

CUET Mathematics Solved Paper-2022

Held on 30 August 2022

1. Assume P , Q , R and S are matrices of order $2 \times m$, $k \times n$, $m \times 2$ and 2×3 respectively. The restrictions on k , m and n , so that $PQ + RS$ is defined are
- (a) $m=3, n=2$ (b) $m=n, k$ is arbitrary
(c) $m=k, n$ is arbitrary (d) $m=k=2, n=3$
2. The system of equations $3x + 4y = 5$, $6x + 7y = -8$ is written in matrix form as

(a) $\begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

(d) $\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -8 \end{bmatrix}$

3. If $2 \begin{bmatrix} a & d \\ b & c \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$, then the value of $|a+b-c-d|$ is
- (a) 3 (b) 24 (c) 6 (d) 16

4. Consider the function $f(x) = x^{\frac{1}{e}}$. Its

- (a) minimum value is e^e
(b) maximum value is e^e
(c) minimum value is e^e
(d) maximum value is $\left(\frac{1}{e}\right)^e$

5. The given function $f(x) = x^5 - 5x^4 + 5x^3 - 1$; has/have

- (a) Local maxima at $x=1$
(b) Local maximum value is 0
(c) Local minimum at $x=3$
(d) Local minimum value is -28
(e) The point of inflexion is $x=1$

Choose the correct answer from the options given below

- (a) (a), (b) only
(b) (a), (b), (c) only
(c) (a), (b), (c), (d) only
(d) (a), (c), (e) only

6. Match List-I with List-II

- List-I
(A) If $x = t^2$ and $y = t^3$

then $\frac{d^2y}{dx^2}$ at $t=1$

- List-II
(i) -2

- (B) If $f(x) = \sqrt{x} + 1$, (ii) -1
the $f''(1)$

- (C) The minimum value (iii) $\frac{3}{4}$
of $f(x) = 9x^2 + 12x + 2$ is

- (D) The point of inflexion (iv) $-\frac{1}{4}$
of the function $f(x) = (x-2)^4$
 $(x+1)^3$ is

Choose the correct answer from the options given below

- (a) (A) - (i), (B) - (iii), (C) - (ii), (D) - (iv)
(b) (A) - (ii), (B) - (iii), (C) - (i), (D) - (iv)
(c) (A) - (iii), (B) - (iv), (C) - (i), (D) - (ii)
(d) (A) - (iv), (B) - (i), (C) - (iii), (D) - (ii)

7. The area enclosed by the curve $y^2 = 4ax$ and its latus-rectum is

- (a) $\frac{8}{3}a^2$ (b) $\frac{4}{3}a^2$ (c) $\frac{1}{3}a^2$ (d) $\frac{1}{12}a^2$

8. $\int \frac{xe^x}{(x+1)^2} dx =$

- (a) $\frac{e^x}{x+1} + c$ (b) $\frac{e^x}{x-1} + c$
(c) $\frac{x}{x+1} + c$ (d) $\frac{x}{x-1} + c$

9. The solution of the differential equation

$(x+1) \frac{dy}{dx} = 1+y$ is

(a) $\frac{1+y+y^2}{1+x^2} = C$

(b) $\log(x+1) - \log\left(y + \frac{1}{2}\right) = C$

(c) $\frac{x+1}{y+1} = C$

(d) $\log(1+y) - \frac{\sqrt{3}}{2} \log(x+1) = C$

10. Order and degree of the differential equation

$y \frac{dy}{dx} + \frac{4}{dx} = 5$ are

- (a) 1, 2 respectively (b) 1, 1 respectively
(c) 1, 0 respectively (d) 2, 1 respectively

11. Derivative of $x^3 + 1$ with respect to $x^2 + 1$ is

- (a) $\frac{2x}{3}$ (b) $\frac{x}{3}$ (c) $\frac{x}{2}$ (d) $\frac{3x}{2}$

12. Solution of the differential equation $(x + xy) dy - y(1-x^2) dx = 0$ is

(a) $y = \log \frac{x}{y} - \frac{x^2}{2} + C$ (b) $y = \log \frac{x}{y} + \frac{x^2}{2} + C$

(c) $y = \log xy - \frac{x^2}{2} + C$ (d) $y = \log xy + \frac{x^2}{2} + C$

13. Two numbers are selected at random (without replacement) from the first three positive integers. Let X denotes the larger of the two integers, then the probability distribution of X is

(a)

x	2	3
$P(X=x)$	1/3	2/3

(b)

x	2	3
$P(X=x)$	1/2	1/2

(c)

x	2	3
$P(X=x)$	1/3	1/3

(d)

x	2	3
$P(X=x)$	1/5	4/5

14. The probability distribution of number of doublets in three throws of a pair of dice is

(a)

x	0	1	2	3
$P(X=x)$	125/216	75/216	15/216	1/216

(b)

x	0	1	2	3
$P(X=x)$	75/216	125/216	1/216	15/216

(c)

x	0	1	2	3
$P(X=x)$	1/216	75/216	15/216	125/216

(d)

x	0	1	2	3
$P(X=x)$	1/216	15/216	75/216	125/216

15. In linear programming, the optimal value of the objective function is attained at the points given by

- (a) intersection of the inequalities with the x -axis only
 (b) intersection of the inequalities with the axes only
 (c) corner points of the feasible region
 (d) intersection of the inequalities with the y -axis only

