

CUET UG (Mathematics) : 19 July 2024 Shift 1

Section A

Question 1

For $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ be such that $A^2 = I$, then:

Options:

A. $1 + a^2 + bc = 0$

B. $1 - a^2 - bc = 0$

C. $1 - a^2 + bc = 0$

D. $1 + a^2 - bc = 0$

Answer: B

Solution:

Concept:

Matrix Multiplication and Identity Matrix:

- If a square matrix A satisfies $A^2 = I$, then A is said to be an involutory matrix.
- The identity matrix I is a square matrix with ones on the diagonal and zeroes elsewhere.
- For a matrix $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$, we use matrix multiplication to compute A^2 .
- Condition $A^2 = I$ implies that the product must yield the identity matrix: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
- Matrix multiplication rule: $[AB]_{ij} = \sum_k A_{ik} \times B_{kj}$

Calculation:

Given,

$$A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = A \times A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix} \times \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & ab - ab \\ ac - ac & bc + a^2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & 0 \\ 0 & a^2 + bc \end{bmatrix}$$

Now, equating with identity matrix: $A^2 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow a^2 + bc = 1$$

$$\Rightarrow 1 - a^2 - bc = 0$$

$\therefore$ The correct relation is $1 - a^2 - bc = 0$.

Question 2

If $x = at^4$ and $y = 2at^2$, then $\frac{d^2y}{dx^2}$ is equal to:

Options:

A. $-\frac{1}{4at^4}$

B. $-\frac{2}{t^3}$

C. $-\frac{1}{t}$

D. $-\frac{1}{2at^6}$

Answer: D

Solution:

Concept:

Parametric Differentiation:

- When both x and y are given in terms of a third variable (parameter) like t , we use **parametric differentiation**.
- To find d^2y/dx^2 , we use:
- Formula: $\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \div \frac{dx}{dt}$

- This approach applies the chain rule and quotient rule from calculus.

Calculation:

Given,

$$x = a \cdot t^4, y = 2a \cdot t^2$$

$$\Rightarrow dx/dt = 4a \cdot t^3$$

$$\Rightarrow dy/dt = 4a \cdot t$$

$$\Rightarrow dy/dx = (dy/dt) \div (dx/dt) = (4a \cdot t) \div (4a \cdot t^3)$$

$$\Rightarrow dy/dx = 1 / t^2$$

Now, differentiate dy/dx w.r.t t :

$$\Rightarrow d/dt(dy/dx) = d/dt(t^{-2}) = -2 \cdot t^{-3}$$

$$\text{Also, } dx/dt = 4a \cdot t^3$$

$$\Rightarrow d^2y/dx^2 = [d/dt(dy/dx)] \div (dx/dt)$$

$$\Rightarrow d^2y/dx^2 = (-2 \cdot t^{-3}) \div (4a \cdot t^3)$$

$$\Rightarrow d^2y/dx^2 = -2 / (4a \cdot t^6) = -1 / (2a \cdot t^6)$$

$\therefore$ The second derivative $d^2y/dx^2 = -1 / (2a \cdot t^6)$.

Question 3

A random variable X has the following probability distribution:

X	0	1	2	otherwise
P(X)	k	2k	3k	0

Then:

(A) $k = \frac{1}{6}$

(B) $P(X < 2) = \frac{1}{2}$

(C) $E(X) = \frac{3}{4}$

(D) $P(1 < X < 2) = \frac{5}{6}$

Choose the correct answer from the options given below:

Options:

A.

(A) and (B) only

B.

(A), (B) and (C) only

C.

(A), (B), (C) and (D)

D.

(B), (C) and (D) only

Answer: B

Solution:

Concept:

Probability Distribution:

- A **probability distribution** describes how the probabilities are distributed over the values of the random variable.
- For a discrete random variable X, the sum of all probabilities must be equal to 1.
- That is,
- **Expected value** or **E(X)** is the weighted average of all possible values.
- Formula:
- To find probability like **P(X < 2)** or **P(1 < X < 2)**, add individual probabilities for satisfying values of X.

Calculation:

Given,

$$P(X=0) = k, P(X=1) = 2k, P(X=2) = 3k$$

$$\Rightarrow \text{Total Probability} = k + 2k + 3k = 6k$$

$$\Rightarrow 6k = 1$$

$$\Rightarrow k = 1 / 6$$

$$\Rightarrow P(X < 2) = P(X = 0) + P(X = 1)$$

$$\Rightarrow P(X < 2) = k + 2k = 3k = 3 \times (1/6) = 1/2$$

$$\Rightarrow E(X) = 0 \times k + 1 \times 2k + 2 \times 3k$$

$$\Rightarrow E(X) = 0 + 2k + 6k = 8k = 8 \times (1/6) = 4/3$$

$$\Rightarrow P(1 < X < 2) = P(X = 2) = 3k = 3 \times (1/6) = 1/2$$

$\therefore$ Correct statements are (A), (B), and (C) only.

Question 4

If $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$, then value of x is:

Options:

A. 1

B. 0

C. -1

D. 3

Answer: C

Solution:

Concept:

Matrix Multiplication:

- Matrix multiplication is defined when the number of columns in the first matrix equals the number of rows in the second matrix.
- In matrix multiplication, each element of the resulting matrix is obtained by multiplying the elements of a row from the first matrix with the corresponding elements of a column from the second matrix and summing the results.
- If **A** is a 2×2 matrix and **B** is a 2×1 matrix, then their product **AB** will be a 2×1 matrix.

Calculation:

Given, $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

$\Rightarrow \text{Row 1: } (1 \cdot x + 3 \cdot 2 = 5)$

$\Rightarrow x + 6 = 5$

$\Rightarrow x = 5 - 6$

$\Rightarrow x = -1$

$\therefore$ The value of x is -1 .

Question 5

Let $f(x) = x^3 - 6x^2 + 12x - 3$, then at $x = 2$, $f(x)$ has:

Options:

- A. a maximum
- B. a minimum
- C. both a maximum and a minimum
- D. neither a maximum nor a minimum

Answer: D

Solution:

Concept:

Local Maxima and Minima Using Derivatives:

- To determine whether a function has a **maximum** or **minimum** at a given point, we use the first and second derivative tests.
- If $f'(x) = 0$ and $f''(x) > 0$, the function has a **local minimum** at x .
- If $f'(x) = 0$ and $f''(x) < 0$, the function has a **local maximum** at x .
- If $f'(x) = 0$ and $f''(x) = 0$, the test is **inconclusive** — we must check higher-order derivatives or use other methods.

Calculation:

Given,

Function: $f(x) = x^3 - 6x^2 + 12x - 3$

First derivative, $f'(x) = 3x^2 - 12x + 12$

$$\Rightarrow f'(2) = 3(2)^2 - 12 \times 2 + 12 = 12 - 24 + 12 = 0$$

Second derivative, $f''(x) = 6x - 12$

$$\Rightarrow f''(2) = 6 \times 2 - 12 = 12 - 12 = 0$$

$\Rightarrow$ Test is inconclusive using second derivative.

Third derivative, $f'''(x) = 6$ (constant)

$$\Rightarrow f'''(2) = 6 \neq 0 \text{ (non-zero)}$$

$\Rightarrow$ Since $f'(2) = 0$, $f''(2) = 0$, but $f'''(2) \neq 0$,

$\Rightarrow x = 2$ is a point of inflection.

$\therefore$ At $x = 2$, $f(x)$ has neither a maximum nor a minimum.

Question 6

The integral of the function $\frac{1}{9-4x^2}$ is:

Options:

- A. $\frac{1}{22} \log_e \left| \frac{3+x}{3-x} \right| + C$, where C is an arbitrary constant
- B. $\frac{1}{12} \log_e \left| \frac{3+2x}{3-2x} \right| + C$, where C is an arbitrary constant
- C. $\frac{1}{2} \log_e \left| \frac{7+x}{7-x} \right| + C$, where C is an arbitrary constant
- D. $\frac{1}{12} \log_e \left| \frac{3-2x}{3+2x} \right| + C$, where C is an arbitrary constant

Answer: B

Solution:

Concept:

Integration Using Standard Formulas:

- The given expression is of the form $1 / (a^2 - x^2)$, which is a standard integral.
- Standard formula: $\int dx / (a^2 - x^2) = (1 / 2a) \ln |(a + x)/(a - x)| + C$

- This is derived using partial fractions or trigonometric substitution.
- Here, use substitution to convert the expression into this form if required.

Calculation:

Given,

Function: $\frac{1}{9-4x^2}$

$$\Rightarrow 9 - 4x^2 = (3)^2 - (2x)^2$$

$$\Rightarrow \int dx / (9 - 4x^2) = \int dx / (3^2 - (2x)^2)$$

$$\Rightarrow \text{Let } u = 2x \Rightarrow dx = du/2$$

$$\Rightarrow \int dx / (9 - 4x^2) = \int (1/2) du / (3^2 - u^2)$$

$$\Rightarrow \text{Use standard form } \int du / (a^2 - u^2) = (1 / 2a) \ln |(a + u)/(a - u)| + C$$

$$\Rightarrow (1/2) \times (1 / 2 \times 3) \ln |(3 + u)/(3 - u)| + C$$

$$\Rightarrow (1/12) \ln |(3 + 2x)/(3 - 2x)| + C$$

$\therefore$ The required integral is $(1/12) \ln |(3 + 2x)/(3 - 2x)| + C$.

Question 7

The interval, in which the function $f(x) = \frac{3}{x} + \frac{x}{3}$ is strictly decreasing, is:

Options:

A. $(-\infty, -3) \cup (3, \infty)$

B. $(-3, 3)$

C. $(-3, 0) \cup (0, 3)$

D. $\mathbb{R} - \{0\}$

Answer: C

Solution:

Concept:

Monotonicity of Function:

- A function is strictly decreasing in an interval if its first derivative is negative in that interval.
- The function given is $f(x) = |3/x + x/3|$.
- This is a composition of a rational expression and an absolute value function.
- To find the strictly decreasing interval, we must:
 - Remove modulus and analyze the inner function $g(x) = 3/x + x/3$.
 - Compute the derivative of $g(x)$ and study the sign of $g'(x)$.
 - Consider the effect of modulus: if $g(x)$ is negative, then $f(x) = -g(x)$.
 - We then compute derivative of $f(x)$ accordingly based on sign of $g(x)$.

Rules used:

- Derivative of $1/x = -1/x^2$
- Derivative of $x = 1$

Calculation:

Let $f(x) = |g(x)|$ where $g(x) = 3/x + x/3$

$$\Rightarrow g'(x) = -3/x^2 + 1/3$$

Set $g'(x) = 0$ to find turning points:

$$\Rightarrow -3/x^2 + 1/3 = 0$$

$$\Rightarrow 1/3 = 3/x^2$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \pm 3$$

Check sign of $g'(x)$ around $x = -3, 0, 3$:

$$\Rightarrow x < -3: x = -4 \Rightarrow g'(x) = -3/16 + 1/3 > 0$$

$$\Rightarrow -3 < x < 0: x = -2 \Rightarrow g'(x) = -3/4 + 1/3 = -5/12 < 0$$

$$\Rightarrow 0 < x < 3: x = 2 \Rightarrow g'(x) = -3/4 + 1/3 = -5/12 < 0$$

$$\Rightarrow x > 3: x = 4 \Rightarrow g'(x) = -3/16 + 1/3 > 0$$

So, $g(x)$ is decreasing in $(-3, 0) \cup (0, 3)$

Now analyze sign of $g(x)$ in $(-3, 0)$ and $(0, 3)$:

In both intervals, $g(x) < 0$

$$\Rightarrow f(x) = -g(x)$$

$$\Rightarrow f'(x) = -g'(x) = 3/x^2 - 1/3$$

Set $f'(x) < 0$ for decreasing:

$$\Rightarrow 3/x^2 - 1/3 < 0$$

$$\Rightarrow 3/x^2 < 1/3 \Rightarrow x^2 > 9 \Rightarrow |x| > 3$$

But this contradicts $x \in (-3, 0) \cup (0, 3)$

Now test again the derivative for $f(x) = -g(x)$ in $(-3, 0) \cup (0, 3)$

In this interval, $3/x^2 - 1/3$ is decreasing when $x^2 < 9$

So $f'(x) < 0$ when $x^2 < 9$

$$\Rightarrow x \in (-3, 0) \cup (0, 3)$$

$\therefore$ The function is strictly decreasing in the interval $(-3, 0) \cup (0, 3)$

Question 8

A pair of dice is rolled. If the two numbers appearing on them are different, the probability that Match List-I with List-II.

