the first quadrant is

1. $3[4 - \sqrt{2}]$
2. $2[4 + \sqrt{2}]$
3. $8[4 - \sqrt{2}]$
4. $4[4 - \sqrt{2}]$

Options:

- A. 1
B. 2
C. 3
D. 4

Answer: D

Solution:

$$\begin{aligned}\text{Area} &= \int_2^4 3\sqrt{x} \, dx \\ &= 2x^{3/2} \Big|_2^4 = 2(4^{3/2} - 2^{3/2}) = 2(8 - 2\sqrt{2}) = 16 - 4\sqrt{2}\end{aligned}$$

Correct answer: (4)

Question30

The direction ratios of the line perpendicular to the lines $\frac{x-5}{2} = \frac{y+11}{-3} = \frac{z+3}{1}$ and $\frac{x-7}{1} = \frac{y+2}{2} = \frac{z-4}{-2}$ are proportional to:

1. $(4, 5, -7)$
2. $(-4, 5, 7)$
3. $(4, -5, 7)$
4. $(4, 5, 7)$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: D

Solution:

$$\begin{aligned}\vec{d}_1 &= \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \quad \vec{d}_2 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \\ \vec{d} &= \vec{d}_1 \times \vec{d}_2 = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{bmatrix} \\ &= \hat{i}((-3)(-2) - 1 \cdot 2) - \hat{j}(2 \cdot (-2) - 1 \cdot 1) + \hat{k}(2 \cdot 2 - (-3) \cdot 1) \\ &= \hat{i}(6 - 2) - \hat{j}(-4 - 1) + \hat{k}(4 + 3) = 4\hat{i} + 5\hat{j} + 7\hat{k}\end{aligned}$$

Correct answer: (4)

Question31

Three students, A, B and C can respectively solve 50%, 25% and 20% of the problems in a book. A particular problem is selected at random from the book. The probability that at least one of them will solve the problem is

1. $\frac{3}{20}$
2. $\frac{17}{20}$
3. $\frac{7}{10}$
4. $\frac{3}{10}$

Options:

- A. 1
- B. 2

C. 3

D. 4

Answer: A

Solution:

$$P(A) = 0.5, \quad P(B) = 0.25, \quad P(C) = 0.2$$

$$P(\text{none}) = (1 - 0.5)(1 - 0.25)(1 - 0.2) = 0.5 \times 0.75 \times 0.8 = 0.3$$

$$P(\text{at least one}) = 1 - 0.3 = 0.7$$

Correct answer: (3)

Question32

The area (in sq. units) of the region enclosed by the curve $9x^2 + 4y^2 = 36$ is

1. 10π

2. 8π

3. 6π

4. 4π

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

$$9x^2 + 4y^2 = 36 \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\text{Area} = \pi ab = \pi \times 2 \times 3 = 6\pi$$

Correct answer: (3)

Question33

Match List-I with List-II

List-I

List-II

(A) $\int \frac{dx}{x^2 - 16}$ (I) $\frac{1}{8} \log \left| \frac{4+x}{4-x} \right| + c$

(B) $\int \frac{dx}{x^2 + 16}$ (II) $\log \left| x + \sqrt{x^2 - 16} \right| + c$

(C) $\int \frac{dx}{16 - x^2}$ (III) $\frac{1}{8} \log \left| \frac{x-4}{x+4} \right| + c$

(D) $\int \frac{dx}{\sqrt{x^2 - 16}}$ (IV) $\frac{1}{4} \tan^{-1} \left(\frac{x}{4} \right) + c$

Choose the correct answer from the options given below:

1. (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
2. (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
3. (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
4. (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Options:

A.

1

B.

2

C.

3

D.

4

Answer: D

Solution:

$$\int \frac{dx}{x^2 - 16} = \frac{1}{8} \log \left| \frac{x-4}{x+4} \right| + C$$

$$\int \frac{dx}{x^2 + 16} = \frac{1}{4} \tan^{-1} \frac{x}{4} + C$$

$$\int \frac{dx}{16 - x^2} = \frac{1}{8} \log \left| \frac{4+x}{4-x} \right| + C$$

$$\int \frac{dx}{\sqrt{x^2 - 16}} = \log \left| x + \sqrt{x^2 - 16} \right| + C$$

Correct answer: (4)

Question34

For the differential equation $x \frac{dy}{dx} + 2y = x^2 \log_e x$

(A) Integrating factor is $2x$

(B) Integrating factor is x^2

(C) General Solution is $y = \frac{x^2}{16} (4 \log_e |x| - 1) + Cx^{-2}$, where C is an arbitrary constant.

(D) General Solution is $y = \frac{x^4}{16} (4 \log_e |x| - 1) + C$, where C is an arbitrary constant.

Choose the correct answer from the options given below:

1. (A) and (C) only

2. (B) and (D) only

3. (B) and (C) only

4. (A) and (D) only

Options:

A.

1

B.

2

C.

3

D.

4

Answer: C

Solution:

$$x \frac{dy}{dx} + 2y = x^2 \log_e x$$

$$\frac{dy}{dx} + \frac{2}{x} y = x \log_e x$$

$$\text{Integrating factor} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$$

$$\frac{d}{dx} (x^2 y) = x^3 \log x$$

$$x^2 y = \int x^3 \log x dx$$

$$\text{Using parts: } u = \log x, dv = x^3 dx \Rightarrow v = \frac{x^4}{4}$$

$$\int x^3 \log x dx = \frac{x^4}{4} \log x - \int \frac{x^4}{4} \frac{dx}{x} = \frac{x^4}{4} \log x - \frac{1}{4} \int x^3 dx = \frac{x^4}{4} \log x - \frac{x^4}{16}$$

$$x^2 y = \frac{x^4}{4} \log x - \frac{x^4}{16} + C$$

$$y = \frac{x^2}{4} \log x - \frac{x^2}{16} + C x^{-2} = \frac{x^2}{16} (4 \log x - 1) + C x^{-2}$$

Correct answer: (3)

Question 35

If $\sin y = x \cos(a + y)$, then $\frac{dy}{dx}$ is equal to

1. $\frac{\cos^2(a + y)}{\cos a}$
2. $\frac{\cos a}{\cos^2(a + y)}$
3. $\frac{\sin^2 y}{\cos a}$
4. $\frac{\sin^2 y}{\cos^2(a + y)}$

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: A

Solution:

$$\sin y = x \cos(a + y)$$

$$\cos y \frac{dy}{dx} = \cos(a + y) + x(-\sin(a + y)) \frac{dy}{dx}$$

$$(\cos y + x \sin(a + y)) \frac{dy}{dx} = \cos(a + y)$$

$$\frac{dy}{dx} = \frac{\cos(a + y)}{\cos y + x \sin(a + y)}$$

$$\text{Since } \sin y = x \cos(a + y), \text{ then } x = \frac{\sin y}{\cos(a + y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a + y)}{\cos a}$$

Correct option: (1)