16. If R is a relation on Z (set of all integers) defined by xRy , iff $|x-y| \leq 1$, then

- (1) R is reflexive (2) R is symmetric
 (3) R is transitive (4) R is not symmetric
 (5) R is not transitive

Choose the most appropriate answer from the options given below

- (a) (1) and (4) only (b) (1), (2) and (3) only
 (c) (2) and (3) only (d) (1), (2) and (5) only

17. If the vertices of a triangle ABC are $A(1, 2, 1)$, $B(4, 2, 3)$ and $C(2, 3, 1)$, then the equation of the median passing through the vertex A , is

(a) $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{2}$

(b) $x-2 = \frac{y-2}{1} = z-1$

(c) $x-1 = 2y-4 = z-1$

(d) $\frac{x-1}{2} = 2y-4 = z-1$

18. A line makes the angle θ with each of the x and z axes. If the angle β which it makes with y -axis is such that $\sin^2 \beta = 3 \sin^2 \theta$, then the value of $\cos^2 \theta$ is

- (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

19. If $x = 2 \sin \theta$ and $y = 2 \cos \theta$, then the value of $\frac{d^2 y}{dx^2}$ at $\theta = 0$ is

- (a) $-\frac{1}{2}$ (b) -1 (c) 0 (d) 1

20. If $x = e^{y+e^{y+e^{y+\dots}}}$, $x > 0$, then $\frac{dy}{dx}$ is equal to

- (a) $\frac{x}{1+x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{1+x}{x}$

21. $\sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$, then x is equal to

- (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

Choose the **most appropriate** answer from the options given below:

- (a) (a) and (b) only (b) (a) and (c) only
 (c) (a) only (d) (c) only

22. The smaller of the areas enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

- (a) $2(\pi-2)$ (b) $\pi-2$
 (c) $2\pi-1$ (d) $2\pi+2$

23. If $0 < x < \pi$ and the matrix $\begin{bmatrix} 4 \sin x & -1 \\ -3 & \sin x \end{bmatrix}$ is singular, then the values of x are:

(a) $\frac{\pi}{3}, \frac{2\pi}{3}$

(b) $\frac{\pi}{6}, \frac{5\pi}{6}$

(c) $\frac{\pi}{6}, \frac{\pi}{3}$

(d) $\frac{\pi}{6}, \frac{2\pi}{3}$

24. $\int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx =$

(a) 3

(b) 4

(c) 6

(d) 0

26. The function $f(x) = e^{|x|}$ is

(A) continuous everywhere on R

(B) not continuous at $x = 0$

(C) Differentiable everywhere on R

(D) not differentiable at $x = 0$

(E) continuous and differentiable on R

Choose the **most appropriate** answer from the options given below:

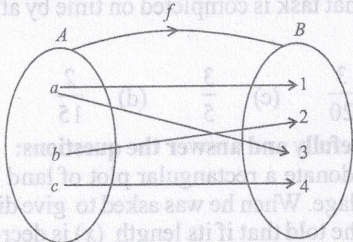
(a) (E) only

(b) (B) and (C) only

(c) (A) and (D) only

(d) (B) and (D) only

26.



Which of the following is **true** on the basis of above diagram?

- (a) ' f ' is a function from $A \rightarrow B$
 (b) ' f ' is one-one function from $A \rightarrow B$
 (c) ' f ' is onto function from $A \rightarrow B$
 (d) ' f ' is not a function from $A \rightarrow B$

27. If the points $(2, -3)$, $(\lambda, -1)$ and $(0, 4)$ are collinear, then the value of λ is

- (a) $\frac{7}{10}$ (b) $\frac{3}{10}$ (c) $\frac{7}{3}$ (d) $\frac{10}{7}$

28. The value of $\sin \left[2 \cot^{-1} \left(\frac{-5}{12} \right) \right]$ is:

- (a) $\frac{120}{169}$ (b) $\frac{-120}{169}$ (c) $\frac{-60}{169}$ (d) $\frac{60}{169}$

29. Let $y = m \sin rx + n \cos rx$. What is the value of $\frac{d^2y}{dx^2}$?

- (a) ry (b) $-ry$ (c) r^2y (d) $-r^2y$

30. The integrating factor of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is

- (a) $\sec x$ (b) $\cos x$
 (c) $\sec x + \tan x$ (d) $\tan x$

31. The order and degree of the differential equation

$$\left[\left(\frac{d^2y}{dx^2} \right) - 3 \right]^{\frac{1}{3}} = 2 \left(\frac{dy}{dx} \right)^{\frac{1}{4}}$$
 are

- (a) order = 2, degree = 2 (b) order = 2, degree = 4
 (c) order = 2, degree = 8 (d) order = 1, degree = 1