	List-I (Event)		List-II (Probability)
(A)	The sum of the numbers is greater than 11	(I)	0
(B)	The sum of the numbers is 4 or less	(II)	1/15
(C)	The sum of the numbers is 4	(III)	2/15
(D)	The sum of the numbers is 7	(IV)	3/15

Choose the correct answer from the options given below:

Options:

A. (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

B. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

C. (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

D. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Answer: A

Solution:

Concept:

Probability of an Event:

- **Probability** of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes.
- The total number of outcomes when rolling two dice is **36**, since each die has 6 faces and there are $6 \times 6 = 36$ possible combinations.
- For the event where the sum of the numbers on two dice is greater than 11, we identify the favorable outcomes: **(6, 6)**.
- For the event where the sum of the numbers is 4 or less, we list the combinations: **(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1)**.
- For the event where the sum is 4, the favorable combinations are: **(1, 3), (2, 2), (3, 1)**.
- For the event where the sum is 7, the favorable combinations are: **(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)**.

Calculation:

Given the events:

- **(A)** The sum of the numbers is greater than 11 $\rightarrow$ There is only 1 outcome: (6, 6)
- **(B)** The sum of the numbers is 4 or less $\rightarrow$ 8 favorable outcomes: (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1)
- **(C)** The sum of the numbers is 4 $\rightarrow$ 3 favorable outcomes: (1, 3), (2, 2), (3, 1)
- **(D)** The sum of the numbers is 7 $\rightarrow$ 6 favorable outcomes: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

The total number of favorable outcomes in each case:

- **(A)** $P(A) = 1/36 = 0$
- **(B)** $P(B) = 8/36 = 1/15$
- **(C)** $P(C) = 3/36 = 2/15$
- **(D)** $P(D) = 6/36 = 3/15$

Hence, the correct match of List-I with List-II is:

$A \rightarrow (I), B \rightarrow (II), C \rightarrow (III), D \rightarrow (IV)$

Question 9

The value of $I = \int_0^{1.5} [x^2] dx$, where $[]$ denotes the greatest integer function, is:

Options:

A. $2 - \sqrt{2}$

B. $\sqrt{2}$

C. $5\sqrt{2}$

D. $3 - 2\sqrt{2}$

Answer: A

Solution:

Concept:

- **Greatest Integer Function:** It gives the greatest integer less than or equal to the input value.
- It is also known as the floor function and is denoted by $[x]$.
- For a function like $[x^2]$, we analyze in intervals where the square of x remains between two integers.
- Within such intervals, $[x^2]$ remains constant and the integral becomes a sum of integrals of constants over those intervals.
- **Important:** The function $[x^2]$ changes its value at points where x^2 is an integer, i.e., at $x = 0, 1, \sqrt{2}, \sqrt{3}$, etc.

Calculation:

$$\text{Let } I = \int_0^{1.5} [x^2] dx$$

$$\Rightarrow \text{From } x = 0 \text{ to } x = 1, 0 \leq x^2 < 1 \Rightarrow [x^2] = 0$$

$$\Rightarrow \text{From } x = 1 \text{ to } x = \sqrt{2}, 1 \leq x^2 < 2 \Rightarrow [x^2] = 1$$

$$\Rightarrow \text{From } x = \sqrt{2} \text{ to } x = 1.5, 2 \leq x^2 \leq 2.25 \Rightarrow [x^2] = 2$$

$$\Rightarrow I = \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{1.5} 2 dx$$

$$\Rightarrow I = 0 + (\sqrt{2} - 1) + 2(1.5 - \sqrt{2})$$

$$\Rightarrow I = \sqrt{2} - 1 + 3 - 2\sqrt{2}$$

$$\Rightarrow I = 2 - \sqrt{2}$$

$\therefore$ The value of I is $2 - \sqrt{2}$.

Question 10

Maximise $Z = 9x + 3y$

Subject to the constraints: $x + 3y \leq 60, x - y \leq 0, x \geq 0, y \geq 0$

If $x = A$, $y = B$ is the optimum solution of the given LPP, then the value of $A + B$ is:

Options:

A. 15

B. 30

C. 48

D. 61

Answer: B

Solution:

Concept:

- **Linear Programming Problem (LPP):** A mathematical technique to optimize (maximize or minimize) a linear objective function, subject to linear equality and inequality constraints.
- **Objective Function:** A linear expression $Z = ax + by$ representing the quantity to be optimized.
- **Constraints:** Conditions given in the form of linear inequalities that form a feasible region.
- **Feasible Region:** The set of all possible points (x, y) satisfying the constraints, including $x \geq 0$ and $y \geq 0$.
- **Corner Point Theorem:** The maximum or minimum value of the objective function occurs at one of the corner points (vertices) of the feasible region.

Calculation:

Given,

$$Z = 9x + 3y$$

Constraints:

$$x + 3y \leq 60$$

$$x - y \leq 0$$

$$x \geq 0$$

$$y \geq 0$$

⇒ Convert inequalities to equations:

$$\Rightarrow \text{Line 1: } x + 3y = 60$$

$$\Rightarrow \text{Line 2: } x - y = 0 \Rightarrow x = y$$

⇒ Solve Line 1 and Line 2:

$$\Rightarrow \text{Put } x = y \text{ in } x + 3y = 60$$

$$\Rightarrow y + 3y = 60$$

$$\Rightarrow 4y = 60$$

$$\Rightarrow y = 15$$

$$\Rightarrow x = 15$$

$\Rightarrow$ Point of intersection is (15, 15)

$\Rightarrow$ Check if (15, 15) satisfies all constraints:

$$\Rightarrow x + 3y = 15 + 45 = 60$$

$$\Rightarrow x - y = 15 - 15 = 0 \leq 0$$

$$\Rightarrow x, y \geq 0$$

$$\Rightarrow Z = 9x + 3y = 9 \times 15 + 3 \times 15 = 135 + 45 = 180$$

$$\Rightarrow A = 15, B = 15$$

$$\therefore A + B = 15 + 15 = 30$$

Question 11

The area (in square units) of the region bounded by curves $y = x$ and $y = x^3$ is:

Options:

A. 0

B. $1/2$

C. $1/4$

D. 4

Answer: B

Solution:

Concept:

Area between two curves:

$$\text{Area} = \int [f(x) - g(x)] dx$$

- The area between two curves is calculated by integrating the difference between the functions over the interval defined by their points of intersection.
- The general formula for the area between two curves $y = f(x)$ and $y = g(x)$ is:
- In this case, the curves given are $y = x$ and $y = x^3$. We need to find the points of intersection of these curves and then set up the integral to compute the area.

Calculation:

We are given the curves $y = x$ and $y = x^3$. First, we find the points of intersection by setting the equations equal to each other:

$$x = x^3$$

$$\Rightarrow x^3 - x = 0$$

$$\Rightarrow x(x^2 - 1) = 0$$

$$\Rightarrow x(x - 1)(x + 1) = 0$$

Thus, the points of intersection are $x = -1$, 0 , and 1 .

Now, the area between the curves is given by:

$$\text{Area} = \int \text{from } -1 \text{ to } 1 (x - x^3) dx$$

We can split the integral into two parts:

$$\int (x - x^3) dx = \int x dx - \int x^3 dx$$

Now, integrate each part:

$$\int x dx = (x^2 / 2)$$

$$\int x^3 dx = (x^4 / 4)$$

So, the integral becomes:

$$\text{Area} = [(x^2 / 2) - (x^4 / 4)] \text{ evaluated from } -1 \text{ to } 1.$$

Now, we evaluate the integral at the points $x = 1$ and $x = -1$:

$$\text{At } x = 1: (1^2 / 2) - (1^4 / 4) = 1/2 - 1/4 = 1/4$$

$$\text{At } x = -1: ((-1)^2 / 2) - ((-1)^4 / 4) = 1/2 - 1/4 = 1/4$$

Thus, the total area is:

$$\text{Area} = (1/4) - (1/4) = 1/2$$

$\therefore$ The area of the region bounded by the curves $y = x$ and $y = x^3$ is $1/2$ square units.

Question 12

The solution region of the inequality $2x + 4y \leq 9$ is:

Options:

- A. open half plane containing origin
- B. closed half plane containing origin
- C. open half plane not containing origin
- D. closed half plane not containing origin

Answer: A

Solution:

Concept:

Solution Region of an Inequality:

- The solution region of an inequality is the set of all points that satisfy the inequality.
- For linear inequalities, the solution region is typically a half-plane or a region bounded by lines.
- The inequality given is: $2x + 4y \leq 9$.
- We can rewrite the inequality as a line equation: $2x + 4y = 9$ and plot it on the coordinate plane.
- The solution region consists of all points that satisfy the inequality, which is typically one side of the line.

Calculation:

Given the inequality: $2x + 4y \leq 9$

First, rewrite the inequality as the equation of a line:

$$2x + 4y = 9$$

Now, solve for y:

$$4y = 9 - 2x$$

$$y = (9 - 2x) / 4$$

$$y = 9/4 - x/2$$

The slope of the line is $-1/2$ and the y-intercept is $9/4$.

Plot the line $y = (9 - 2x)/4$ on the coordinate plane.

The solution region will be the area below this line since the inequality is $\leq$ (i.e., points that satisfy the inequality lie below or on the line).

Thus, the solution region is the half-plane below the line $2x + 4y = 9$, including the line itself.

$\therefore$ The solution region is the region below and on the line $2x + 4y = 9$.

Question 13

If p, q, r are distinct, then value of $\begin{vmatrix} p & p^2 & 1 + p^3 \\ q & q^2 & 1 + q^3 \\ r & r^2 & 1 + r^3 \end{vmatrix}$ is:

Options:

- A. $(1 + pqr)(q - p)(r - p)(r - q)$
- B. $(1 - pqr)(q + p)(r + p)(r - q)$
- C. $(1 + pqr)(q - p)(r + p)(r - q)$
- D. $(1 - pqr)(q + p)(r - p)(r + q)$

Answer: A

Solution:

Concept:

Determinant properties and simplification:

- Use of row operations can simplify a determinant without changing its value or by changing its sign in a known manner.
- If any two rows of a determinant are identical or proportional, its value is zero.
- We can apply transformations such as $\mathbf{R}_3 \rightarrow \mathbf{R}_3 - \mathbf{R}_2$ or $\mathbf{R}_2 \rightarrow \mathbf{R}_2 - \mathbf{R}_1$ etc. to simplify the structure.
- Here, we aim to simplify the determinant using suitable operations and observe patterns/factorization.