32. $\int \sqrt{1-49x^2} dx$ is equal to

(a) $\frac{x}{2} \left(\sqrt{1-49x^2} \right) + \frac{1}{98} \sin^{-1} 7x + C$

(b) $\frac{7x}{2} \sqrt{1+49x^2} + \frac{1}{49} \sin^{-1} x + C$

(c) $\frac{x}{2} \sqrt{1+\frac{1}{7x^2}} - \frac{1}{49} \sin^{-1} 7x + C$

(d) $\frac{x}{2} \sqrt{1-49x^2} + \frac{1}{14} \sin^{-1} 7x + C$

33. The shortest distances of the point $(1, 2, 3)$ from x , y , z axes respectively are

- (a) 1, 2, 3 (b) $\sqrt{5}, \sqrt{13}, \sqrt{10}$
 (c) $\sqrt{10}, \sqrt{13}, \sqrt{5}$ (d) $\sqrt{13}, \sqrt{10}, \sqrt{5}$

34. Distance between two planes $x + 2y - z = 5$ and $2x + 4y - 2z + 2 = 0$ is

- (a) $\sqrt{6}$ unit (b) 7 unit
 (c) $\frac{5}{\sqrt{6}}$ unit (d) $\frac{4}{\sqrt{6}}$ unit

35. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two non zero vectors inclined at an angle θ , then identify the correct option out of the given options

- (A) $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$
 (B) $\vec{a}$ and $\vec{b}$ are perpendicular, if $a_1b_1 + a_2b_2 + a_3b_3 = 0$
 (C) $\vec{a}$ and $\vec{b}$ are perpendicular, $\frac{a_1}{b_1} = \frac{a_2}{b_2} \neq \frac{c_1}{c_2}$
 (D) for $\theta = \pi$, $\vec{a} \times \vec{b} = 0$
 (E) $\cos \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| \cdot |\vec{b}|}$

Choose the **most appropriate** answer from the options given below

- (a) (A), (B) and (D) only
 (b) (A), (B) and (E) only
 (c) (B), (D) and (E) only
 (d) (A) and (B) only

36. If $\vec{p} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{q} = 2\hat{i} + \hat{j} - \hat{k}$, then the area of parallelogram having diagonals $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$ is

- (a) $4\sqrt{11}$ sq. unit (b) $\sqrt{44}$ sq. unit
 (c) $\sqrt{11}$ sq. unit (d) $3\sqrt{11}$ sq. unit

37. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is

- (a) 3 (b) $-\frac{3}{2}$ (c) $\frac{3}{2}$ (d) -3

38. The corner points of the feasible region for an L.P.P. are $(0, 10)$, $(5, 5)$, $(15, 15)$ and $(0, 20)$. If the objective function is $z = px + qy$; $p, q > 0$, then the condition on p and q so that the maximum of z occurs at $(15, 15)$ and $(0, 20)$ is

- (a) $p = q$ (b) $p = 2q$
 (c) $q = 3p$ (d) $q = 2p$

39. $\int x\sqrt{x+2}$ is equal to:

(a) $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

(b) $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

(c) $\frac{1}{5}(x+2)^{\frac{5}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + C$

(d) $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

40. Three urns contain 6 red, 4 black; 4 red, 6 black and 5 red, 5 black marbles respectively. One of the urns is selected at random and a marble is drawn from it. If the marble drawn is red, then the probability that it is drawn from the first urn is
- (a) $\frac{6}{10}$ (b) $\frac{4}{10}$ (c) $\frac{5}{10}$ (d) $\frac{2}{5}$
41. Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently. The probability that exactly one of them complete the task on time is
- (a) $\frac{2}{15}$ (b) $\frac{2}{5}$ (c) $\frac{3}{20}$ (d) $\frac{13}{20}$
42. **Read the text carefully and answer the questions:**
Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently. The probability that exactly two of them complete the task on time is
- (a) $\frac{3}{20}$ (b) $\frac{13}{20}$ (c) $\frac{1}{5}$ (d) $\frac{2}{15}$
43. **Read the text carefully and answer the questions:**
Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently. The probability that B alone complete the task on time is:
- (a) $\frac{13}{30}$ (b) $\frac{3}{30}$ (c) $\frac{2}{5}$ (d) $\frac{2}{15}$
44. **Read the text carefully and answer the questions:**
Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently. The probability that the task is completed on time by none of them is
- (a) $\frac{3}{20}$ (b) $\frac{2}{5}$ (c) $\frac{13}{20}$ (d) $\frac{2}{15}$
45. **Read the text carefully and answer the questions:**
Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.

The probability that task is completed on time by at least one of them is:

- (a) $\frac{2}{5}$ (b) $\frac{3}{20}$ (c) $\frac{3}{5}$ (d) $\frac{2}{15}$

46. **Read the text carefully and answer the questions:**
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m².
The equations in terms of x and y are:
- (a) $x - y = 50, 2x + y = 550$
(b) $x + y = 40, 2x - y = 550$
(c) $x - y = 10, 2x + y = 50$
(d) $x - y = 30, 2x + y = 505$
47. **Read the text carefully and answer the questions:**
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m².
The value x is:
- (a) 150 m (b) 100 m (c) 200 m (d) 300 m
48. **Read the text carefully and answer the questions:**
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m².
The value of y is
- (a) 50 m (b) 100 m (c) 240 m (d) 150 m
49. **Read the text carefully and answer the questions:**
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m².
The value of the expression $\frac{x^2 + y^2}{x - y}$ is:
- (a) 625 (b) 1250 (c) 312.5 (d) 3125
50. **Read the text carefully and answer the questions:**
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m².
The area of rectangular field is:
- (a) 30000 sq. m (b) 3000 sq. m
(c) 300000 sq. m (d) 60000 sq. m

Hints & Explanations

1. (d) Order of $P = 2 \times m$

Order of $Q = k \times n$

Order of $R = m \times 2$

Order of $S = 2 \times 3$

Now, for $PQ + RS$ to be defined,

PQ and RS is to be defined and PQ and RS should be of same order.

For PQ to be defined $m = k$

$\Rightarrow$ Order of $PQ = 2 \times n$

For RS to be defined $2 = 3$

$\Rightarrow$ Order of $RS = m \times 3$

If order of $PQ =$ Order of RS

$\Rightarrow 2 \times n = m \times 3$

$\Rightarrow m = 2, n = 3, k = 2$

2. (d) Given the equations

$$3x + 4y = 5$$

$$6x + 7y = -8$$

In matrix form

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

3. (c) Given, $2 \begin{bmatrix} a & d \\ b & c \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 2a+3 & 2d-3 \\ 2b & 2c+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow 2a+3=9 \Rightarrow a=3$$

$$2d-3=15 \Rightarrow d=9$$

$$2b=12 \Rightarrow b=6$$

$$2c+6=18 \Rightarrow c=6$$

$$|3+6-9-6| = |-6| = 6$$

4. (b) Given the function

$$f(x) = x^x \Rightarrow \log f(x) = \frac{1}{x} \log x$$

$$\frac{f'(x)}{f(x)} = \frac{1}{x^2} + (\log x) \left(-\frac{1}{x^2} \right)$$

$$f'(x) = \frac{x^x [1 - \log x]}{x^2}, \text{ Now, } f'(x) = 0$$

$$\Rightarrow 1 - \log x = 0 \Rightarrow \log x = 1$$

$$\therefore x = e$$

Since, $f'(e) > 0$ and $f'(e^+) < 0$

So, $f(x)$ has max. at e .

$$\therefore f(x)_{\max} = e^e$$

5. (c) Given, $f(x) = x^5 - 5x^4 + 5x^3 - 1$

$$f'(x) = 5x^4 - 20x^3 + 15x^2$$

$$= 5x^2(x^2 - 4x + 3) = 0$$

$$= 5x^2(x-3)(x-1) = 0$$

$$x = 0, 3, 1$$

$$\text{Now, } f''(x) = 20x^3 - 60x^2 + 30x$$

$$= 10x(2x^2 - 6x + 3)$$

$$f''(x) = 0 \Rightarrow x = 0$$

$\therefore x = 0$ is point of inflexion.

$$f''(1) < 0 \Rightarrow x = 1$$

point of maxima.

$$f''(3) > 0 \Rightarrow x = 3$$

point of minima.

$$f(3) = (3)^5 - 5(3)^4 + 5(3)^3 - 1 = -28$$

$$f(1) = 1 - 5 + 5 - 1 = 0$$

6. (c) (a) Given, $x = t^2, y = t^3$

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 3t^2$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{dy}{dx} = \frac{3}{2}t$$

$$\frac{d^2y}{dx^2} = \frac{3}{2} \times \frac{dt}{dx} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

$$(b) f(x) = \sqrt{x} + 1$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f''(x) = -\frac{1}{4x^{\frac{3}{2}}} \Rightarrow f''(1) = -\frac{1}{4}$$

$$(c) f(x) = 9x^2 + 12x + 2$$

$$f'(x) = 18x + 12 = 0 \Rightarrow x = -\frac{2}{3}$$

$$\text{Now, } f''(x) = 18 \text{ and } f''\left(-\frac{2}{3}\right) > 0$$

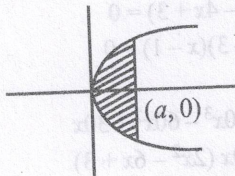
$$\text{So, } f(x)_{\min} = 9\left(-\frac{2}{3}\right)^2 + 12\left(-\frac{2}{3}\right) + 2$$

$$= 9 \times \frac{4}{9} + (-8) + 2 = 4 - 8 + 2 = -2$$

(d) Given, $f(x) = (x-2)^4(x+1)^3$
 $f'(x) = 3(x+1)^2(x-2)^4 + 4(x-2)^3(x+1)^3$
 $f''(x) = 6(x+1)(x-2)^4 + 12(x-2)^3(x+1)^2$
 $+ 12(x+1)^2(x-2)^3 + 12(x-2)^2(x+1)^3$
 $= (x+1)[6(x-2)^4 + 12(x-2)^3(x+1)$
 $+ 12(x+1)(x-2)^3 + 12(x-2)^2(x+1)^2] = 0$
 $\Rightarrow x = -1$ is point of inflexion.