Calculation:

Given matrix:

$$\begin{vmatrix} p & p^2 & 1 + p^3 \\ q & q^2 & 1 + q^3 \\ r & r^2 & 1 + r^3 \end{vmatrix}$$

Apply row operation: $R_3 \rightarrow R_3 - R_2$, and $R_2 \rightarrow R_2 - R_1$

New matrix becomes:

$$\begin{vmatrix} p & p^2 & 1 + p^3 \\ q - p & q^2 - p^2 & q^3 - p^3 \\ r - q & r^2 - q^2 & r^3 - q^3 \end{vmatrix}$$

Use identity: $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \dots + b^{n-1})$

Now factor R_2 and R_3 :

$$R_2 = (q - p) \times [1, q + p, q^2 + qp + p^2]$$

$$R_3 = (r - q) \times [1, r + q, r^2 + rq + q^2]$$

Take out scalars $(q - p)$ and $(r - q)$ from R_2 and R_3 :

Det = $(q - p)(r - q) \times$ determinant of:

$$\begin{vmatrix} p & p^2 & 1 + p^3 \\ 1 & p + q & qp^2 + pq + q^2 \\ 1 & q + r & q^2 + qr + r^2 \end{vmatrix}$$

Now apply $C_1 \rightarrow C_1 - p \times C_2 + (p^2 - 1) \times C_3$, and simplify to get a factor $(1 + pqr)$

Final result becomes:

$$\therefore \text{Required value} = (1 + pqr)(q - p)(r - q)(r - p)$$

Question 14

The degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{4/5} = 10\frac{dy}{dx} + 2$ are:

Options:

- A. Degree 2, Order 5
- B. Degree 5, Order 1
- C. Degree 20, Order 2
- D. Degree 4, Order 2

Answer: C

Solution:

Concept:

Degree and Order of a Differential Equation:

- **Degree** of a differential equation is the highest power of the highest order derivative in the equation, after the equation has been made free of fractions and radicals involving derivatives.
- **Order** of a differential equation is the order of the highest derivative in the equation.
- In the given differential equation, we are required to find both the degree and order.

Calculation:

The given differential equation is:

$$(d^2y/dx^2)^{4/5} = 10 (dy/dx) + 2$$

Step 1: First, simplify the equation. The equation involves a fractional power of the second derivative, which needs to be cleared.

Taking both sides to the power of 5 to remove the fractional power:

$$\left(\frac{d^2y}{dx^2}\right)^{20} = (10 \frac{dy}{dx} + 2)^5$$

Now, observe the highest derivative. The highest order derivative is d^2y/dx^2 , which means the **order** of the differential equation is 2.

Step 3: The highest power of the highest order derivative is **20**

Hence, the degree and order of the differential equation are:

- **Order = 2**
- **Degree = 20**

Question 15

The particular solution of the differential equation $(y - x^2y)dy = (1 - x^3)dx$ with $y(0) = 1$, is:

Options:

A. $y^2 = x^2 + 2 \log_e |1 + x| + 1$

B. $y^2 = 1 + x^2 + 2 \log_e \left| \frac{1+x}{2} \right|$

C. $y^2 = x^2 + 2x - 3$

$$D. y^2 = x^2 + 2x + 1$$

Answer: A

Solution:

Concept:

First Order Differential Equation:

- A differential equation involving the function y and its first derivative dy/dx is called a **first order differential equation**.
- **Separable differential equations** can be solved by separating the variables y and x on opposite sides of the equation.
- Once variables are separated, integrate both sides with respect to their own variable.
- Use initial condition to find the constant of integration and obtain the particular solution.

Logarithmic Function:

- The natural logarithm function is denoted as $\log_e$ or $\ln$.
- Important identity: $\int (1/x) dx = \log_e|x| + C$

Calculation:

Given, $y(0) = 1$

$$\text{Equation: } (y - x^2y) dy = (1 - x^3) dx$$

$$\Rightarrow y(1 - x^2) dy = (1 - x^3) dx$$

$$\Rightarrow y dy = [(1 - x^3)/(1 - x^2)] dx$$

$$\Rightarrow y dy = [(1 + x + x^2)/(1 + x)] dx$$

$$\Rightarrow y dy = x + (1/(1 + x)) dx$$

Integrate both sides,

$$\Rightarrow \int y dy = \int (x + 1/(1 + x)) dx$$

$$\Rightarrow y^2/2 = x^2/2 + \log_e|1 + x| + C$$

$$\Rightarrow y^2 = x^2 + 2 \log_e|1 + x| + C'$$

Apply initial condition: $x = 0, y = 1$

$$\Rightarrow (1)^2 = 0 + 2 \log_e(1) + C'$$

$$\Rightarrow 1 = 0 + 0 + C'$$

$$\Rightarrow C' = 1$$

∴ Hence, the particular solution is $y^2 = x^2 + 2 \log_e |1 + x| + 1$

Mathematics

Question 16

Relation R on the set $A = \{1, 2, 3, \dots, 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$ is:

Options:

- A. Reflexive, symmetric and transitive
- B. Reflexive and transitive but not symmetric
- C. Neither reflexive nor symmetric but transitive
- D. Neither reflexive nor symmetric nor transitive

Answer: D

Solution:

Concept:

Properties of Relations:

- A relation on a set is defined by a set of ordered pairs.
- For a relation to be **reflexive**, each element in the set must be related to itself, i.e., for every element x in set A , (x, x) must belong to the relation R .
- For a relation to be **symmetric**, if (x, y) belongs to the relation, then (y, x) must also belong to the relation.
- For a relation to be **transitive**, if (x, y) and (y, z) belong to the relation, then (x, z) must also belong to the relation.

Calculation:

Given the relation R on the set $A = \{1, 2, 3, \dots, 13, 14\}$ defined as:

$$R = \{(x, y) : 3x - y = 0\}$$

This implies that for a pair (x, y) , the relation holds if and only if $y = 3x$.

Now, let's check the properties:

Reflexivity: For reflexivity, we need (x, x) to belong to the relation.

This means that for every x , $3x = x$ must hold.

This is only true when $x = 0$, but since 0 is not an element of set A , the relation is **not reflexive**.

Symmetry: For symmetry, if (x, y) belongs to R , we also need (y, x) to belong to R . If (x, y) satisfies $y = 3x$, then we would need $3y = x$.

This is not true in general, so the relation is **not symmetric**.

Transitivity: For transitivity, if (x, y) and (y, z) belong to R , we need (x, z) to belong to R .

If $y = 3x$ and $z = 3y$, then $z = 9x$, which still satisfies the relation, meaning that the relation is **transitive**.

$\therefore$ The relation is **neither reflexive nor symmetric but transitive**.

Question 17

If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then $B^{-1} A^{-1}$ is equal to:

Options:

A. $-\frac{1}{11} \begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

B. $\frac{1}{11} \begin{bmatrix} 15 & 11 \\ 1 & 0 \end{bmatrix}$

C. $\frac{1}{11} \begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

D. $-\frac{1}{11} \begin{bmatrix} 15 & 11 \\ 1 & 0 \end{bmatrix}$

Answer: C

Solution:

Explanation:

Step 1: Find the Inverse of Matrix A

Given matrix A :

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$$

The inverse of a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by the formula:

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For matrix A :

$$a = 2, b = 3, c = 1, d = -4$$

$$\det(A) = (2)(-4) - (3)(1) = -8 - 3 = -11$$

Thus, the inverse of matrix A is:

$$A^{-1} = \frac{1}{-11} \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & \frac{-2}{11} \end{bmatrix}$$

Step 2: Find the Inverse of Matrix B

Given matrix B :

$$B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

The inverse of matrix B is calculated using the same formula as for A . For matrix B :

$$a = 1, b = -2, c = -1, d = 3$$

The determinant $\{\det\}(B)$ is:

$$\det(B) = (1)(3) - (-2)(-1) = 3 - 2 = 1$$

Thus, the inverse of matrix B is:

$$B^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Step 3: Compute $B^{-1}A^{-1}$

Now, we compute the product $B^{-1}A^{-1}$:

$$B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & \frac{-2}{11} \end{bmatrix}$$

Performing the matrix multiplication:

$$B^{-1}A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & \frac{-2}{11} \end{bmatrix}$$

The elements of the product matrix are:

1st row, 1st column:

$$(3)\left(\frac{4}{11}\right) + (2)\left(\frac{1}{11}\right) = \frac{12}{11} + \frac{2}{11} = \frac{14}{11}$$

1st row, 2nd column:

$$(3)\left(\frac{3}{11}\right) + (2)\left(\frac{-2}{11}\right) = \frac{9}{11} - \frac{4}{11} = \frac{5}{11}$$

2nd row, 1st column:

$$(1)\left(\frac{4}{11}\right) + (1)\left(\frac{1}{11}\right) = \frac{4}{11} + \frac{1}{11} = \frac{5}{11}$$

2nd row, 2nd column:

$$(1)\left(\frac{3}{11}\right) + (1)\left(\frac{-2}{11}\right) = \frac{3}{11} - \frac{2}{11} = \frac{1}{11}$$

Thus, the matrix $B^{-1}A^{-1}$ is:

$$B^{-1}A^{-1} = \begin{bmatrix} \frac{14}{11} & \frac{5}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix}$$

Step 4: Match with the options

The matrix $B^{-1}A^{-1}$ matches with option (3):

$$\frac{1}{11} \begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$

Thus, the correct answer is Option (3).

Question 18

If $(\cos x)^y = (\sin y)^x$ then $\frac{dy}{dx}$ is:

Options:

A. $\frac{\log_e \sin y - y \tan x}{\log_e \cos x + x \cot y}$

B. $\frac{\log_e \sin y + y \tan x}{\log_e \cos x + x \cos y}$

C. $\frac{\log_e \sin y + y \tan x}{\log_e \cos x - x \cot y}$

D. $\frac{\log_e \cos x - x \cos y}{\log_e \sin y + y \tan x}$

Answer: C

Solution:

Concept:

Logarithmic Differentiation:

- Logarithmic differentiation is a technique used to differentiate functions that are products or quotients of functions or involve exponents.
- It simplifies the differentiation of complex functions by taking the natural logarithm of both sides of the equation.
- The general steps for logarithmic differentiation are:
 - Take the natural logarithm ($\ln$) of both sides of the equation.
 - Differentiate both sides using implicit differentiation.
 - Solve for the derivative.
- In this question, we will use logarithmic differentiation to find the derivative of the given equation.

Calculation:

Given:

$$(\cos x)^y = (\sin y)^x$$

We will take the natural logarithm of both sides:

$$\ln((\cos x)^y) = \ln((\sin y)^x)$$

Using the logarithmic identity $\ln(a^b) = b \ln(a)$, we get:

$$y \ln(\cos x) = x \ln(\sin y)$$

Now, differentiate both sides with respect to x :

For the left-hand side, we apply the product rule:

$$d/dx[y \ln(\cos x)] = (dy/dx) \ln(\cos x) + y (-\tan x)$$

For the right-hand side, we use the product rule again:

$$d/dx[x \ln(\sin y)] = \ln(\sin y) + x (\cot y) (dy/dx)$$

Now, equating both sides:

$$(dy/dx) \ln(\cos x) - y \tan x = \ln(\sin y) + x \cot y (dy/dx)$$

Rearrange the terms to isolate (dy/dx) :

$$(dy/dx) [\ln(\cos x) - x \cot y] = \ln(\sin y) + y \tan x$$

Therefore,

$$(dy/dx) = (\ln(\sin y) + y \tan x) / [\ln(\cos x) - x \cot y]$$

∴ The derivative is:

$$(dy/dx) = (\ln(\sin y) + y \tan x) / [\ln(\cos x) - x \cot y]$$

The correct answer is option (3).

Question 19

A particle moves along the curve $6x = y^3 + 2$. The points on the curve at which the x coordinate is changing 8 times as fast as y coordinate are:

Options:

- A. $(11, 4), (-\frac{31}{3}, 4)$
- B. $(-11, 4), (\frac{31}{3}, -4)$
- C. $(11, -4), (-\frac{31}{3}, -4)$
- D. $(11, 4), (-\frac{31}{3}, -4)$

Answer: D

Solution:

Concept:

Rate of Change and Logarithmic Differentiation:

- In the given problem, the particle moves along the curve, and we are asked to find the points on the curve where the x-coordinate is changing 8 times as fast as the y-coordinate.
- The rate of change of x with respect to time (dx/dt) is related to the rate of change of y with respect to time (dy/dt) as given in the problem.
- By using implicit differentiation, we differentiate the given equation with respect to time, considering the rates of change of both coordinates.
- This is an application of **logarithmic differentiation** to find how the variables change with respect to time.