$\therefore$ (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (i), (d) $\rightarrow$ (ii)

7. (d) Graph of given curves



$$\text{Area} = 2 \int_0^a \sqrt{4ax} \, dx = 2.2\sqrt{a} \int_0^a \sqrt{x} \, dx$$

$$= 4\sqrt{a} \frac{x^{3/2}}{3/2} \times 2 \Big|_0^a = \frac{4a^{3/2}}{3} \left[\frac{3}{2} - 0 \right] \times 2$$

$$= \frac{4a^{3/2} \cdot \frac{3}{2}}{3} \times 2 = \frac{8}{3} a^2$$

8. (a) $\int \frac{xe^x}{(x+1)^2} dx = \int e^x \left[\frac{x+1-1}{(x+1)^2} \right] dx$

$$\therefore \int e^x [f(x) + f'(x)] dx = e^x f(x) + C$$

$$= \int e^x \left[\frac{1}{x+1} - \frac{1}{(x+1)^2} \right] dx = \frac{e^x}{1+x} + C$$

9. (c) Given differential equation,

$$(x+1) \frac{dy}{dx} = 1+y \Rightarrow \frac{dx}{1+x} = \frac{dy}{1+y}$$

$$\Rightarrow \ln(1+x) = \ln(1+y) + \ln C \Rightarrow \ln \left(\frac{1+x}{1+y} \right) = \ln C$$

$$\Rightarrow \frac{1+x}{1+y} = C$$

10. (a) Given differential equation,

$$y \frac{dy}{dx} + \frac{4}{\frac{dy}{dx}} = 5$$

$$y \left(\frac{dy}{dx} \right)^2 + 4 = 5 \left(\frac{dy}{dx} \right)$$

$\therefore$ Order = 1
Degree = 2

11. (d) Let $f(x) = x^3 + 1$
 $g(x) = x^2 + 1$

$$\frac{f'(x)}{g'(x)} = \frac{3x^2}{2x} = \frac{3}{2}x$$

12. (a) The given differential equation

$$(x+xy) dy - y(1-x^2) dx = 0$$

$$x(1+y) dy - y(1-x^2) dx = 0$$

$$\int \frac{(1+y)}{y} dy = \int \frac{(1-x^2)}{x} dx$$

$$\log y + y = \log x - \frac{x^2}{2} + C$$

$$y = \log \frac{x}{y} - \frac{x^2}{2} + C$$

13. (a) Sample space

$$S: \{1, 2, 3\}$$

Two numbers can be select as (1, 2) (2, 3) (1, 3)

Now,

x	2	3
P(X=x)	1/3	2/3

where X denotes larger of two integers.

14. (d) X = getting number of doubles

$$P = \frac{6}{36} = \frac{1}{6}$$

$$P(X=0) = {}^3C_0 \left(\frac{1}{6} \right)^0 \left(\frac{5}{6} \right)^3 = \frac{125}{216}$$

$$P(X=1) = {}^3C_1 \left(\frac{1}{6} \right)^1 \left(\frac{5}{6} \right)^2 = \frac{75}{216}$$

$$P(X=2) = {}^3C_2 \left(\frac{1}{6} \right)^2 \left(\frac{5}{6} \right)^1 = \frac{15}{216}$$

$$P(X=3) = {}^3C_3 \left(\frac{1}{6} \right)^3 \left(\frac{5}{6} \right)^0 = \frac{1}{216}$$

15. (c) The optimal values of the objective function is attained at the corner points of the feasible region.

16. (d) Since, $xRy, |x-y| \leq 1$

For reflexive $(a, a) \Rightarrow |a-a| = 0 \leq 1$.

$\therefore$ Relation is reflexive,

For symmetric $(a, b) \Rightarrow |a-b| \leq 1$.

$(b, a) \Rightarrow |b-a| \leq 1$.

$\therefore$ Relation is symmetric.

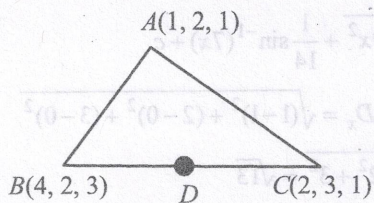
For transitive $(a, b) \Rightarrow |a-b| \leq 1$

$$(b, c) \Rightarrow |b - c| \leq 1$$

$$\nRightarrow |a - c| \leq 1$$

$\therefore$ Only reflexive and symmetric and not transitive.

17. (d)



D is mid-point of line segment BC.

$$D = \left(\frac{4+2}{2}, \frac{2+3}{2}, \frac{3+1}{2} \right) = \left(3, \frac{5}{2}, 2 \right)$$

Equation of median through A.

$$\left(\frac{x-1}{3-1} = \frac{y-2}{\frac{5}{2}-2} = \frac{z-1}{2-1} \right) \Rightarrow \frac{x-1}{2} = \frac{y-2}{\frac{1}{2}} = \frac{z-1}{1}$$

18. (c) We know that

$$\cos^2 \theta + \cos^2 \beta + \cos^2 \theta = 1$$

$$\text{or } 2 \cos^2 \theta + 1 - 3 \sin^2 \theta = 1$$

$$\text{or } 2 \cos^2 \theta = 3 \sin^2 \theta$$

$$\text{or } 2 \cos^2 \theta = 3 - 3 \cos^2 \theta \Rightarrow \cos \theta = \frac{3}{5}$$