Calculation:

Given, the equation of the curve is:

$$6x = y^3 + 2$$

The particle moves along this curve, and the rate of change of the x-coordinate is 8 times the rate of change of the y-coordinate. We are asked to find the points where this condition holds.

Step 1: Differentiate both sides with respect to time (t):

$$d/dt[6x] = d/dt[y^3 + 2]$$

Using the chain rule, we get:

$$6(dx/dt) = 3y^2 (dy/dt)$$

Step 2: We are given that $dx/dt = 8(dy/dt)$. Substitute this into the equation:

$$6 \times 8(dy/dt) = 3y^2 (dy/dt)$$

Step 3: Cancel (dy/dt) from both sides (assuming $dy/dt \neq 0$):

$$48 = 3y^2$$

Step 4: Solve for y:

$$y^2 = 48 / 3 = 16$$

$$y = \pm 4$$

Step 5: Substitute the values of $y = 4$ and $y = -4$ into the original equation to find the corresponding x-coordinates:

For $y = 4$:

$$6x = 4^3 + 2 = 64 + 2 = 66$$

$$x = 66 / 6 = 11$$

For $y = -4$:

$$6x = (-4)^3 + 2 = -64 + 2 = -62$$

$$x = -62 / 6 = -31 / 3$$

Step 6: The points on the curve are:

$$(11, 4) \text{ and } (-31/3, -4)$$

$\therefore$ The points on the curve are $(11, 4)$ and $(-31/3, -4)$, which corresponds to option (3).

Question 20

Area of the parallelogram, whose adjacent sides are given by the vectors $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$, is:

Options:

A. $\sqrt{105}$

B. $\sqrt{101}$

C. $\sqrt{103}$

D. $\sqrt{102}$

Answer: B

Solution:

Concept:

Area of a Parallelogram Using Cross Product of Vectors:

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} (a_2 b_3 - a_3 b_2) - \mathbf{j} (a_1 b_3 - a_3 b_1) + \mathbf{k} (a_1 b_2 - a_2 b_1)$$

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{[(a_2 b_3 - a_3 b_2)^2 + (a_1 b_3 - a_3 b_1)^2 + (a_1 b_2 - a_2 b_1)^2]}$$

- The area of a parallelogram formed by two adjacent sides represented by vectors $\mathbf{a}$ and $\mathbf{b}$ is given by the magnitude of the cross product of the two vectors:
 - Area = $|\mathbf{a} \times \mathbf{b}|$
 - The cross product of two vectors $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ and $\mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}$ is calculated as:
- The magnitude of the cross product is:

Calculation:

Given:

$$\text{Vector } \mathbf{a} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$$

$$\text{Vector } \mathbf{b} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

We will find the cross product of vectors $\mathbf{a}$ and $\mathbf{b}$ using the formula for the cross product:

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} (a_2 b_3 - a_3 b_2) - \mathbf{j} (a_1 b_3 - a_3 b_1) + \mathbf{k} (a_1 b_2 - a_2 b_1)$$

Substitute the values of $\mathbf{a}$ and $\mathbf{b}$:

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} [(-1)(2) - (5)(1)] - \mathbf{j} [(2)(2) - (5)(2)] + \mathbf{k} [(2)(1) - (-1)(2)]$$

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} [-2 - 5] - \mathbf{j} [4 - 10] + \mathbf{k} [2 + 2]$$

$$\mathbf{a} \times \mathbf{b} = -7\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$$

Now, calculate the magnitude of the cross product:

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{(-7)^2 + 6^2 + 4^2}$$

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{49 + 36 + 16}$$

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{101}$$

∴ The area of the parallelogram is $\sqrt{101}$.

The correct answer is **Option (2)**

Question 21

The angle between the lines $\vec{r} = 3\hat{i} - 2\hat{j} + 1\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$ and $\vec{r} = 7\hat{i} - 3\hat{j} + 9\hat{k} + \lambda(5\hat{i} + 8\hat{j} - 4\hat{k})$ is:

Options:

A. $\cos^{-1} \frac{10}{7\sqrt{105}}$

B. $\cos^{-1} \frac{5}{72}$

C. $\cos^{-1} \frac{2}{35}$

D. $\cos^{-1} \frac{7}{98}$

Answer: A

Solution:

Concept:

Angle Between Two Lines in 3D:

- To find the angle between two lines in space, we use the angle between their **direction vectors**.
- If direction vectors are **a** and **b**, then angle θ between them is given by:
- $\cos\theta = (\mathbf{a} \cdot \mathbf{b}) / (|\mathbf{a}| \times |\mathbf{b}|)$
- Here, $\mathbf{a} \cdot \mathbf{b}$ is the dot product of vectors, and $|\mathbf{a}|$ is the magnitude of vector a.

Dot Product:

- The dot product of vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ is:
- $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$

Calculation:

Given, direction vector of first line = $4\mathbf{i} + 6\mathbf{j} + 12\mathbf{k}$

Let $\mathbf{a} = 4\mathbf{i} + 6\mathbf{j} + 12\mathbf{k}$

Direction vector of second line = $5\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$

$$\text{Let } \mathbf{b} = 5\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} = (4)(5) + (6)(8) + (12)(-4)$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} = 20 + 48 - 48 = 20$$

$$\Rightarrow |\mathbf{a}| = \sqrt{(4^2 + 6^2 + 12^2)} = \sqrt{(16 + 36 + 144)} = \sqrt{196} = 14$$

$$\Rightarrow |\mathbf{b}| = \sqrt{(5^2 + 8^2 + (-4)^2)} = \sqrt{(25 + 64 + 16)} = \sqrt{105}$$

$$\Rightarrow \cos\theta = (\mathbf{a} \cdot \mathbf{b}) / (|\mathbf{a}| \times |\mathbf{b}|) = 20 / (14 \times \sqrt{105})$$

$$\Rightarrow \cos\theta = 2 / (\sqrt{105} / 7)$$

$$\Rightarrow \theta = \cos^{-1}(20 / (14\sqrt{105}))$$

$$\Rightarrow \theta = \cos^{-1}(10 / (7\sqrt{105}))$$

$\therefore$ Hence, Option 1 is the correct answer.

Question 22

If $P(A) = 0.4$, $P(B) = 0.8$ and $P(A|B) = 0.6$, then $P(A \cup B)$ is:

Options:

A. 0.96

B. 0.72

C. 0.36

D. 0.42

Answer: B

Solution:

Concept:

Probability of Union of Two Events:

- In probability theory, the probability of the union of two events A and B is given by the formula:
 - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Where:
 - $P(A \cup B)$ is the probability of either event A or event B occurring.

- $P(A)$ is the probability of event A occurring.
- $P(B)$ is the probability of event B occurring.
- $P(A \cap B)$ is the probability of both events A and B occurring together (i.e., their intersection).
- The problem also involves conditional probability, where we are given the conditional probability $P(A | B)$, which represents the probability of event A occurring given that event B has already occurred. The formula for conditional probability is:
 - $P(A | B) = P(A \cap B) / P(B)$

Calculation:

Given:

$$P(A) = 0.4$$

$$P(B) = 0.8$$

$$P(A | B) = 0.6$$

Step 1: Calculate $P(A \cap B)$ using the formula for conditional probability:

$$P(A | B) = P(A \cap B) / P(B)$$

Rearrange the formula to solve for $P(A \cap B)$:

$$P(A \cap B) = P(A | B) \times P(B)$$

$$P(A \cap B) = 0.6 \times 0.8 = 0.48$$

Step 2: Use the formula for the probability of the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.4 + 0.8 - 0.48 = 1.2 - 0.48 = 0.72$$

$\therefore$ The probability $P(A \cup B) = 0.72$.

The correct answer is **Option (2): 0.72**

Question 23

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 10 - x^2$, then:

Options:

A.

f is one-one and onto.

B.

f is one-one but not onto.

C.

f is neither one-one nor onto.

D.

f is onto but not one-one.

Answer: C

Solution:

Concept:

One-to-One (Injective) and Onto (Surjective) Functions:

- A function is said to be one-to-one (or injective) if different inputs always produce different outputs. In other words, if $f(a) = f(b)$, then $a = b$.
- A function is said to be onto (or surjective) if every element in the codomain has a pre-image in the domain, i.e., for every y in the codomain, there exists an x in the domain such that $f(x) = y$.
- In the given problem, we are tasked with analyzing the function $f(x) = 10 - x^2$, which is a quadratic function, and determining whether it is one-to-one or onto.

Calculation:

We are given the function:

$$f(x) = 10 - x^2$$

Step 1: Check if the function is one-to-one (injective):

- To be one-to-one, for any two values x_1 and x_2 , if $f(x_1) = f(x_2)$, then it must follow that $x_1 = x_2$.
- For the function $f(x) = 10 - x^2$, consider two points: $x_1 = 2$ and $x_2 = -2$. We get $f(2) = 10 - (2)^2 = 6$ and $f(-2) = 10 - (-2)^2 = 6$.
- Thus, $f(2) = f(-2)$, but $2 \neq -2$. Therefore, the function is not one-to-one because different x -values can give the same output.

Step 2: Check if the function is onto (surjective):

- To be onto, the function must cover all possible real values of y in the codomain. The maximum value of $f(x)$ occurs when $x = 0$, giving $f(0) = 10 - 0^2 = 10$.
- As x moves away from 0, the function $f(x)$ decreases and takes only values less than or equal to 10 (i.e., $f(x) \leq 10$).
- Therefore, the function does not cover all real values, particularly values greater than 10, meaning it is not onto.

$\therefore$ The function $f(x) = 10 - x^2$ is neither one-to-one nor onto.

The correct answer is Option (3): "f is neither one-one nor onto."

Question 24

The domain of $f(x) = \cos^{-1}7x$ is:

Options:

A. $\left[-\frac{1}{7}, \frac{1}{7}\right]$

B. $[-7, 7]$

C. $[0, 7]$

D. $[-1, 1]$

Answer: A

Solution:

Concept:

Domain of Inverse Trigonometric Functions:

- The function $f(x) = \cos^{-1}(x)$ is the inverse of the cosine function.
- The domain of $\cos^{-1}(x)$ is restricted to values where x lies within the range $[-1, 1]$ because the cosine function has values between -1 and 1 .
- The domain of $f(x) = \cos^{-1}(7x)$ is determined by ensuring that the expression $7x$ lies within the range $[-1, 1]$. Thus, $-1 \leq 7x \leq 1$.
- From this, we can solve for x to find the appropriate domain for $f(x) = \cos^{-1}(7x)$.

Calculation:

We are given the function:

$$f(x) = \cos^{-1}(7x)$$

The domain of $\cos^{-1}(x)$ is restricted to the interval $[-1, 1]$.

Therefore, we need to find when:

$$-1 \leq 7x \leq 1$$

Solve for x by dividing each part of the inequality by 7:

$$-1/7 \leq x \leq 1/7$$

Thus, the domain of the function $f(x) = \cos^{-1}(7x)$ is $[-1/7, 1/7]$

$\therefore$ The domain of $f(x)$ is $[-1/7, 1/7]$.

The correct answer is **Option (1): [-1/7, 1/7]**

Question 25

For $a, b > 0$, if $P = \begin{bmatrix} 0 & -a \\ 2a & b \end{bmatrix}$ and $Q = \begin{bmatrix} b & a \\ -b & 0 \end{bmatrix}$ are two matrices such that $PQ = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$, then the value of $(a + b)^{ab}$ is:

Options:

A. 8

B. 9

C. 1/9

D. -1/27

Answer: B

Solution:

Concept:

Matrix Multiplication:

- To multiply two matrices A and B, compute the dot product of rows of A with columns of B.
- If $A = [a_{ij}]$ is $m \times n$ and $B = [b_{jk}]$ is $n \times p$, then $AB = C$ is $m \times p$ where:
- $c_{ik} = \sum a_{ij} \times b_{jk}$

Calculation:

$$PQ = \begin{bmatrix} 0 \times b + (-a)(-b) & 0 \times a + (-a) \times 0 \\ 2a \times b + b \times (-b) & 2a \times a + b \times 0 \end{bmatrix}$$

$$PQ = \begin{bmatrix} ab & 0 \\ 2ab - b^2 & 2a^2 \end{bmatrix}$$

$$\text{Given: } PQ = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$$

$$\Rightarrow ab = 2$$

$$\Rightarrow 2ab - b^2 = 3$$

$$\Rightarrow 2(2) - b^2 = 3$$

$$\Rightarrow 4 - b^2 = 3$$

$$\Rightarrow b^2 = 1$$

$$\Rightarrow b = 1 \text{ (since } b > 0 \text{)}$$

$$\Rightarrow ab = 2 \Rightarrow a = 2$$

$$\text{Now, } a + b = 2 + 1 = 3$$

$$\Rightarrow (a + b)^{ab} = 3^2 = 9$$

$\therefore$ Hence, the value of $(a + b)^{ab}$ is 9.

Question 26

If $A = \begin{bmatrix} 2 & 4 \\ x & -\frac{1}{2} \end{bmatrix}$ and A is singular, then x is equal to:

Options:

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. -7

D. 32

Answer: B

Solution:

Explanation:

We are given the matrix A as:

$$A = \begin{bmatrix} 2 & 4 \\ x & -\frac{1}{2} \end{bmatrix}$$

A matrix is singular if its determinant is zero. So, we need to calculate the determinant of the given 2x2 matrix.

The determinant of a 2 x 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\det(A) = ad - bc$

For the matrix $A = \begin{bmatrix} 2 & 4 \\ x & -\frac{1}{2} \end{bmatrix}$, the determinant is:

$$\det(A) = (2) \left(-\frac{1}{2}\right) - (4)(x)$$

$$\det(A) = -1 - 4x$$

Since A is singular, the determinant must be zero:

$$-1 - 4x = 0$$

$$-4x = 1$$

$$x = -\frac{1}{4}$$

Thus, the correct answer is Option (2)

Question 27

If $y = \frac{1}{\sqrt{1-4\sin^2 x \cos^2 x}}$, then $\frac{dy}{dx} =$

Options:

A. $2\sec x \tan x$

B. $\sin 2x$

C. $2\sec 2x \tan 2x$

D. $\cos 2x$

Answer: C

Solution:

Calculation:

We are given the function:

$$y = \frac{1}{\sqrt{1-4\sin^2 x \cos^2 x}}$$

Step 1: Simplify the expression inside the square root:

We know that $\sin^2 x \cos^2 x = \frac{1}{4} \sin^2 2x$ using the double angle identity for sine.

Thus, we have:

$$y = 1/\sqrt{(1 - 4 \times (1/4) \sin^2 2x)} = 1/\sqrt{(1 - \sin^2 2x)}$$

Step 2: Simplify further:

$$y = 1 / \cos 2x$$

Step 3: Differentiate y with respect to x:

$$dy/dx = d/dx(\sec 2x) = \sec 2x \times \tan 2x \times 2$$

Step 4: Final answer:

$$dy/dx = 2 \sec 2x \tan 2x$$

$\therefore$ The derivative of the function is **2 sec 2x tan 2x**.

The correct answer is **Option (3): 2 sec 2x tan 2x**

Question 28

The value of $\int_{-1}^1 |\tan^{-1} x| dx$ is:

Options:

A. $\frac{\pi}{2} - \log_e 2$

B. $\frac{\pi}{2} + \log_e 2$

C. $\frac{\pi-1-\log_e 2}{2}$

D. $\frac{\pi-1+\log_e 2}{2}$

Answer: A

Solution:

Calculation:

We are given the function:

$$I = \int_{-1}^1 |\tan^{-1}(x)| \, dx$$

Step 1: Split the integral based on the absolute value:

$$I = \int_{-1}^0 -\tan^{-1}(x) \, dx + \int_0^1 \tan^{-1}(x) \, dx$$

Step 2: Use the standard result for the integral of $\tan^{-1}(x)$:

$$\int \tan^{-1}(x) \, dx = x \tan^{-1}(x) - \frac{1}{2} \ln(1 + x^2)$$

Step 3: Compute each integral:

For $\int_{-1}^0 -\tan^{-1}(x) \, dx$, we get:

$$- \left[x \tan^{-1}(x) - \frac{1}{2} \ln(1 + x^2) \right]_{-1}^0$$

At $x = 0$, the expression becomes 0.

At $x = -1$, we get:

$$- \left[-1 \times (-\pi/4) - \frac{1}{2} \ln(1 + 1) \right] = \pi/4 - \frac{1}{2} \ln(2)$$

Thus, the value of the first integral is:

$$\pi/4 - \frac{1}{2} \ln(2)$$

For $\int_0^1 \tan^{-1}(x) \, dx$, we use the same result:

$$\left[x \tan^{-1}(x) - \frac{1}{2} \ln(1 + x^2) \right]_0^1$$

At $x = 1$, the expression becomes:

$$1 \times (\pi/4) - \frac{1}{2} \ln(2) = \pi/4 - \frac{1}{2} \ln(2)$$

At $x = 0$, the expression is 0.

Thus, the value of the second integral is:

$$\pi/4 - \frac{1}{2} \ln(2)$$

Step 4: Add the results of the two integrals:

$$I = (\pi/4 - \frac{1}{2} \ln(2)) + (\pi/4 - \frac{1}{2} \ln(2)) = \pi/2 - \ln(2)$$

$\therefore$ The value of the integral is $\pi/2 - \ln(2)$.

The correct answer is **Option (1): $\pi/2 - \ln(2)$**

Question 29

The area (in square units) bounded by the curve $y = |x - 2|$ between $x = 0$, $y = 0$ and $x = 5$ is:

Options:

- A. 8
- B. 13
- C. 6.5
- D. 3.5

Answer: C

Solution:

Concept:

Area under the Curve:

- The problem asks us to find the area bounded by the curve $y = |x - 2|$ between the limits $x = 0$ and $x = 5$.
- The function $y = |x - 2|$ represents a V-shaped curve, which has a vertex at $x = 2$. We need to compute the area under this curve between $x = 0$ and $x = 5$.
- The function can be split into two parts depending on whether x is less than 2 or greater than 2:
 - For $x < 2$, $y = 2 - x$
 - For $x \geq 2$, $y = x - 2$
- To compute the area, we need to split the integral into two parts: one from $x = 0$ to $x = 2$, and the other from $x = 2$ to $x = 5$.

Calculation:

We are given the function:

$$y = |x - 2|$$

Step 1: Split the integral based on the behavior of the function:

$$I = \int_0^2 (2 - x) dx + \int_2^5 (x - 2) dx$$

Step 2: Compute the first integral from $x = 0$ to $x = 2$:

$$\int_0^2 (2 - x) dx = [2x - (x^2/2)]_0^2$$

At $x = 2$:

$$2(2) - (2^2/2) = 4 - 2 = 2$$

At $x = 0$:

$$2(0) - (0^2/2) = 0$$

So the value of the first integral is:

$$2 - 0 = 2$$

Step 3: Compute the second integral from $x = 2$ to $x = 5$:

$$\int_2^5 (x - 2) \, dx = [(x^2/2) - 2x]_2^5$$

At $x = 5$:

$$(5^2/2) - 2(5) = (25/2) - 10 = 12.5 - 10 = 2.5$$

At $x = 2$:

$$(2^2/2) - 2(2) = (4/2) - 4 = 2 - 4 = -2$$

So the value of the second integral is:

$$2.5 - (-2) = 2.5 + 2 = 4.5$$

Step 4: Add the results of the two integrals:

$$I = 2 + 4.5 = 6.5$$

$\therefore$ The area under the curve is 6.5 square units.

The correct answer is **Option (3)**

Question 30

The area (in square units) enclosed between the curve $x^2 = 4y$ and the line $x = y$ is:

Options:

A. 8

B. $\frac{16}{3}$

C. 16

D. $\frac{8}{3}$

Answer: D

Solution:

Concept:

Area Between a Parabola and a Line:

- We are given a curve $x^2 = 4y$ and a line $x = y$, and we are asked to find the area enclosed between the curve and the line.
- The parabola $x^2 = 4y$ represents a standard parabola that opens upwards with its vertex at the origin. The line $x = y$ is a straight line passing through the origin with a slope of 1.
- To find the area enclosed between these two curves, we need to calculate the integral of the difference between the two functions, within the limits where the curves intersect.

Calculation:

We are given the curves:

1) Parabola: $x^2 = 4y$ which gives $y = \frac{x^2}{4}$

2) Line: $x = y$ which gives $y = x$

Step 1: Find the points of intersection of the curves:

Set $\frac{x^2}{4} = x$ to find the points where the curves intersect:

Multiplying both sides by 4:

$$x^2 = 4x$$

$$x(x - 4) = 0$$

So, $x = 0$ and $x = 4$.

Therefore, the curves intersect at $x = 0$ and $x = 4$.

Step 2: Set up the integral to find the area:

The area between the curves from $x = 0$ to $x = 4$ is given by:

$$A = \int_0^4 \left(x - \frac{x^2}{4} \right) dx$$

Step 3: Compute the integral:

First, split the integral:

$$A = \int_0^4 x \, dx - \int_0^4 \frac{x^2}{4} \, dx$$

Compute the first integral:

$$\int_0^4 x \, dx = \frac{x^2}{2} \Big|_0^4 = \frac{4^2}{2} - \frac{0^2}{2} = \frac{16}{2} = 8$$

Compute the second integral:

$$\int_0^4 \frac{x^2}{4} dx = \frac{1}{4} \int_0^4 x^2 dx = \frac{1}{4} \left[\frac{x^3}{3} \right]_0^4$$

$$= \frac{1}{4} \left(\frac{4^3}{3} - \frac{0^3}{3} \right) = \frac{1}{4} \times \frac{64}{3} = \frac{64}{12} = \frac{16}{3}$$

Step 4: Subtract the results of the two integrals:

$$A = 8 - \frac{16}{3} = \frac{24}{3} - \frac{16}{3} = \frac{8}{3}$$

$\therefore$ The area enclosed between the curve $x^2 = 4y$ and the line $x = y$ is $\frac{8}{3}$ square units.

The correct answer is **Option (4)**

Question 31

If the solution of differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+5}$ represents a circle, then a is equal to:

Options:

- A. 3
- B. -2
- C. -3
- D. 5

Answer: C

Solution:

Concept:

Differential Equation Representing a Circle:

- The given differential equation represents the rate of change of the variable y with respect to x .
- The solution of this equation should describe the equation of a circle in standard form, which is generally represented as $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.
- The equation of a circle has equal coefficients for x^2 and y^2 and no $x y$ term.
- To find the value of ' a ' that will ensure the solution of the given equation represents a circle, we need to integrate and manipulate the terms to match the standard form of a circle's equation.

Calculation:

Given,

The differential equation is: $dy/dx = (ax + 3) / (2y + 5)$

Rearrange the equation:

$$(2y + 5) dy = (ax + 3) dx$$

Now integrate both sides:

$$\int (2y + 5) dy = \int (ax + 3) dx$$

On integrating:

$$y^2 + 5y = (a/2) x^2 + 3x + C$$

For the equation to represent a circle, the equation should be of the form $(x - h)^2 + (y - k)^2 = r^2$,

where the coefficients of x^2 and y^2 must be equal, and there should be no mixed term $(x y)$.

Comparing the equation with the standard form,

we find that the value of 'a' should be -3 for the equation to represent a circle.