19. (a) Now, $\frac{dy}{dx} = \frac{dy}{d\theta} \Rightarrow \frac{dy}{dx} = \frac{-2 \sin \theta}{2 \cos \theta} = -\tan \theta$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta} \left(\frac{dy}{dx} \right) \cdot \frac{d\theta}{dx} \Rightarrow \frac{d^2 y}{dx^2} = -\sec^2 \theta \cdot \frac{d\theta}{dx}$$

$$= \frac{-\sec^2 \theta}{2 \cos \theta} = -\frac{1}{2} \sec^3 \theta$$

$$\text{At } \theta = 0 \Rightarrow \frac{d^2 y}{dx^2} = -\frac{1}{2}$$

20. (c) Since, $x = e^{y+x}$

Differentiating w.r.t. x, we get

$$1 = e^{x+y} \left(1 + \frac{dy}{dx} \right) \Rightarrow \frac{1}{x} = 1 + \frac{dy}{dx}$$

$$\text{or } \frac{dy}{dx} = \frac{1-x}{x}$$

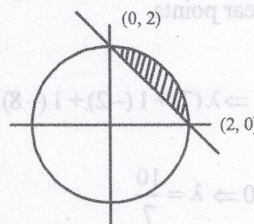
21. (c) Domain for $\sin^{-1}(1-x)$ is $[0, 2]$ and domain for $\sin^{-1}x$ is $[-1, 1]$.

So, Domain for x is $[0, 1]$.

$$\sin^{-1}(1-x)|_{\max} = \frac{\pi}{2}; \sin^{-1}(x)|_{\max} = 0$$

So, only possible condition is when $x = 0$.

22. (b) Graph of given curves



Area of smaller region

$$= \frac{1}{4} \times 4\pi - \frac{1}{2} \cdot 2 \cdot 2 = (\pi - 2) \text{ sq. units}$$

23. (a) Since, $\begin{vmatrix} 4 \sin x & -1 \\ -3 & \sin x \end{vmatrix} = 0$

$$\Rightarrow 4 \sin^2 x - 3 = 0 \Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

24. (c) Let

$$I = \int_{1/3}^1 \frac{x \left(\frac{1}{x^2} - 1 \right)^{1/3}}{x^4} dx = \int_{1/3}^1 \frac{\left(\frac{1}{x^2} - 1 \right)^{1/3}}{x^3} dx$$

$$\text{Let } \frac{1}{x^2} - 1 = t \Rightarrow \frac{-2}{x^3} dx = dt$$

$$\text{If } x = \frac{1}{3} \Rightarrow t = 8$$

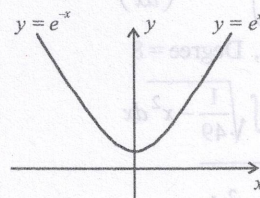
$$\text{If } x = 1 \Rightarrow t = 0$$

$$\text{So, } I = \int_8^0 \frac{-t^{1/3}}{2} dt = \frac{1}{2} \int_0^8 t^{1/3} dt = \frac{1}{2} \cdot \frac{t^{4/3}}{4/3} \Big|_0^8$$

$$= \frac{3}{8} \left(8^{4/3} - 0 \right) = \frac{3}{8} \cdot 8^{4/3} = 6$$

25. (d) Given function $f(x) = e^{|x|}$

$$f(x) = \begin{cases} e^x & x \geq 0 \\ e^{-x} & x < 0 \end{cases}$$



$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0) = 1$$

So, $f(x)$ is continuous.

But RHD at $x=0$ is 1.

LHD at $x=0$ is -1 .

So, $f(x)$ is not differentiable at $x=0$.

26. (d) $f(a)=1$ and $f(a)=3$, which is not possible for any function.

27. (d) For collinear points,

$$\begin{vmatrix} \lambda & -1 & 1 \\ 0 & 4 & 1 \\ 2 & -3 & 1 \end{vmatrix} = 0 \Rightarrow \lambda(7) + 1(-2) + 1(-8) = 0$$

$$\Rightarrow 7\lambda - 10 = 0 \Rightarrow \lambda = \frac{10}{7}$$

28. (b) $\sin\left(2\pi - 2\cot^{-1}\frac{5}{12}\right) = -\sin\left(2\cot^{-1}\frac{5}{12}\right)$

Since, $\sin(2\pi - x) = -\sin x$

$$\text{Let } \cot^{-1}\frac{15}{12} = \theta \Rightarrow \cot \theta = \frac{5}{12}$$

$$\Rightarrow \tan \theta = \frac{12}{5}$$

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2 \cdot \frac{12}{5}}{1 + \frac{144}{25}} = \frac{\frac{24}{5}}{\frac{169}{25}} = \frac{24}{169}$$

29. (d) Given, $y = m \sin rx + r \cos rx$

$$\frac{dy}{dx} = m \cdot r (\cos rx) - nr (\sin rx)$$

$$\Rightarrow \frac{d^2y}{dx^2} = mr^2 [(-\sin rx) - nr^2 (\cos rx)]$$

$$= -r^2y$$

30. (a) Given, the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = 1$$

$$\frac{dy}{dx} + y \tan x = \sec x$$

$$\text{So, I.F.} = e^{\int \tan x dx} = e^{\ln(\sec x)} = \sec x$$

31. (c) Since, $\left[\left(\frac{d^2y}{dx^2}\right)^2 - 3\right]^{\frac{1}{3}} = 2\left(\frac{dy}{dx}\right)^{\frac{1}{4}}$

$$\left[\left(\frac{d^2y}{dx^2}\right)^2 - 3\right]^4 = 2^{12}\left(\frac{dy}{dx}\right)^3$$

So, Order = 2, Degree = 8

32. (d) Let $I = 7 \int \sqrt{\frac{1}{49} - x^2} dx$

$$= 7 \int \sqrt{\left(\frac{1}{7}\right)^2 - x^2} dx$$

$$= 7 \left[\frac{x}{2} \sqrt{\frac{1}{49} - x^2} + \frac{1}{98} \sin^{-1} \frac{x}{\frac{1}{7}} \right] + c$$

$$= \frac{x}{2} \sqrt{1 - 49x^2} + \frac{1}{14} \sin^{-1}(7x) + c$$

33. (d) Since, $D_x = \sqrt{(1-1)^2 + (2-0)^2 + (3-0)^2}$

$$\Rightarrow D_x = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$D_y = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$D_z = \sqrt{1^2 + 2^2} = \sqrt{5}$$

34. (a) Given the planes

$$P_1: 2x + 4y - 2z = 10$$

$$P_2: 2x + 4y - 2z = -2$$

$$d = \frac{12}{\sqrt{2^2 + 4^2 + 2^2}} = \frac{12}{\sqrt{24}} = \sqrt{6} \text{ units}$$

35. (a) Given the vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}; \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

If angle between $\vec{a}$ and $\vec{b}$ is θ

$$\text{So, } \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

and if $\vec{a}$ is perpendicular to $\vec{b}$ then

$$\vec{a} \cdot \vec{b} = 0 \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$$

and if $\theta = \pi$, then

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n} = \vec{0}$$

So, option (1) is correct.

36. (c) Given the vectors

$$p = \hat{i} + \hat{j} - 2\hat{k} \text{ and } q = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Let } \vec{d}_1 = \vec{p} + \vec{q} = 3\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{d}_2 = \vec{p} - \vec{q} = \hat{i} - \hat{k}$$

$$\therefore \text{Area of parallelogram} = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

$$\text{Now, } \vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -3 \\ -1 & 0 & -1 \end{vmatrix} = -2\hat{i} + 6\hat{j} + 2\hat{k}$$

$$\Rightarrow |\vec{d}_1 \times \vec{d}_2| = \sqrt{(-2)^2 + 6^2 + 2^2}$$

$$\therefore \text{Area} = \frac{1}{2} \sqrt{4 + 36 + 4} = \sqrt{11} \text{ sq. units}$$

37. (b) Given,
- $\vec{a} + \vec{b} + \vec{c} = 0$

$$\text{Now, } (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\text{Since, } |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\Rightarrow 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3$$

$$\therefore (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -\frac{3}{2}$$

38. (c) Given,
- z
- has max at
- $(15, 15)$
- and
- $(0, 20)$
- .

$$\text{So, } z_{(15, 15)} = z_{(0, 20)}$$

$$\Rightarrow 15p + 15q = 20q \Rightarrow q = 3p$$

39. (a)
- $I = \int x\sqrt{x+2} dx$

$$\text{Let } x+2 = t^2 \Rightarrow dx = 2t dt$$

$$\therefore I = \int (t^2 - 2)t(2t) dt$$

$$= \int (2t^4 - 4t^2) dt = \frac{2}{5}t^5 - \frac{4}{3}t^3 + C$$

$$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$$

40. (d) Let

$$6R \quad 4R \quad 5R$$

$$4B' \quad 6B' \quad 5B$$

$$\frac{1}{U-1} \quad \frac{1}{U-2} \quad \frac{1}{U-3}$$

E : Drawn marble is red

E_1 : Drawn marble from urn I

E_2 : Drawn marble from urn II

E_3 : Drawn marble from urn III

$$\text{So, } P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

So, by conditional probability

$$P\left(\frac{E_1}{E}\right) = \frac{P(E \cap E_1)}{P(E)}$$

$$\begin{aligned} &= \frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right) + P(E_3)P\left(\frac{E}{E_3}\right)} \\ &= \frac{\frac{6}{10}}{\frac{6}{10} + \frac{4}{10} + \frac{5}{10}} = \frac{6}{15} = \frac{2}{5} \end{aligned}$$

41. (d) Given,
- $P(A) = \frac{1}{3}$
- ;
- $P(B) = \frac{1}{4}$
- ;
- $P(C) = \frac{1}{5}$

So, required probability

$$\begin{aligned} P &= \frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} \\ &= \frac{12+8+6}{60} = \frac{26}{60} = \frac{13}{30} \end{aligned}$$