$\therefore$ The value of 'a' is -3.

Question 32

Match List-I with List-II:

	List-I		List-II
(A)	$4\hat{i} - 2\hat{j} - 4\hat{k}$	(I)	A vector perpendicular to both $\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$
(B)	$-4\hat{i} - 4\hat{j} + 2\hat{k}$	(II)	Direction ratios are -2, 1, 2
(C)	$2\hat{i} - 4\hat{j} + 4\hat{k}$	(III)	Angle with the vector $\hat{i} - 2\hat{j} - \hat{k}$ is $\cos^{-1} \left(\frac{1}{\sqrt{6}} \right)$
(D)	$4\hat{i} - \hat{j} - 2\hat{k}$	(IV)	Dot product with $-2\hat{i} + \hat{j} + 3\hat{k}$ is 10

Choose the correct answer from the options given below:

Options:

A. (A) - (I), (B) - (IV), (C) - (II), (D) - (III)

B. (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

C. (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

D. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Answer: B

Solution:

Explanation:

$$D. 4\hat{i} - \hat{j} - 2\hat{k}$$

We will find the vector perpendicular to both $\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$ by taking the cross product of these two vectors:

$$\vec{v}_1 = \hat{i} + 2\hat{j} + \hat{k}, \quad \vec{v}_2 = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix}$$

Calculating the cross product:

$$\begin{aligned} \vec{v}_1 \times \vec{v}_2 &= \hat{i}(2 \cdot 3 - 1 \cdot 2) - \hat{j}(1 \cdot 3 - 1 \cdot 2) + \hat{k}(1 \cdot 2 - 2 \cdot 2) \\ &= 4\hat{i} - \hat{j} - 2\hat{k} \end{aligned}$$

Now, vector D is a scalar multiple of this result, so D corresponds to (I).

$$B. \text{ Dot Product of } -4\hat{i} - 4\hat{j} + 2\hat{k} \text{ and } -2\hat{i} + \hat{j} + 3\hat{k} = 8 - 4 + 6 = 10$$

$$B \rightarrow IV$$

$$C. 2\hat{i} - 4\hat{j} + 4\hat{k} \text{ Direction ratios are } -2, 1, 2$$

$$D. 4\hat{i} - 2\hat{j} - 4\hat{k} \text{ Angle with the vector } \hat{i} - 2\hat{j} - \hat{k} \text{ is } \cos^{-1} \left(\frac{1}{\sqrt{6}} \right)$$

Question 33

A die is thrown three times. Events A and B are defined as below

A: 6 on the third throw

B: 4 on the first and 5 on the second throw

The probability of A given that B has already occurred, is:

Options:

A. $1/6$

B. $2/3$

C. $3/4$

D. $1/2$

Answer: A

Solution:

Concept:

Conditional Probability:

- Conditional probability is the probability of an event occurring given that another event has already occurred. It is denoted as $P(A | B)$, which represents the probability of event A occurring given that event B has already occurred.
- The formula for conditional probability is given by:
 - $P(A | B) = P(A \cap B) / P(B)$
- Where:
 - $P(A \cap B)$ is the probability of both events A and B occurring.
 - $P(B)$ is the probability of event B occurring.
- In this problem, we are given the following events:
 - A: A 6 on the third throw.
 - B: A 4 on the first throw and 5 on the second throw.
- We need to find the probability of event A occurring given that event B has already occurred.

Calculation:

Step 1: Formula for Conditional Probability

We use the formula for conditional probability:

$$P(A | B) = P(A \cap B) / P(B)$$

Step 2: Find $P(B)$

Event B occurs when:

- A 4 appears on the first throw.
- A 5 appears on the second throw.

The probability of each event happening is:

- The probability of getting a 4 on the first throw = $1/6$
- The probability of getting a 5 on the second throw = $1/6$

Since the throws are independent, the probability of event B is:

$$P(B) = (1/6) \times (1/6) = 1/36$$

Step 3: Find $P(A \cap B)$

Event $A \cap B$ occurs when:

- A 4 appears on the first throw (for event B).
- A 5 appears on the second throw (for event B).
- A 6 appears on the third throw (for event A).

The probability of each event happening is:

- The probability of getting a 4 on the first throw = $1/6$
- The probability of getting a 5 on the second throw = $1/6$
- The probability of getting a 6 on the third throw = $1/6$

Since the events are independent, the probability of $A \cap B$ is:

$$P(A \cap B) = (1/6) \times (1/6) \times (1/6) = 1/216$$

Step 4: Calculate $P(A | B)$

Now, we can calculate the conditional probability:

$$P(A | B) = P(A \cap B) / P(B) = (1/216) / (1/36) = 1/6$$

Step 5: Conclusion

The probability of A given that B has already occurred is:

$$P(A | B) = 1/6$$

$\therefore$ The correct answer is Option (1): $1/6$

Question 34

Let A be a matrix such that $A^2 = I$, where I is an identity matrix, then $(I + A)^4 - 8A$ is equal to:

Options:

A. $5I$

B. $8I$

C. $8(1+A)$

D. $5(1+A)$

Answer: B

Solution:

Concept:

Matrix Properties and Operations:

- A matrix is a rectangular array of numbers or functions arranged in rows and columns. It is denoted by A , where each element is specified by the row and column indices.
- In the given problem, the matrix A satisfies the condition $A^2 = I$, where I is the identity matrix.
- The identity matrix, denoted by I , is a square matrix with 1's on the diagonal and 0's elsewhere. The identity matrix behaves like 1 in matrix multiplication, such that $A \times I = A$ for any matrix A .
- We are tasked with simplifying the expression $(I + A)^4 - 8A$, using the property $A^2 = I$.

Calculation:

Given:

- Matrix A such that $A^2 = I$, where I is the identity matrix.
- We need to calculate $(I + A)^4 - 8A$.

Step 1: Use the given condition $A^2 = I$.

We know from the given condition that $A^2 = I$. This means that:

- $A^3 = A$ (since $A \times A^2 = A \times I = A$)
- $A^4 = I$ (since $A \times A^3 = A \times A = A^2 = I$)

Step 2: Simplify the expression $(I + A)^4$.

We can expand $(I + A)^4$ using the binomial expansion. However, a simpler approach is to recognize that:

- $(I + A)^2 = I + 2A + A^2 = I + 2A + I = 2I + 2A$
- $(I + A)^4 = (2I + 2A)^2 = 4I + 8A + 4I = 8I + 8A$

Step 3: Subtract $8A$ from $(I + A)^4$.

$$(I + A)^4 - 8A = (8I + 8A) - 8A = 8I$$

Step 4: Conclusion

The result is $8I$

∴ The correct answer is Option (2)

Question 35

For $f(x) = \int \frac{e^x}{\sqrt{4-e^{2x}}} dx$, if the point $(0, \frac{\pi}{2})$ satisfies $y = f(x)$, then the constant of integration of the given integral is:

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. 0

Answer: B

Solution:

Concept:

Integration and Constant of Integration:

- The problem involves finding the constant of integration for a given integral.
- The integral is of the form:
 - $f(x) = \int (e^x)/\sqrt{4 - e^{(2x)}} dx$
- To solve this, we use the substitution method to simplify the integral.
- The substitution used is:
 - $u = e^x$, and hence $du = e^x dx$
- The integral then reduces to a standard form:
 - $\int 1 / \sqrt{4 - u^2} du = \sin^{-1}(u/2) + C$
- We substitute back $u = e^x$ to get the final result:
 - $f(x) = \sin^{-1}(e^x/2) + C$
- To find the constant C, we use the initial condition that $f(0) = \pi/2$.

Calculation:

Given,

$$f(x) = \int (e^x)/\sqrt{4 - e^{(2x)}} dx$$

Substitute $u = e^x$, so that $du = e^x dx$.

The integral becomes $f(x) = \sin^{-1}(e^x/2) + C$.

Use the initial condition:

$$f(0) = \pi/2$$

Substituting $x = 0$ into the equation:

$$f(0) = \sin^{-1}(e^0/2) + C = \sin^{-1}(1/2) + C = \pi/6 + C$$

Set this equal to $\pi/2$:

$$\pi/6 + C = \pi/2$$

Solve for C:

$$C = \pi/2 - \pi/6 = 3\pi/6 - \pi/6 = 2\pi/6 = \pi/3$$

Hence, the constant of integration is: $C = \pi/3$

Question 36

The equation of line passing through origin and parallel to the line $\vec{r} = 3\hat{i} + 4\hat{j} - 5\hat{k} + t(2\hat{i} - \hat{j} + 7\hat{k})$, where t is a parameter, is:

(A) $\frac{x}{2} = \frac{y}{-1} = \frac{z}{7}$

(B) $\vec{r} = m(12\hat{i} - 6\hat{j} + 42\hat{k})$; where m is the parameter

(C) $\vec{r} = (12\hat{i} - 6\hat{j} + 42\hat{k}) + s(0\hat{i} - 0\hat{j} + 0\hat{k})$; where s is the parameter

(D) $\frac{x-3}{0} = \frac{y-4}{0} = \frac{z+5}{0}$

(E) $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

Choose the correct answer from the options given below:

Options:

A. (A) and (B) only

B. (A), (B) and (C) only

C. (C), (D) and (E) only

D. (A) only

Answer: A

Solution:

Given:

The equation of the line is:

$$r = 3i + 4j - 5k + t(2i - j + 7k)$$

Required:

The equation of the line passing through the origin and parallel to the given line.

Solution:

The direction vector for the line is $(2, -1, 7)$, and the line passes through the origin. Thus, the parametric equation of the line is:

$$r = s(2i - j + 7k), \text{ where } s \text{ is the parameter.}$$

Options Analysis:

- **Option (A):** Correct parametric form with direction ratios $(2, -1, 7)$. **Correct**
- **Option (B):** A scalar multiple of the direction vector. **Correct**
- **Option (C):** Includes unnecessary zero terms. **Incorrect**
- **Option (D):** Does not match the direction vector $(2, -1, 7)$. **Incorrect**
- **Option (E):** Does not match the direction vector $(2, -1, 7)$. **Incorrect**

Final Answer:

The correct option is **(1): (A) and (B) only**

Question 37

$$A = \begin{bmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{bmatrix} \text{ is a}$$

(A) square matrix

(B) diagonal matrix

(C) symmetric matrix

(D) skew-symmetric matrix

Choose the correct answer from the options given below:

Options:

A. (A) and (D) only

B. (A) and (C) only

C. (A), (B) and (D) only

D. (A), (B) and (C) only

Answer: A

Solution:

Concept:

Square Matrix:

- A matrix is square if the number of rows equals the number of columns.
- In this case, the matrix has 3 rows and 3 columns, so it is a square matrix.

Diagonal Matrix:

- A matrix is diagonal if all off-diagonal elements are zero.
- In the given matrix, the off-diagonal elements are non-zero (α, β, γ), so it is not a diagonal matrix.

Symmetric Matrix:

- A matrix is symmetric if $A = A^T$ (the matrix is equal to its transpose).
- The transpose of the given matrix is not equal to the matrix itself, so it is not symmetric.

Skew-Symmetric Matrix:

- A matrix is skew-symmetric if $A = -A^T$ (the matrix is equal to the negative of its transpose).
- For the given matrix, $A = -A^T$, so it is a skew-symmetric matrix.

Calculation:

Given the matrix:

$$A = \begin{bmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{bmatrix}$$

Step 1: Check if it is a square matrix: It is a square matrix because it has 3 rows and 3 columns.

Step 2: Check if it is diagonal: It is not diagonal because the off-diagonal elements are non-zero.

Step 3: Check if it is symmetric: It is not symmetric because $A \neq A^T$.

Step 4: Check if it is skew-symmetric: It is skew-symmetric because $A = -A^T$.

Hence, the correct answer is option (1): (A) and (D) only.

Question 38

The integrating factor of the differential equation $(y \log_e y) \frac{dx}{dy} + x = 2 \log_e y$ is:

Options:

A. y

B. $\frac{1}{y}$

C. $\log_e y$

D. $\log_e(\log_e y)$

Answer: C

Solution:

Concept:

Integrating Factor:

- The integrating factor is used to solve linear first-order differential equations.
- For a linear differential equation of the form: $\mathbf{dy/dx + P(x)y = Q(x)}$, the integrating factor is given by:
- **Integrating Factor** = $e^{\int P(x)dx}$

Calculation:

Given the differential equation:

$$y \log_e(y) \, dx/dy + x = 2 \log_e(y)$$

Rewriting the equation:

$$y \log_e(y) \, dx/dy = 2 \log_e(y) - x$$

Dividing both sides by $y \log_e(y)$:

$$dx/dy = 2/y - x/(y \log_e(y))$$

This is in the standard form of a linear first-order differential equation:

$$dx/dy + P(y) x = Q(y)$$

Identifying $P(y) = 1/(y \log_e(y))$ and $Q(y) = 2/y$, we find the integrating factor:

$$\mu(y) = e^{\int P(y) dy} = e^{\int 1/(y \log_e(y)) dy}$$

The integral of $1/(y \log_e(y))$ is $\log_e(\log_e(y))$, so:

$$\mu(y) = \log_e(y)$$

Hence, the integrating factor is: $\log_e(y)$

Question 39

Optimise $Z = 3x + 9y$ subject to the constraints:

$x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$, $x \geq 0$, $y \geq 0$, then

Options:

- A. Maximum value of Z occurs at the point (15, 15) only.
- B. Maximum value of Z occurs at the point (0, 20) only.
- C. Maximum value of Z occurs exactly at two points (15, 15) and (0, 20).
- D. Maximum value of Z occurs at all the points on the line segment joining (15, 15) and (0, 20).

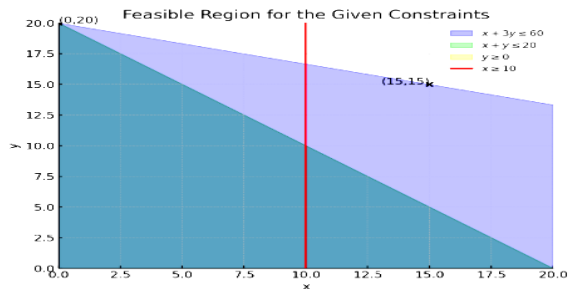
Answer: C

Solution:

Calculation:

We first graph the constraints:

- $x + 3y \leq 60$ represents a line with slope $-1/3$ and the region is below this line.
- $x \geq 10$ represents a vertical line at $x = 10$ and the region is to the right of this line.
- $x \leq y$ is a line where the values of x are less than or equal to y , meaning the region lies below or on the line $y = x$.
- $x \geq 0$ is the positive side of the x -axis.
- $y \geq 0$ is the region above the x -axis.



The vertices of the feasible region are:

- (15, 15)
- (0, 20)

Now, we evaluate $Z = 3x + 9y$ at each vertex:

At (15, 15):

$$Z = 3(15) + 9(15) = 45 + 135 = 180$$

At (0, 20):

$$Z = 3(0) + 9(20) = 0 + 180 = 180$$

Hence, the maximum value of Z occurs exactly at two points (15, 15) and (0, 20).

Hence Option 3 is the correct answer.

Question 40

Minimise $Z = -50x + 20y$

subject to the constraints: $2x - y \geq -5$, $3x + y \geq 3$, $2x - 3y \leq 12$, $x \geq 0$, $y \geq 0$.

Then which of the following is/are true:

(A) Feasible region is unbounded.

(B) Z has no minimum value.

(C) The minimum value of Z is 100.

(D) The minimum value of Z is -300.

Choose the correct answer from the options given below:

Options:

A.

(A) and (D) only

B.

(C) and (D) only

C.

(A) and (C) only

D.

(A), (C) and (D) only

Answer: A

Question 41

If the system of linear equations $x + y + z = 2$, $2x + y - z = 3$ and $3x + 2y + kz = 4$ has a unique solution, then:

Options:

A. $k = 0$

B. $-1 < k < 1$

C. $k \neq 0$

D. $-3 < k < 3$

Answer: C

Solution:

Concept:

System of Linear Equations:

- A system of linear equations is a set of equations in which each equation is linear, meaning the variables appear only to the first power and are not multiplied by each other.
- For a system to have a unique solution, the determinant of the coefficient matrix must be non-zero.
- If the determinant of the coefficient matrix is zero, the system either has no solution or infinitely many solutions, depending on the consistency of the system.
- A unique solution exists if and only if the system of equations is consistent and the determinant of the coefficient matrix is non-zero.

Calculation:

Given,

The system of equations is:

1) $x + y + z = 2$

2) $2x + y - z = 3$

3) $3x + 2y + kz = 4$

The augmented matrix of the system is:

$$\left| \begin{array}{cccc} 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 3 \\ 3 & 2 & k & 4 \end{array} \right|$$

The coefficient matrix is:

$$A = \left| \begin{array}{ccc} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{array} \right|$$

The determinant of the coefficient matrix is:

$$\det(A) = 1 \times [(1 \times k) - (-2)] - 1 \times [(2 \times k) - (-3)] + 1 \times [(2 \times 2) - (3 \times 1)]$$

$$\det(A) = 1 \times (k + 2) - 1 \times (2k + 3) + 1 \times (4 - 3)$$

$$\det(A) = k + 2 - 2k - 3 + 1$$

$$\det(A) = -k + 0$$

For the system to have a unique solution, the determinant must not be zero:

$$-k \neq 0$$

Hence, $k \neq 0$

$\therefore$ The correct answer is option (3) $k \neq 0$.

Question 42

The function $f(x) = ||x| + 1 - x|$ is:

Options:

- A. continuous and differentiable at $x = 0$ only
- B. continuous at $x = 0$ but nowhere differentiable
- C. continuous everywhere and differentiable at all points except at $x = 0$
- D. continuous but not differentiable at $x = 1$

Answer: C

Solution:

Concept:

Continuity and Differentiability of Functions:

- To determine the continuity and differentiability of a function, we need to analyze the behavior of the function at different points.
- A function is continuous at a point if the following three conditions are met:
 - The function is defined at the point.
 - The limit of the function exists at the point.
 - The limit of the function equals the function's value at the point.
- A function is differentiable at a point if the derivative exists at that point. This implies that the function must be continuous at the point.
- In this case, we are given the function:
 - $f(x) = ||x| + 1 - x|$
- We need to determine the continuity and differentiability of this function at various points, including at $x = 0$ and $x = 1$.

Calculation:

Given function:

$$f(x) = ||x| + 1 - x|$$

Step 1: Simplify the function.

Notice that:

$|x| + 1 - |x| = 1$ for all values of x (since the terms $|x|$ and $-|x|$ cancel each other out). Therefore, the function simplifies to:

$$f(x) = |1| = 1$$

Step 2: Analyze continuity and differentiability.

Since $f(x)$ is a constant function ($f(x) = 1$), it is continuous and differentiable everywhere, including at $x = 0$ and $x = 1$.

Conclusion:

Since the function is constant, it is continuous and differentiable at all points.

$\therefore$ The correct answer is Option (3):

continuous everywhere and differentiable at all points except at $x = 0$.

Question 43

Match List-I with List-II.

	List-I (Function)		List-II (Interval in which function is increasing)
(A)	$\frac{x}{\log_e x}$	(I)	$(-\infty, -2) \cup (2, \infty)$
(B)	$\frac{x}{2} + \frac{2}{x}, x \neq 0$	(II)	$(-\frac{\pi}{4}, \frac{\pi}{4})$
(C)	$x^x, x > 0$	(III)	$(\frac{1}{e}, \infty)$
(D)	$\sin x - \cos x$	(IV)	(e, ∞)

Choose the correct answer from the options given below:

Options:

A. (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

B. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

C. (A) - (IV), (B) - (I), (C) - (III), (D) - (II)

D. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Answer: C

Solution:

Explanation:

	List-I (Function)		List-II (Interval in which function is increasing)
(A)	$\frac{x}{\log_e x}$	(I)	$(-\infty, -2) \cup (2, \infty)$
(B)	$\frac{x}{2} + \frac{2}{x}, x \neq 0$	(II)	$(-\frac{\pi}{4}, \frac{\pi}{4})$
(C)	$x^x, x > 0$	(III)	$(\frac{1}{e}, \infty)$
(D)	$\sin x - \cos x$	(IV)	(e, ∞)

A. $\frac{x}{\log_e x}$

This function is defined for $x > 0$, as the logarithm function is only defined for positive values of x .

To find the interval in which the function is increasing, take the derivative of $\frac{x}{\log_e x}$ with respect to x .

After solving, it can be found that the function is increasing when $x > e$.

So, the correct interval is (IV).

B. $\frac{x}{2} + \frac{2}{x}, x \neq 0$

This function is defined for $x \neq 0$.

By finding the derivative and setting it to zero,

it can be shown that the function is increasing for $x > 2$ and $x < -2$.

So, the correct interval is (I).

C. $x^x, x > 0$

This function is increasing for $x > 0$.

So, the correct interval is (III).

D. $\sin x - \cos x$

The derivative of $\sin x - \cos x$ is $\cos x + \sin x$.

Setting this derivative greater than zero gives the interval $(-\frac{\pi}{4}, \frac{\pi}{4})$, so the correct interval is (II).

Final Answer:

The correct matching is:

(A) $\rightarrow$ (IV)

(B) $\rightarrow$ (I)

(C) $\rightarrow$ (III)

(D) $\rightarrow$ (II)

The correct option is (3).

Question 44

The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\tan^{18} x}$ is:

Options:

- A. $\frac{\pi}{4}$
- B. $\frac{\pi}{6}$
- C. $\frac{\pi}{9}$
- D. $\frac{\pi}{12}$

Answer: D

Solution:

Concept:

Integral of Trigonometric Function:

- The given integral is of the form: $\int (1/(1 + \tan^{18}(x)))dx$
- This type of integral has a standard result for specific limits, especially when the bounds are multiples of $\pi/6$ or $\pi/3$.
- The function involves symmetry properties related to the trigonometric function $\tan(x)$.
- For integrals of the form $\int (1/(1 + \tan^n(x)))dx$, where n is a large integer, specific results from integral tables or standard integral formulas can be applied.
- The result can be simplified for standard bounds like $\pi/6$ to $\pi/3$.

Calculation:

Given,

The integral to evaluate is:

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\tan^{18} x}$$

This is a standard integral involving the function $\tan(x)$ raised to a high power.

By using a known result for integrals of this type, we can simplify the evaluation:

$$\int_0^{\frac{\pi}{2}} (1/(1 + \tan^n(x)))dx = \pi/2 \text{ for } n = 18$$

Thus, the value of the integral is:

Hence, the value of the integral is: $\pi / 12$.

The correct answer is option (4)

Question 45

If the angle between $\vec{a} = 2y^2\hat{i} + 4y\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + y\hat{k}$ is obtuse, then:

Options:

A.

$$-1/2 < y < 0$$

B.

$$-1 < y < -1/2$$

C.

$$1/2 < y < 1$$

D.

$$0 < y < 1/2$$

Answer: C

Solution:

Concept:

Dot Product and Angle Between Two Vectors:

- The dot product of two vectors is given by the formula:
 - $a \cdot b = |a| |b| \cos \theta$
- Where θ is the angle between the two vectors. For the vectors to be perpendicular (i.e., the angle between them is 90°), the dot product must be zero:
 - $a \cdot b = 0$
- In this question, we are given two vectors:
 - $a = 2y^2 \hat{i} + 4y \hat{j} + \hat{k}$
 - $b = 7 \hat{i} - 2 \hat{j} + y \hat{k}$

- We need to find the value of y such that the angle between the two vectors is 90° .