42. (a) Given,
- $P(A) = \frac{1}{3}$
- ;
- $P(B) = \frac{1}{4}$
- ;
- $P(C) = \frac{1}{5}$

Required probability

$$\begin{aligned} P &= \frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} \times \frac{1}{5} + \frac{1}{3} \times \frac{3}{4} \times \frac{1}{5} \\ &= \frac{4+2+3}{60} = \frac{9}{60} = \frac{3}{20} \end{aligned}$$

43. (d) Given,
- $P(A) = \frac{1}{3}$
- ;
- $P(B) = \frac{1}{4}$
- ;
- $P(C) = \frac{1}{5}$

So, required probability

$$P = \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{2}{15}$$

44. (b) Given,
- $P(A) = \frac{1}{3}$
- ;
- $P(B) = \frac{1}{4}$
- ;
- $P(C) = \frac{1}{5}$

So, required probability

$$P = \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$$

45. (c) Given,
- $P(A) = \frac{1}{3}$
- ;
- $P(B) = \frac{1}{4}$
- ;
- $P(C) = \frac{1}{5}$

So, required probability

$$P = 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = 1 - \frac{2}{5} = \frac{3}{5}$$

46. (a) Let length and breadth of plot be
- x
- and
- y
- respectively.

$$\text{Then, } (x-50)(y+50) = xy \quad \dots(i)$$

$$\text{and, } (x-10)(y-20) = xy - 5300 \quad \dots(ii)$$

$$\text{From (i), } 50x - 50y - 2500 = 0$$

$$\Rightarrow x - y = 50$$

$$\text{From (ii), } -20x - 10y + 200 = -5300$$

$$\Rightarrow 20x + 10y = 5500$$

$$\Rightarrow 2x + y = 550$$

47. (c) Let length and breadth of plot be
- x
- and
- y
- respectively.

$$\text{Then, } (x-50)(y+50) = xy \quad \dots(i)$$

$$\text{and } (x-10)(y-20) = xy - 5300 \quad \dots(ii)$$

$$\text{From (i), } 50x - 50y - 2500 = 0$$

$$\Rightarrow x - y = 50$$

$$\text{From (ii), } -20x - 10y + 200 = -5300$$

$$\Rightarrow 20x + 10y = 5500$$

$$\Rightarrow 2x + y = 550$$

$$x - y = 50 \quad \dots(iii)$$

$$2x + y = 550 \quad \dots(iv)$$

$$\text{Equation (iii) + (iv),}$$

$$\Rightarrow 3x = 600 \Rightarrow x = 200$$

48. (d) Let length and breadth of plot be
- x
- and
- y
- respectively.

$$\text{Then, } (x-50)(y+50) = xy \quad \dots(i)$$

$$\text{and } (x-10)(y-20) = xy - 5300 \quad \dots(ii)$$

$$\text{From (i), } 50x - 50y - 2500 = 0$$

$$\Rightarrow x - y = 50$$

$$\text{From (ii), } -20x - 10y + 200 = -5300$$

$$\Rightarrow 20x + 10y = 5500$$

$$\Rightarrow 2x + y = 550$$

$$x - y = 50$$

$$2x + y = 550$$

$$\text{Equation (iii) + (iv)}$$

$$\Rightarrow 3x = 600 \Rightarrow x = 200$$

$$x - y = 50 \text{ and } x = 200$$

$$\text{then } y = 150 \text{ m}$$

49. (b) Let length and breadth of plot be x and y respectively.

$$\text{Then, } (x - 50)(y + 50) = xy \quad \dots(i)$$

$$\text{and } (x - 10)(y - 20) = xy - 5300 \quad \dots(ii)$$

$$\text{From (i), } 50x - 50y - 2500 = 0$$

$$\Rightarrow x - y = 50$$

$$\text{From (ii), } -20x - 10y + 200 = -5300$$

$$\Rightarrow 20x + 10y = 5500$$

$$\Rightarrow 2x + y = 550$$

$$x - y = 50$$

$$2x + y = 550$$

$$\text{Equation (iii) + (iv),}$$

$$\Rightarrow 3x = 600 \Rightarrow x = 200$$

$$x = 200, y = 150$$

$$\frac{x^2 + y^2}{x - y} = \frac{(200)^2 + (150)^2}{50} = \frac{62500}{50} = 1250$$

50. (a) Let length and breadth of plot be x and y respectively.

$$\text{Then, } (x - 50)(y + 50) = xy \quad \dots(i)$$

$$\text{and } (x - 10)(y - 20) = xy - 5300 \quad \dots(ii)$$

$$\text{From (i), } 50x - 50y - 2500 = 0$$

$$\Rightarrow x - y = 50$$

$$\text{From (ii), } -20x - 10y + 200 = -5300$$

$$\Rightarrow 20x + 10y = 5500$$

$$\Rightarrow 2x + y = 550$$

$$x - y = 50$$

$$2x + y = 550$$

$$\text{Equation (iii) + (iv)}$$

$$\Rightarrow 3x = 600 \Rightarrow x = 200$$

$$x = 200, y = 150$$

$$\text{Area} = x \times y$$

$$\text{Area} = 200 \times 150 = 30000 \text{ sq. m.}$$