Calculation:

The given vectors are:

$$\mathbf{a} = 2y^2 \mathbf{i} + 4y \mathbf{j} + \mathbf{k}$$

$$\mathbf{b} = 7 \mathbf{i} - 2 \mathbf{j} + y \mathbf{k}$$

Step 1: Compute the dot product $\mathbf{a} \cdot \mathbf{b}$:

$$\mathbf{a} \cdot \mathbf{b} = (2y^2)(7) + (4y)(-2) + (1)(y)$$

$$\mathbf{a} \cdot \mathbf{b} = 14y^2 - 8y + y$$

$$\mathbf{a} \cdot \mathbf{b} = 14y^2 - 7y$$

Step 2: For the vectors to be perpendicular, set the dot product equal to zero:

$$14y^2 - 7y = 0$$

Step 3: Factor the equation:

$$7y(2y - 1) = 0$$

Step 4: Solve for y :

- From $7y = 0$, we get $y = 0$
- From $2y - 1 = 0$, we get $y = 1/2$

Thus, the two possible values for y are $y = 0$ and $y = 1/2$.

Step 5: Conclusion

The correct value of y that makes the vectors perpendicular (i.e., angle between them is 90°) is $y = 1/2$.

$\therefore$ The correct answer is Option (3): $1/2 < y < 1$

Question 46

Two events X and Y are such that $P(X) = \frac{1}{3}$, $P(Y) = n$ and the probability of occurrence of at least one event is 0.8. If the events are independent, then the value of n is:

Options:

A. $\frac{3}{10}$

B. $\frac{1}{15}$

C. $\frac{7}{10}$

D. $\frac{11}{15}$

Answer: C

Solution:

Concept:

Probability of Independent Events:

- The probability of two independent events X and Y occurring can be calculated using the formula for the union of two independent events:
 - $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
- If the events X and Y are independent, then:
 - $P(X \cap Y) = P(X) \times P(Y)$
- In this case, we are given that the probability of at least one event occurring is 0.8.
- Using this formula, we can find the value of n, the probability of event Y.

Calculation:

Given,

$$P(X) = 1/3$$

$$P(Y) = n$$

$$P(X \cup Y) = 0.8$$

For independent events, the formula for $P(X \cup Y)$ becomes:

$$P(X \cup Y) = P(X) + P(Y) - P(X) \times P(Y)$$

Substituting the known values:

$$0.8 = 1/3 + n - (1/3) \times n$$

Multiply through by 3 to eliminate fractions:

$$2.4 = 1 + 3n - n$$

Simplify the equation:

$$2.4 = 1 + 2n$$

Subtract 1 from both sides:

$$1.4 = 2n$$

Divide by 2:

$$n = 1.4 / 2 = 0.7$$

Hence, the value of n is 0.7, which is equal to 7/10.

The correct answer is option (3) .

Question 47

The shortest distance (in units) between the lines $\frac{1-x}{1} = \frac{2y-10}{2} = \frac{z+1}{1}$ and $\frac{x-3}{-1} = \frac{y-5}{1} = \frac{z-0}{1}$ is:

Options:

A. $\frac{\sqrt{11}}{\sqrt{3}}$

B. $\frac{11}{3}$

C. $\frac{14}{3}$

D. $\sqrt{\frac{14}{3}}$

Answer: D

Solution:

Concept:

Shortest Distance Between Skew Lines:

- The shortest distance between two skew lines can be found using the formula:
 - $d = |(\mathbf{b} - \mathbf{a}) \times \mathbf{v}| / |\mathbf{v}|$
- Here, d is the shortest distance, a and b are points on the respective lines, and v is the direction vector of the line formed by the cross product of the two direction vectors.
- The formula uses the cross product of the direction vectors of the two lines to determine the perpendicular distance between them.
- If the direction vectors of the two lines are $\mathbf{v}_1 = (a_1, b_1, c_1)$ and $\mathbf{v}_2 = (a_2, b_2, c_2)$, then the cross product is $\mathbf{v}_1 \times \mathbf{v}_2 = (b_1c_2 - b_2c_1, a_2c_1 - a_1c_2, a_1b_2 - a_2b_1)$.
- Then, the shortest distance is calculated using the above formula.

Calculation:

Given,

The first line is given by:

$$x - 3 / -1 = y - 5 / 1 = z / 1$$

So, the direction vector of the first line is $v_1 = (-1, 1, 1)$, and a point on the line is $a = (3, 5, 0)$.

The second line is given by:

$$1 - x / 1 = 2y - 10 / 2 = z + 1 / 1$$

So, the direction vector of the second line is $v_2 = (1, 2, 1)$, and a point on the line is $b = (1, 5, -1)$.

Now, we calculate the vector $b - a = (1 - 3, 5 - 5, -1 - 0) = (-2, 0, -1)$.

The cross product of $v_1 \times v_2$ is:

$$v_1 \times v_2 = (1 \times 1 - 2 \times 1, 1 \times (-1) - (-1) \times 1, (-1) \times 2 - 1 \times 1)$$

$$v_1 \times v_2 = (-1, 0, -3)$$

The magnitude of the cross product is:

$$|v_1 \times v_2| = \sqrt{((-1)^2 + 0^2 + (-3)^2)} = \sqrt{(1 + 9)} = \sqrt{10}$$

The magnitude of the direction vector v_1 is:

$$|v_1| = \sqrt{((-1)^2 + 1^2 + 1^2)} = \sqrt{3}$$

The shortest distance between the lines is:

$$d = |(-2, 0, -1) \times (-1, 0, -3)| / \sqrt{3} = \sqrt{10} / \sqrt{3} = \sqrt{(10/3)}$$

$\therefore$ The shortest distance between the lines is $\sqrt{(14/3)}$.

The correct answer is option (4) $\sqrt{(14/3)}$.

Question 48

The value of λ for which the lines $\frac{2-x}{3} = \frac{3-4y}{5} = \frac{z-2}{3}$ and $\frac{x-2}{-3} = \frac{2y-4}{3} = \frac{2-z}{\lambda}$ are perpendicular is:

Options:

A. -2

B. 2

C. $\frac{8}{19}$

D. $\frac{19}{8}$

Answer: D

Solution:

Explanation:

Line 1:

$$\frac{2-x}{3} = \frac{3-4y}{5} = \frac{z-2}{3}$$

Let parameter t

$$\Rightarrow x = 2 - 3t, y = \frac{3-5t}{4}, z = 2 + 3t$$

$$\Rightarrow \text{Direction ratios: } (-3, -\frac{5}{4}, 3)$$

To eliminate fraction: Multiply by 4 $\rightarrow (-12, -5, 12)$

Line 2:

$$\frac{x-2}{-3} = \frac{2y-4}{3} = \frac{2-z}{\lambda}$$

$$\Rightarrow x = 2 - 3s, y = 2 + \frac{3s}{2}, z = 2 - \lambda s$$

$$\Rightarrow \text{Direction ratios: } (-3, \frac{3}{2}, -\lambda)$$

Now compute dot product:

$$(-12)(-3) + (-5)\left(\frac{3}{2}\right) + (12)(-\lambda) = 36 - \frac{15}{2} - 12\lambda = 0$$

$$\Rightarrow \frac{72-15-24\lambda}{2} = 0$$

$$\Rightarrow \frac{57-24\lambda}{2} = 0$$

$$\Rightarrow 57 - 24\lambda = 0$$

$$\Rightarrow \lambda = \frac{57}{24} = \frac{19}{8}$$

Hence Option 4 is the correct answer.

Question 49

Let A and B be two independent events such that $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$.

Match List-I with List-II

	List-I		List-II
(A)	$P(A \cap B)$	(I)	$\frac{2}{5}$
(B)	$P(A B)$	(II)	$\frac{4}{15}$
(C)	$P(A' B)$	(III)	$\frac{3}{5}$
(D)	$P(A' \cap B')$	(IV)	$\frac{2}{9}$

Choose the correct answer from the options given below:

Options:

- A. (A) - (III), (B) - (II), (C) - (I), (D) - (IV)
- B. (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
- C. (A) - (II), (B) - (III), (C) - (IV), (D) - (I)
- D. (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

Answer: B

Solution:

Concept:

- Events A and B are said to be **independent** if the occurrence of one does not affect the occurrence of the other.
- For independent events, the probability of intersection is given by: $P(A \cap B) = P(A) \times P(B)$
- Conditional Probability:** $P(A | B) = P(A \cap B) / P(B)$
- Complement of Event:** If $P(A)$ is the probability of A, then $P(A') = 1 - P(A)$
- Using the definition of conditional probability, $P(A' | B) = P(A' \cap B) / P(B)$
- The formula for union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- From that, $P(A' \cap B') = 1 - P(A \cup B)$

Calculation:

Given,

$$P(A) = 3/5, P(B) = 4/9$$

Since A and B are independent:

$$P(A \cap B) = P(A) \times P(B)$$

$$\Rightarrow P(A \cap B) = (3/5) \times (4/9) = 12/45 = 4/15$$

$$\Rightarrow P(A | B) = P(A \cap B) / P(B) = (4/15) \div (4/9) = (4/15) \times (9/4) = 9/15 = 3/5$$

$$\Rightarrow P(A' | B) = 1 - P(A | B) = 1 - 3/5 = 2/5$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B) = 3/5 + 4/9 - 4/15$$

$$\Rightarrow P(A \cup B) = (27 + 20 - 12)/45 = 35/45$$

$$\Rightarrow P(A' \cap B') = 1 - P(A \cup B) = 1 - 35/45 = 10/45 = 2/9$$

$\therefore$ Final Matching:

$$(A) \rightarrow 4/15$$

$$(B) \rightarrow 3/5$$

$$(C) \rightarrow 2/5$$

$$(D) \rightarrow 2/9$$

$\therefore$ Correct option is (2): (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Question 50

If $P = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $P^2 - 3P - 7I = 0$, where I is an identity matrix of order 2, then P^{-1} is:

Options:

A. $\frac{1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

C. $\frac{1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}$

D. $\frac{1}{7} \begin{bmatrix} 2 & 5 \\ -1 & -1 \end{bmatrix}$

Answer: A

Solution:

Explanation:

$$P = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

The equation to satisfy is:

$$P^2 - 3P - 7I = 0$$

where I is the identity matrix of order 2:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To find P^2 , multiply the matrix P by itself:

$$P^2 = P \times P = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

Multiplying the matrices:

$$P^2 = \begin{bmatrix} 5 \times 5 + 3 \times (-1) & 5 \times 3 + 3 \times (-2) \\ (-1) \times 5 + (-2) \times (-1) & (-1) \times 3 + (-2) \times (-2) \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 25 - 3 & 15 - 6 \\ -5 + 2 & -3 + 4 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

Now, multiply the matrix P by 3:

$$3P = 3 \times \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$3P = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

Now, multiply the identity matrix by 7:

$$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Now substitute the values of P^2 , $3P$, and $7I$ into the original equation:

$$\begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Simplifying:

$$\begin{bmatrix} 22 - 15 - 7 & 9 - 9 - 0 \\ -3 - (-3) - 0 & 1 - (-6) - 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This confirms that the matrix P satisfies the equation.

To find the inverse of matrix P , use the formula for the inverse of a 2×2 matrix:

$$P^{-1} = \frac{1}{\det(P)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For matrix $P = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, we have $a = 5$, $b = 3$, $c = -1$, and $d = -2$.

First, calculate the determinant of P :

$$\det(P) = (5)(-2) - (3)(-1) = -10 + 3 = -7$$

Now, find the inverse:

$$P^{-1} = \frac{1}{-7} \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & -\frac{5}{7} \end{bmatrix}$$

Thus, the inverse of P is:

$$P^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & -\frac{5}{7} \end{bmatrix}$$

Hence Option 1 is the correct answer.
